Numerical studies of nonlinear two fluid tearing modes in cylindrical RFPs

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APS-DPP
Nov. 2008

Motivation

$$\eta J \approx \langle \tilde{v} \times \tilde{B} \rangle - \frac{1}{n_0 e} \langle \tilde{J} \times \tilde{B} \rangle = \langle \tilde{v}_e \times \tilde{B} \rangle$$

mhd & Hall dynamos

- The mhd dynamo modification of the mean parallel current profile has been extensively studied
- How does the nonlinear Hall dynamo scale with the two fluid parameter, the ion sound gyroradius, ρ_s, and what is its structure?

$$\rho_s = \frac{c_s}{\omega_{ci}} = \sqrt{\frac{5}{6}\beta}\, d_i$$

- What is the fine scale reconnection structure in 3D cylindrical geometry?
- What are the resolution requirements for a $S=10^3$-10^5 study with two core modes, and what is the condition (in terms of λ_0 and ρ_s) under which an edge mode and a reversed state obtained?

Outline

- Equilibrium & two fluid parameters
- Moderate parallel current, low S results (λ_0=3.3, S= 5x10^3)
  - linear growth, eigenfunctions & dynamo vs ρ_s
  - nonlinearly saturated eigenfunctions, dynamo and profile modification vs ρ_s
- Low parallel current, moderate S results (λ_0=2.7, S = 10^5)
  - linear growth
  - nonlinearly saturated eigenfunctions, dynamo and profile modification vs ρ_s
- Fine scale reconnection structure
- Progress towards multihelicity studies & reversal
- Conclusions

The paramagnetic pinch configuration is an ohmic equilibrium

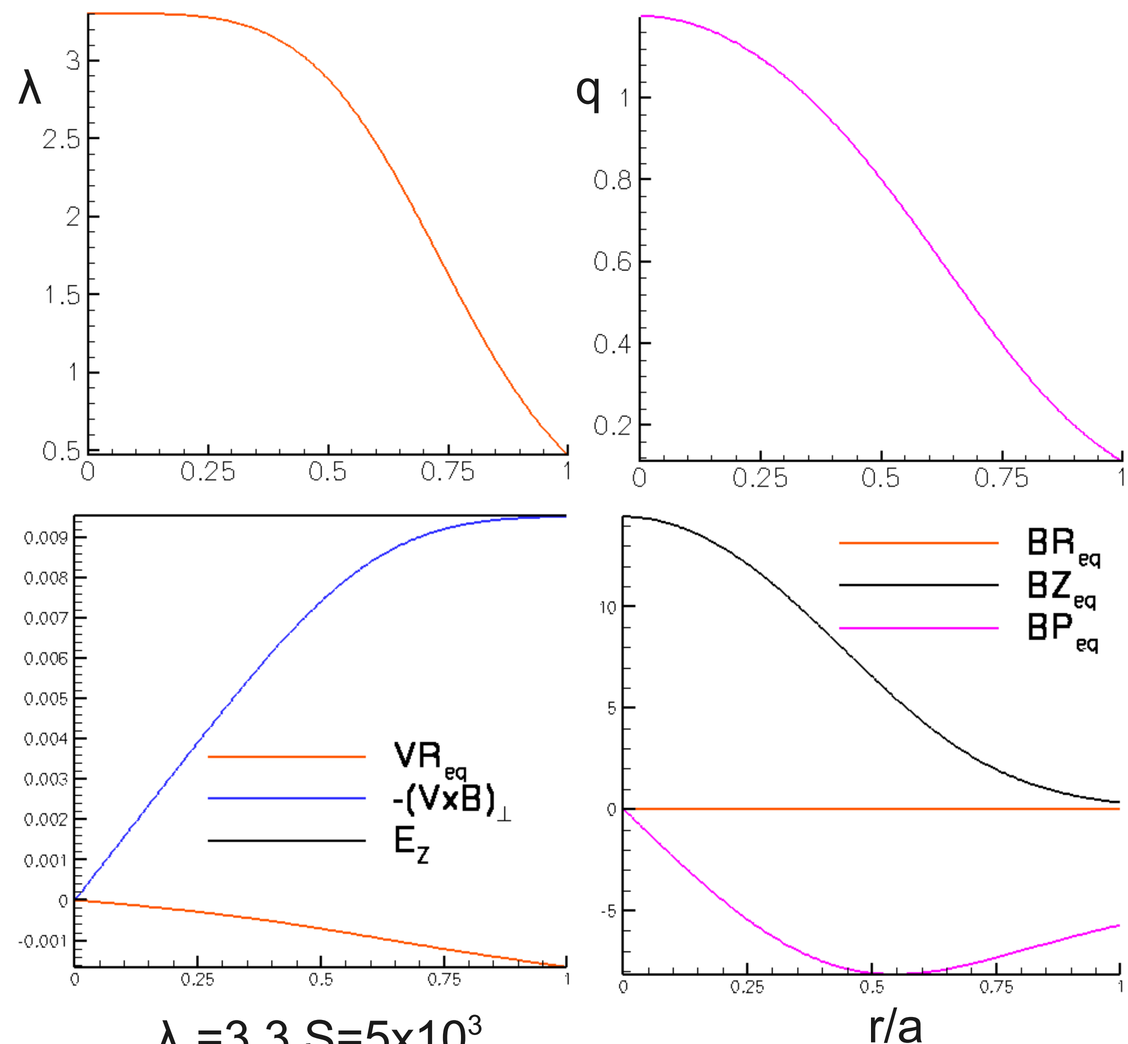
- Force free equilibrium
- Resistive diffusion is balanced by a small pinch flow, vxB electric field
- The inward density flux from the pinch flow is not evolved but is small on the time scales of interest
- This creates a self-consistent equilibrium suitable for nonlinear studies

