Dynamic Behavior of Peeling-Ballooning Modes in a Shifted-Circle Tokamak Equilibrium

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ELM onset and growth linked to the coupling between ballooning and kink modes

- “Ideal-like”/ “Halo” defined in NIMROD
- ELITE benchmarks with NIMROD
- Single Linear case examined as precursor to NL studies
- Technique developed to isolate ballooning and kink drives
- Preliminary nonlinear results guide future analysis
- Summary
Detailed “ideal” study in NIMROD

Define ideal and halo in NIMROD with the intent of using the halo placement to “dial in” kink/ballooning drive:

1) kink: halo region just outside pedestal
2) ballooning: halo region far from pedestal
Halo region defined with an imposed resistivity and density transition

- $\eta$ transitions from low, “ideal” to a large value at a specified $\rho_{\text{Halo}}$
  - Tanh function used as an $\eta$ multiplier
- Density decreased by a factor of 100, transitions at a specified $\rho_{\text{dens}}$
  - Tanh function also used, sharp transition not possible

![Graph showing density and pressure transitions with Halo parameters]

- $\rho_{\text{Halo}} = 0.775$
- $\rho_{\text{dens}} = 0.7$
- $\Delta\eta_{\text{width}} = 0.002$
- $\Delta\eta_{\text{width}} = 0.02$
- $\Delta\eta_{\text{width}} = 0.04$
Quantifying “Ideal” in NIMROD requires high spatial resolution

- Start with purely ideal case
  - S = \infty everywhere
    - no halo region

- linear ideal MHD, n = 12
- no dissipation in system
- k_{\text{visc}}, k_{\text{perp}} = 0
Lundquist scans define critical, “ideal-like” value in NIMROD

- Systematically decreased $S$ from $\infty$
- Critical value defined, below which plasma behaves ideally

Ideal Plasma behavior in NIMROD

$S_{\text{crit-ideal}} \sim 5 \times 10^7$
Beyond a critical halo-resistivity the modes are not affected

- Define $S_{\text{crit-Halo}}$: increase vacuum resistivity until no effect is produced
  - Lundquist ratio not a good characterization parameter
  - Introduction of halo region doesn’t affect $S_{\text{crit-ideal}}$

\[
\begin{align*}
\eta_{\text{crit-Halo}} & \sim 1-10 \,(\Omega \cdot \text{m}) \\
S_{\text{crit-Halo}} & \sim 0.5 \\
S_{\text{crit-ideal}} / S_{\text{crit-Halo}} & \sim 10^8
\end{align*}
\]
Ballooning unstable equilibrium generated for ELITE benchmarking

- TOQ-generated series of equilibria scanning across stability boundary
- shape = simple circle
- pedestal is wide
- interface at $\Psi_N \sim 0.7$ ($P'$ and $J_\parallel = 0.0$)
- plasma vacuum interface has 0 pressure and current
- ELITE growth rates weakly sensitive to vacuum location
Results show excellent spectral agreement with ELITE

- Equilibrium generated to have little variation with vacuum placement
- Results without halo region show little variation at high-n
n = 8 mode structure in NIMROD and ELITE

- $\gamma T_{a\text{ELITE}} \sim 0.132$
- $\gamma T_{a\text{NIMROD}} = 0.132$
- $S_{\text{in}} = 10^8$, $S_{\text{out}} = 10$
- $\rho_{\text{halo}} = 0.82$, $d_{\text{vac}} = 500$
- $\rho_{\text{dens}} = 0.82$, $n_{\text{pp}(1)} = 50$
- $k_{\text{perp}} = k_{\text{visc}} = 0$

Vn contours NIMROD

ELITE contours
Nonlinear calculations of $n = 10$

**duration = 100$T_A$**

- 22 modes included: $n=0-21$, initialized with linear $n=10$ mode
- nonlinear $n=0$ & $n=20$ mode growth at twice linear $n=10$ rate expected
- The transition to nonlinear dynamics is expected when $\frac{\xi}{\Delta x} \sim O(1)$
  - For an initial velocity perturbation $V_0 \sim 1 \times 10^{-4}$ this occurs after $\sim 30T_A$

- Results show linear growth rates well into NL regime,
  - (as expected, Ping Zhu -- see APS poster)
Currently working on benchmarking the stability threshold with ELITE

- Similar equilibrium
- Lower pedestal pressure and edge current
- \( n > 10 \) converged
- \( n \ (1-10) \) appears to be slowly growing oscillating modes... in progress

![Graph showing \( \gamma_n \) vs. Toroidal mode number]

**dens6**

same halo parameters as dens8 case
New equilibrium allows the study of peeling & ballooning mode drives

TOQ-generated shifted-circle tokamak equilibrium
~S. Kruger & P. Snyder

\[ R_0 = 3m, \ a=1m \]
\[ B_0 = 2T \]
\[ \beta_{to} = .005 \]
\[ n = 1.06 \times 10^{20} (m^{-3}) \]
- no density transition