$$\frac{dn}{dt} = -n\nabla\cdot\vec{v}$$

$$\frac{2\pi}{\tau_n} \simeq \frac{dv_r}{dr} \simeq \frac{\Delta v}{\Delta x} \simeq 0.0015$$

$$\tau_n \simeq 4200\,\tau_a$$

Simulation length ~ 8000 τ_a



λ_0=3.3 S=5x10^3

A two fluid calculation has inherent separation of scales and introduces new dimensions in parameter space

Fixing the equilibrium q_0, and the minor radius a (1 m)

Dimensionless	Physical
Resistive MHD Model	
τ_a (1 s)	B_0
	n_0
λ_0	J_0
β (0.1)	T_0
S	η
P_m (0.1)	ν
Two Fluid Model	
ρ_s/a (or d_i)	(n_0)
m_e/m_i	m_e

In a single fluid calculation, it is often chosen that B_0=1, and n_0 is adjusted so τ_a=1.

In a two fluid calculation n_0 is set to adjust the scale of the ion gyroradius (or ion skin depth) and then B_0 sets τ_a=1.

Analytical theory has produced many linear two fluid results

(1) The growth rate in the two fluid regime is predicted to be enhanced as the magnetic flux is now carried by the more mobile electrons

(2) The physics transitions from single to two fluid behavior when the ion sound gyroradius, ρ_s, is larger than the resistive layer ~ $S^{-2/5}$

(3) The Hall dynamo calculated in the small ρ_s limit is zero as a result of the phase relations between B_1 and J_1

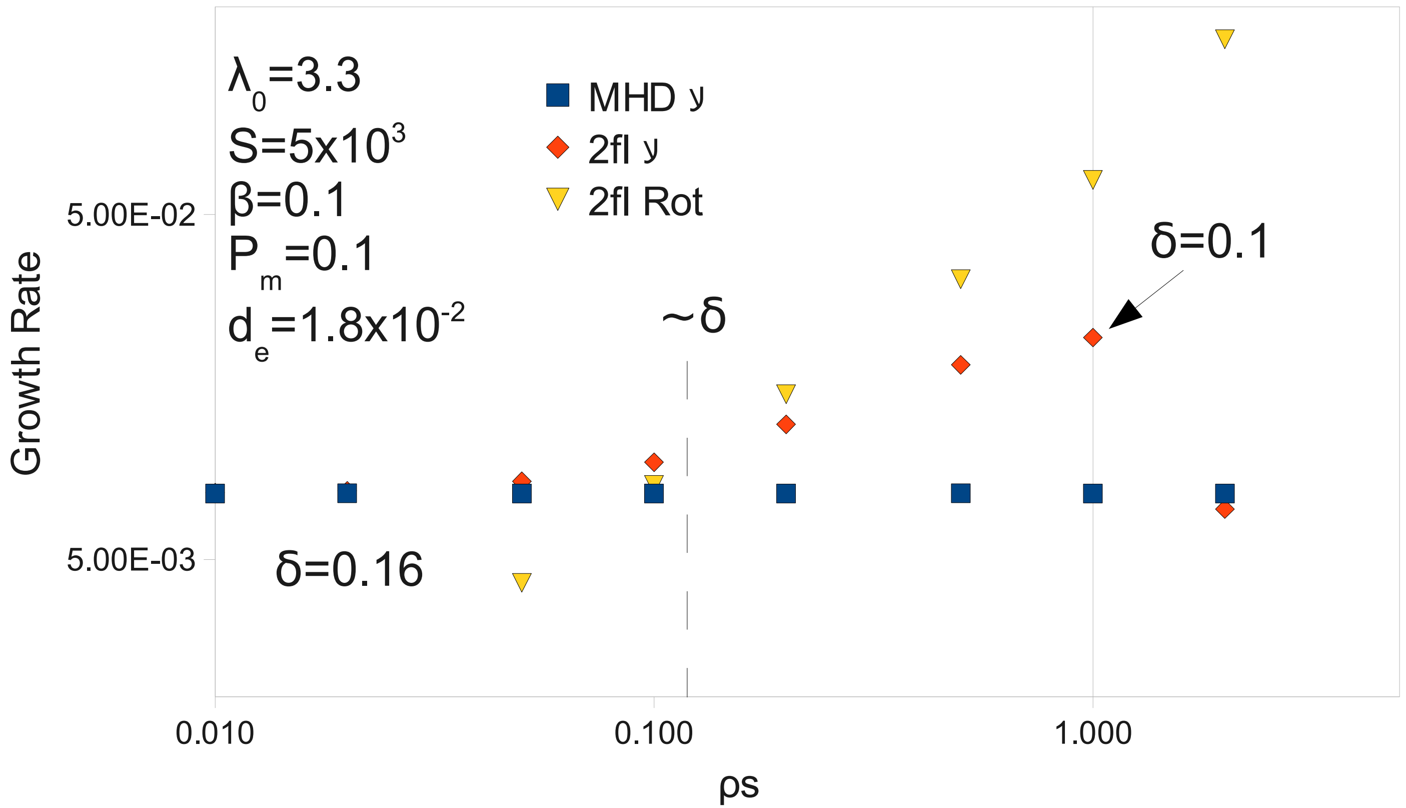
(4) At large ρ_s two fluid effect produce out of phase components of the eigenfunctions that create a Hall dynamo

(5) Cylindrical curvature, with the two fluid model, leads to complex growth rates (mode phase rotation)

References:

F. Porcelli, Phys. Rev. Let. 66, 425 (1991)
V. V. Mirnov et al, Physics of Plasmas, Vol 11, (2004)
V. V. Mirnov et al, Proc. of 21st IAEA Fusion Energy Conf, TH/P3-18, (2006)

The growth rate and mode rotation of the λ_0=3.3 case scales with ρ_s



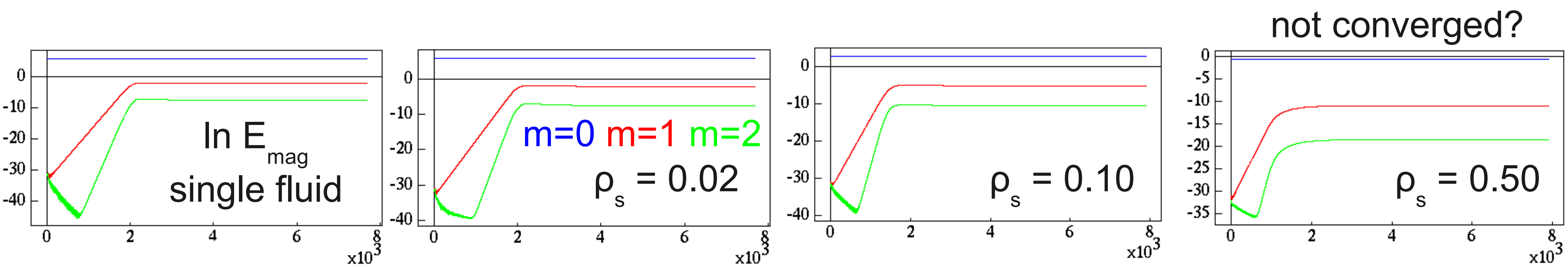
$$\delta=\sqrt{\frac{\eta}{\mu_0\gamma}+d_e^2}\simeq\sqrt{\frac{\eta}{\mu_0\gamma}}$$

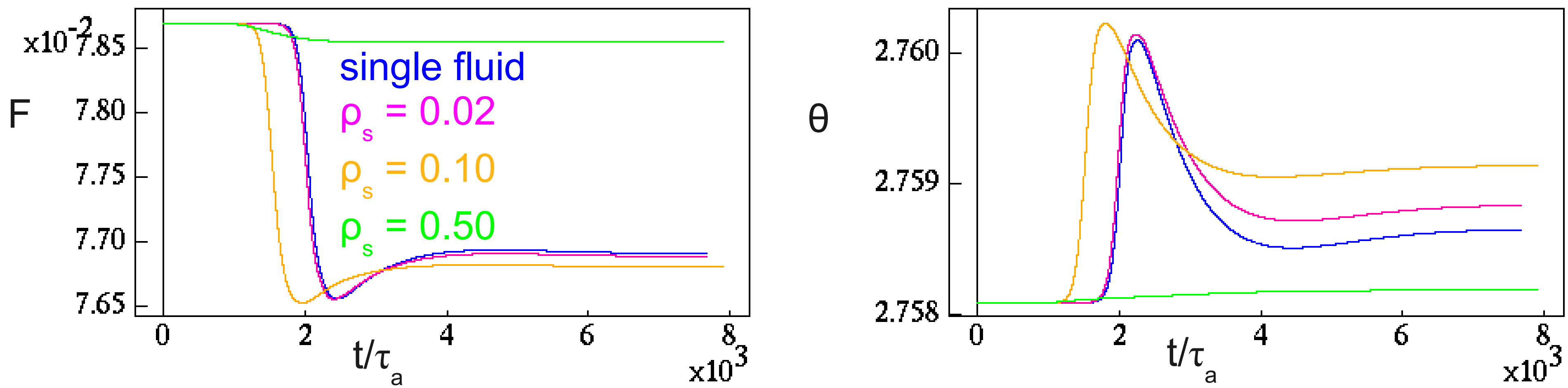
Since d_e is small compared to the resistive term in δ, these case are in a collisional regime, where resistivity is responsible for the breaking of the magnetic flux tubes at the resonant surface