- Modified TOQ
  - currents in edge set to 0
  - minimizes numerical errors (no separatrix)
  - pedestal region
  - ~67-75cm on midplane
Equilibrium profiles show peeling-ballooning instability drive source

- Steep pressure gradients drive ballooning modes (DCON)
  - Pedestal width twice experimental value, simplify vacuum transition region

- Self-consistent edge currents & $2 < q_{\text{edge}} < 5$ to provide increased kink drive
  - comparable to ballooning drive
Halo location relative to the q rational surfaces affects instability drives

- using q profile to identify mode rational surfaces

- adjusting the halo location "dials in" kink, ballooning, & peeling-ballooning behavior
Low-n modes are sensitive to location of halo transition

- When $\rho_{\text{ped}} < \rho_{\text{Halo}} < \rho_{\text{rat}} = 8/3$
  $n = 3$ kink mode is driven
- compared to ideal spectrum: more clearly ballooning dominant
- convergence challenging
Kink & ballooning drives are adjusted within a single equilibrium

- Developed a technique where relative rates of ballooning / kink drive are changed by adjusting the location of the halo region relative to the plasma pedestal region.
  - *actual NIMROD calculations*

**No Halo: Ballooning Dominant Spectra**

**Halo: Increases Low-n Kink Drive**

![Graphs showing the relationship between mag and A.U. and n values for different halo conditions.](image-url)
Summary

- Currently developing/documenting detailed linear peeling-balloonning analysis in NIMROD
  - Defined critical Lundquist values for defining an “ideal-like” plasma and halo region in NIMROD
    - \( S_{\text{crit-ideal}} \sim 5 \times 10^7; S_{\text{crit-halo}} \sim 0.5 \)
    - Ratio of these values are greater than in experiment
  - Demonstrated a technique that varies the linear spectral properties of a single equilibrium
    - scans show extreme spectral sensitivity to halo location
      - convergence in this region is quite challenging
        - *(especially when \( \rho_{\text{Halo}} \sim \rho_{\text{qmn}} \))
    - edge ballooning & kink effects can be “diale in” by using a sharp resistivity transition region located at relevant flux positions

- Preliminary NL results show qualitatively needed resolution and expected energy growth rates for a single NL filament growth
Eigenfunctions have peeling-ballooning structure

- $n=12$ Halo-free mode structure, ballooning
- $n=3 \ \rho_{\text{vac}}=0.751$ mode structure, peeling-ballooning

![n=3 mode](image)

$n=3$ mode
$V_n$ eigenfunction

![n=12 mode](image)

$n=12$ mode
$V_n$ eigenfunction
no halo
• In addition to the linear, began preliminary NL calculations in NIMROD

• Purely a demonstration of technique
  ✦ not ideal \( S_{\text{crit}}: S_{\text{in}} \sim 5 \times 10^5 \)
  ✦ not resistivity independent halo: \( \eta_{\text{out}} \sim 10^{-2} \) (\( \Omega \cdot \text{m} \))
    - \( S_{\text{out}} \sim 5 \times 10^2 \)
  ✦ \( \rho_{\text{vac}} = 0.84 \)
  ✦ calculation grid points not packed

• Used to:
  ✦ guide future studies
  ✦ use results to design analysis tools
    - develop method to estimate transition between NL stages
    - determine growth regime to compare with analytic studies
Increasing $S$ (constant) increases problem

- For $S = 10^6$ the sawtoothing is seen up to $n = 3$ mode
- Not entirely sure if it is the exact same behavior
- Ping and Chris believe this is converged growth, I am not sure

Magnetic Energy vs. $t$
Low-n modes have oscillatory behavior

- Strange “saw-tooth-like” growth
  - occurs in low-n (stable?) modes
  - may be real physics
    - two modes (resistive & ideal) may simultaneously exist
    - Scott also saw this sawtoothing
    - perhaps nimrod bounces between two solutions
  - Moving the vacuum region out seems to eliminate the issue...
    - without a vacuum modes don’t grow
  - Modes appear to be rotating/oscillating?
Linear n=9 eigenmode used to excite NL growth

- 22 modes included: n=0-21
- initialized with linear n=9
- nonlinear beating expected to produce n=0 & n=18 mode growth at twice linear n=9 rate
Lundquist/Resistivity ratio is not a good characterization parameter

\[ \frac{\eta_{\text{out}}}{\eta_{\text{in}}} \]

\[ S_{\text{in}} / S_{\text{out}} \sim 10^8 \]

Note: the two criteria, Sin and Sout, must be simultaneously and separately satisfied

\[ \rho_{\text{ped}} = 0.75 \]
\[ \rho_{\text{vac}} = 0.84 \]