Some possible plasma parameters:

B_0=0.4 T	B_0=0.4 T	B_0=0.2 T	B_0=0.2 T
n=1x10^{19} m^{-3}	n=1x10^{18} m^{-3}	n=1x10^{19} m^{-3}	n=1x10^{18} m^{-3}
β=0.1	β=0.02	β=0.2	β=0.1
a=0.51 m	a=0.51 m	a=0.51 m	a=0.51 m
ρ_s/a=5.76x10^{-2}	ρ_s/a=8.16x10^{-2}	ρ_s/a=8.16x10^{-2}	ρ_s/a=1.8x10^{-1}

The λ_0 = 3.3 large ρ_s case exhibits significantly different behavior than the single fluid limit





Mean field modification takes place long after the mode saturation

The large ρ_s (0.50) island requires much less mean field modification for saturation

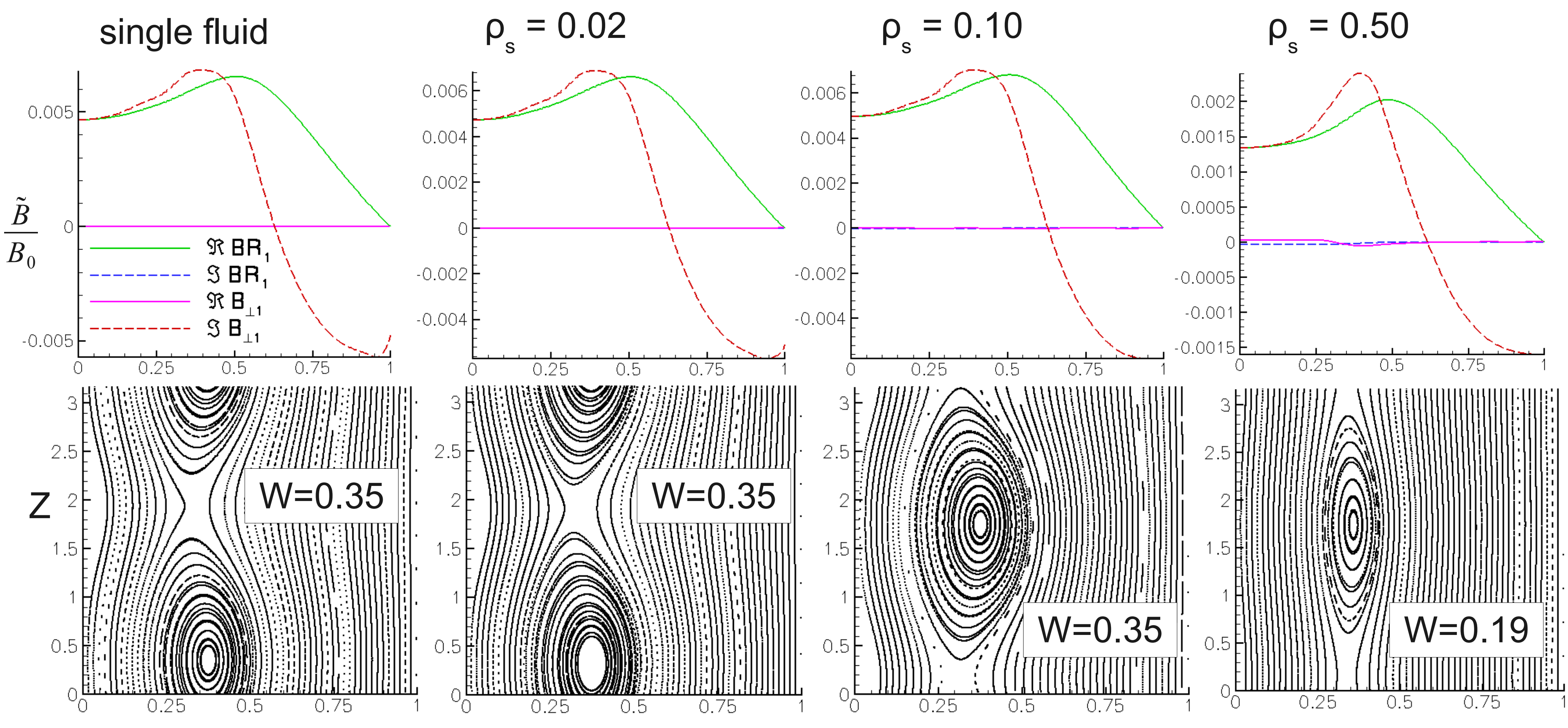
λ_0=3.3 S=5x10^3 β=0.1 r_s=0.34 Δ'=16 P_m=0.1 d_e=1.8x10^{-2}

The saturated magnetic island size is larger than the resistive layer

the saturation amplitude is significantly less at $\rho_s = 0.5$

and unlike the linear cases, the out-of-phase components are small

at r_s: Re B_{R1} ~ Im $B_{\perp 1}$ >> Im B_{R1} ~ Re $B_{\perp 1}$



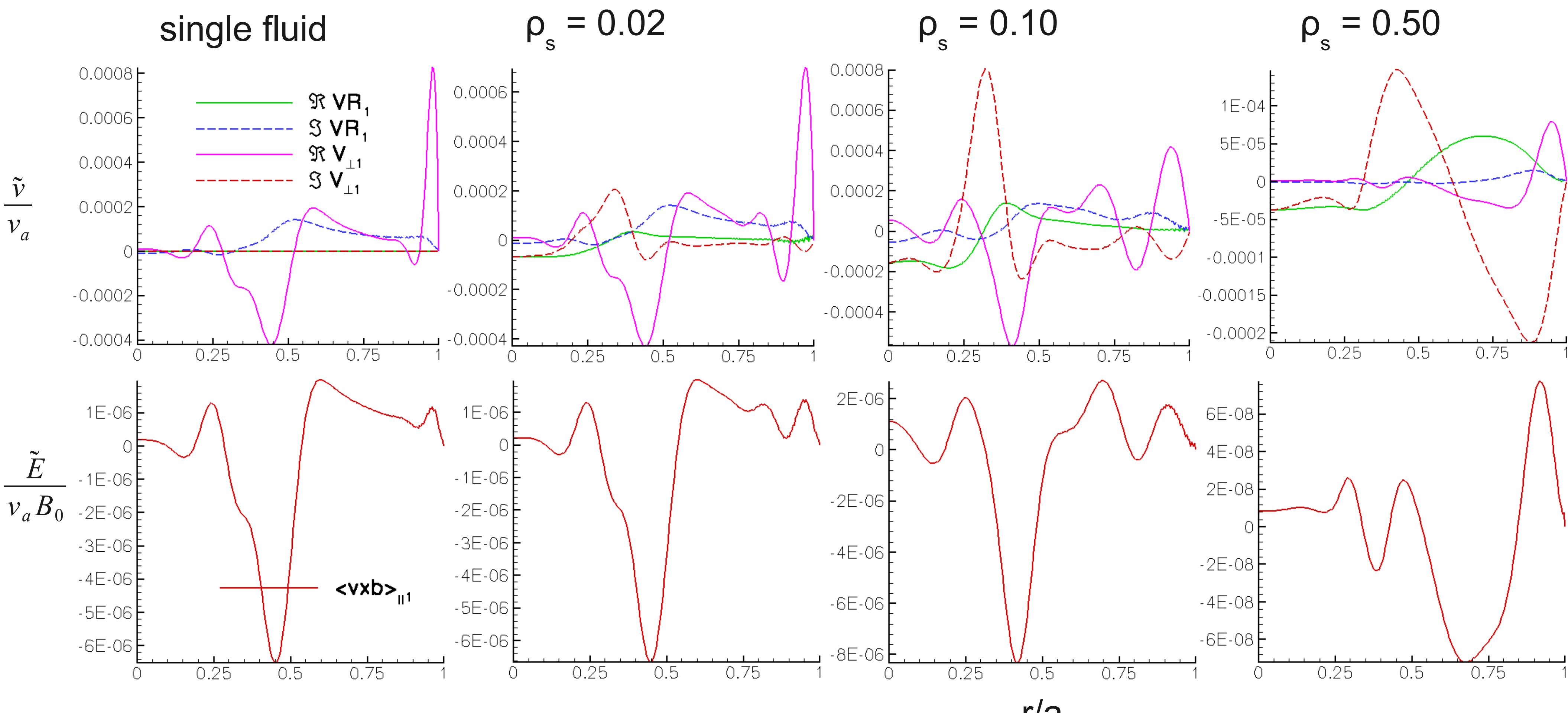
λ_0=3.3 S=5x10^3 β=0.1 r_s=0.34 Δ'=16 P_m=0.1 d_e=1.8x10^{-2}

at saturation

Again the MHD dynamo is broadened and determined by Re $V_{\perp 1}$

at r_s: Re B_{R1} ~ Im $B_{\perp 1}$ >> Im B_{R1} ~ Re $B_{\perp 1}$

$$\langle \tilde{v}\,\tilde{B} \rangle = 2\,\Re(\tilde{v})\,\Re(\tilde{B}) + 2\,\Im(\tilde{v})\,\Im(\tilde{B})$$



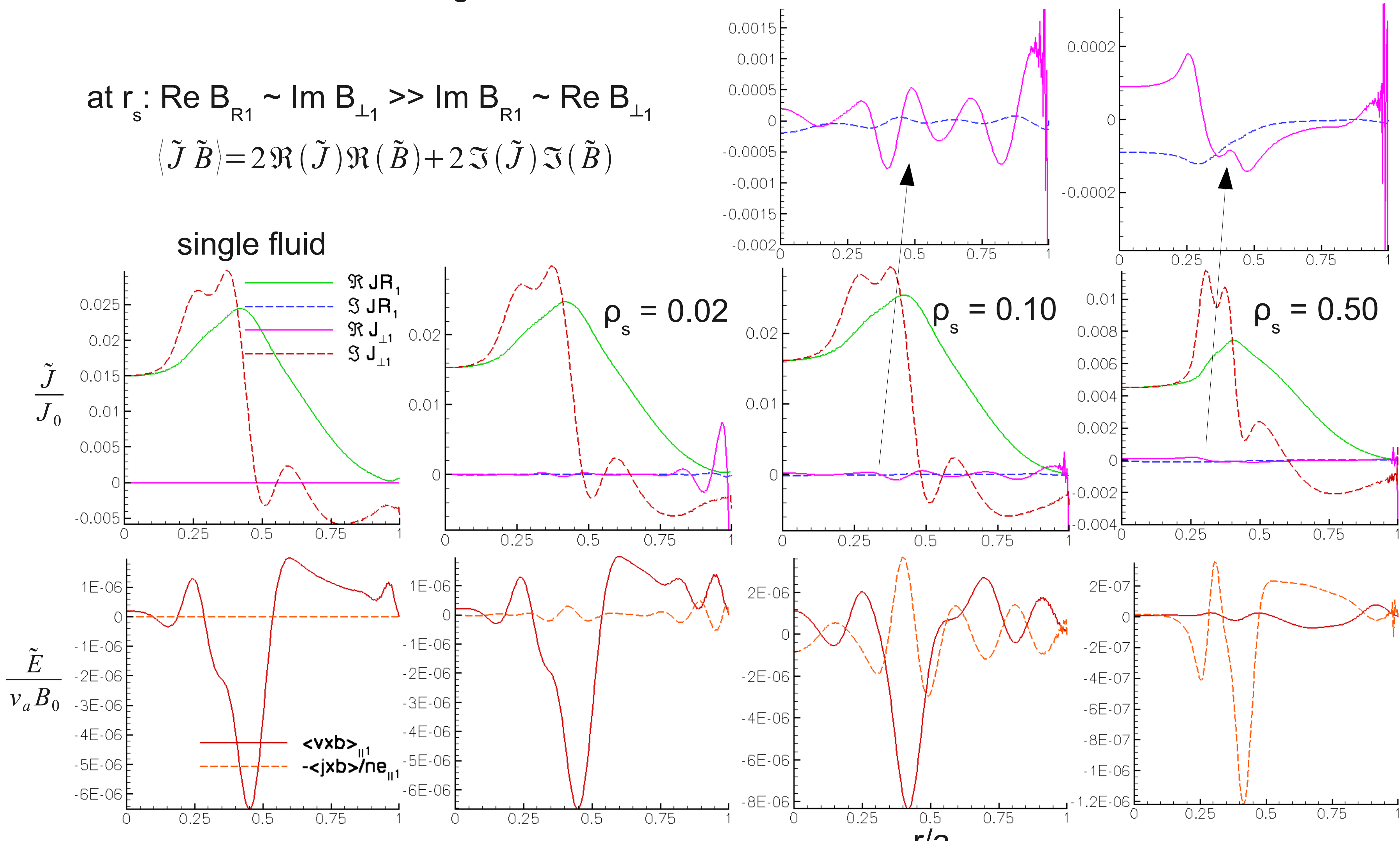
λ_0=3.3 S=5x10^3 β=0.1 r_s=0.34 Δ'=16 P_m=0.1 d_e=1.8x10^{-2}

at saturation

Like the linear result, the Hall dynamo is dominant at ρ_s=0.5, but broadened

at r_s: Re B_{R1} ~ Im $B_{\perp 1}$ >> Im B_{R1} ~ Re $B_{\perp 1}$

$$\langle \tilde{J}\,\tilde{B}\rangle = 2\Re(\tilde{J})\Re(\tilde{B}) + 2\Im(\tilde{J})\Im(\tilde{B})$$



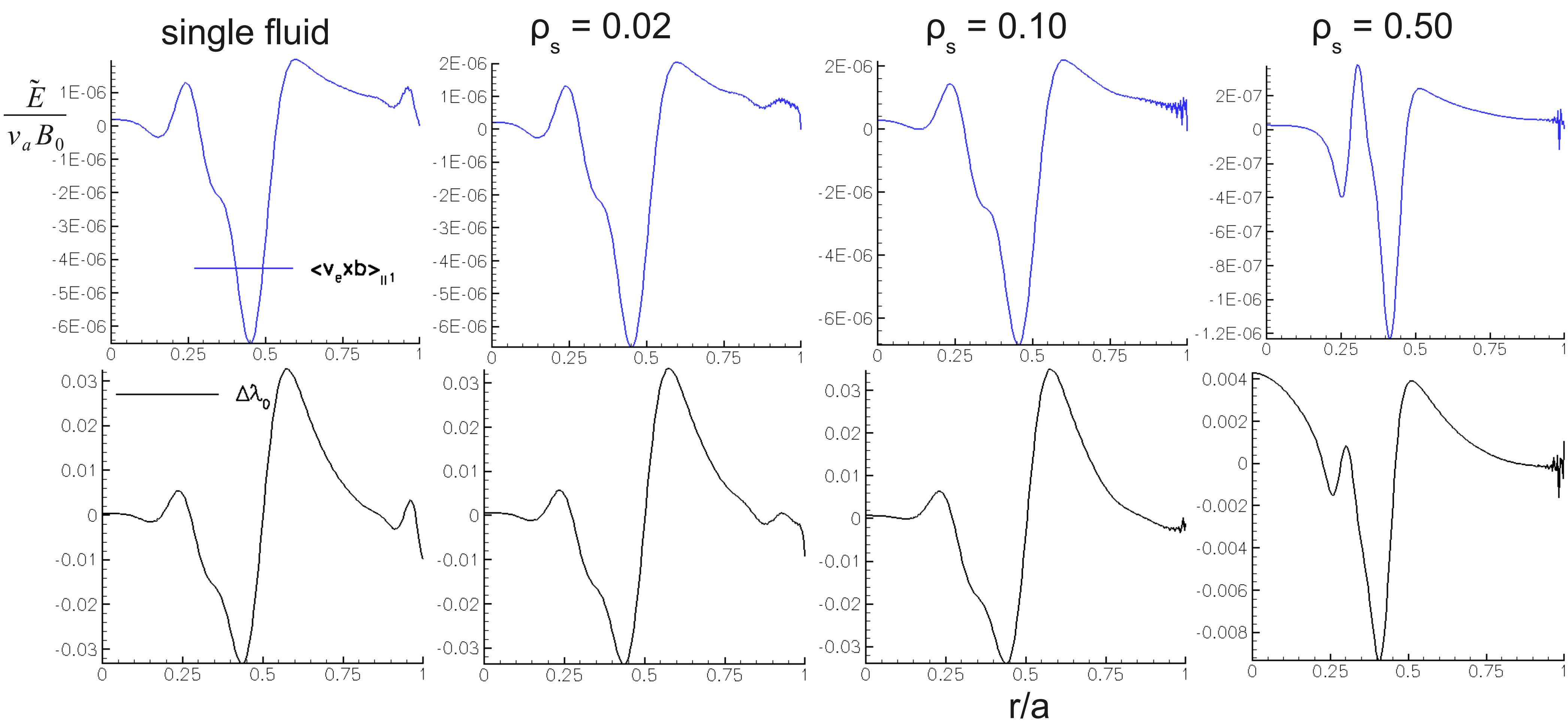
λ_0=3.3 S=5x10^3 β=0.1 r_s=0.34 Δ'=16 P_m=0.1 d_e=1.8x10^{-2}

at saturation

The large ρ_s case requires a smaller $\Delta\lambda$ to saturate

The overall shape of the parallel current modification remains the same in all cases. The saturated island reduces the parallel current inside the island, and enhances the parallel current on the outboard side of the island.

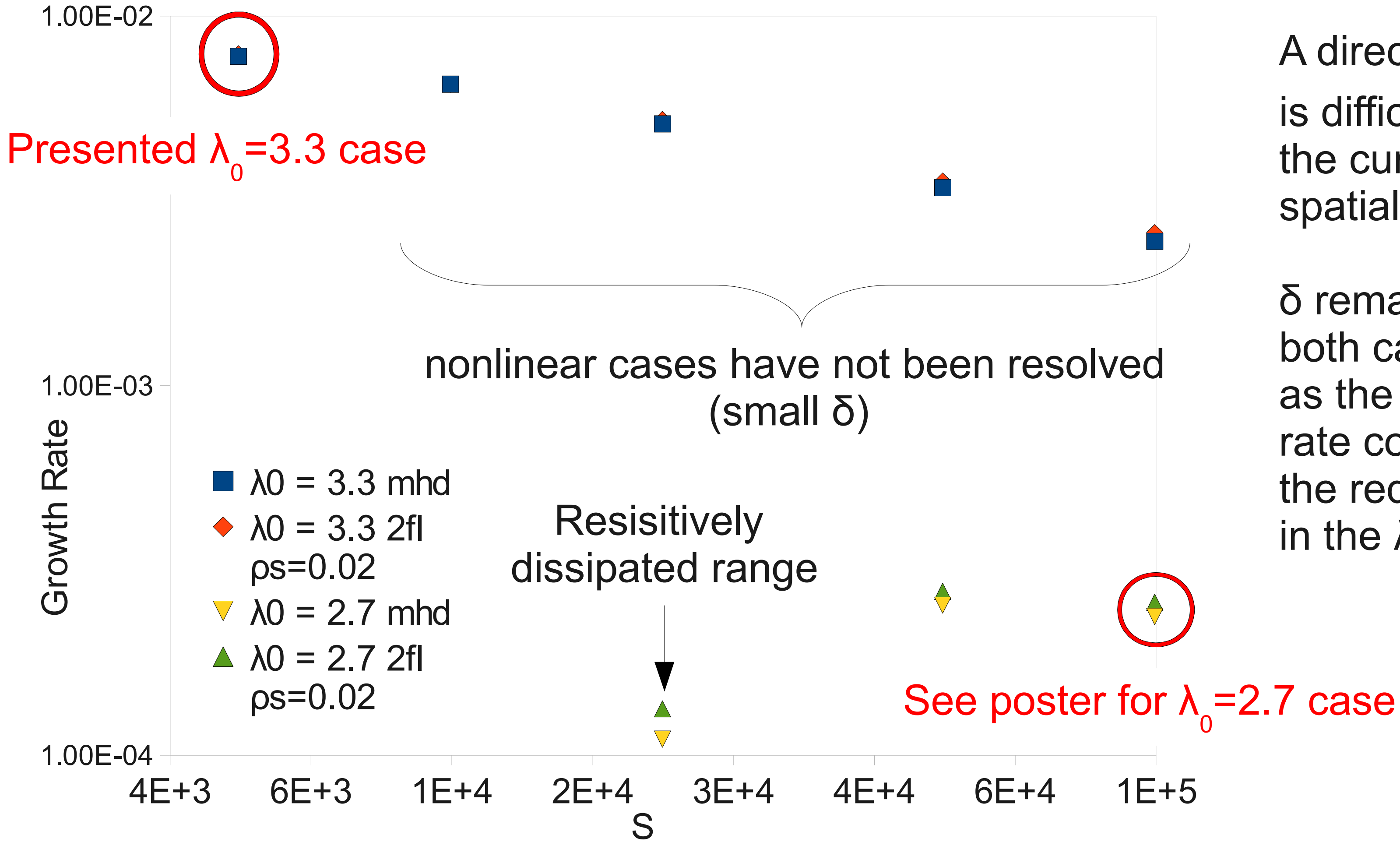
$$\eta J \approx \left\langle \tilde{v} \times \tilde{B} \right\rangle - \frac{1}{n_0 e} \left\langle \tilde{J} \times \tilde{B} \right\rangle = \left\langle \tilde{v}_e \times \tilde{B} \right\rangle$$



λ_0=3.3 S=5x10^3 β=0.1 r_s=0.34 Δ'=16 P_m=0.1 d_e=1.8x10^{-2}

at saturation

With current spatial resolution, high S cases (10^5) must be run at low λ_0 (2.7)



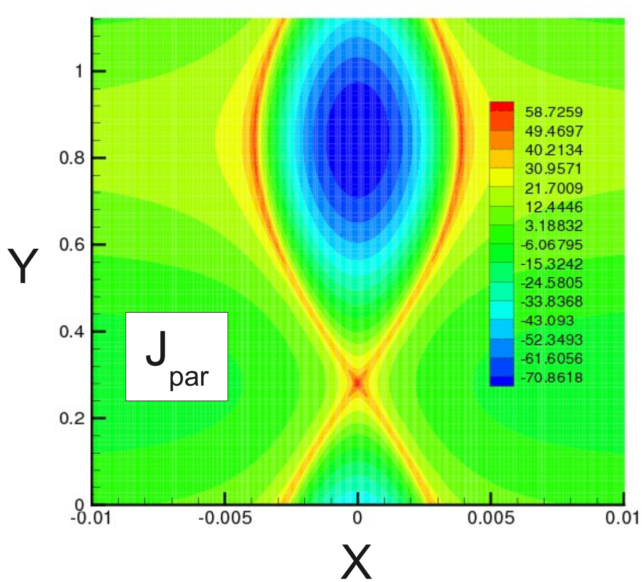
A direct scan of S or λ_0 is difficult to do with the current amount of spatial resolution

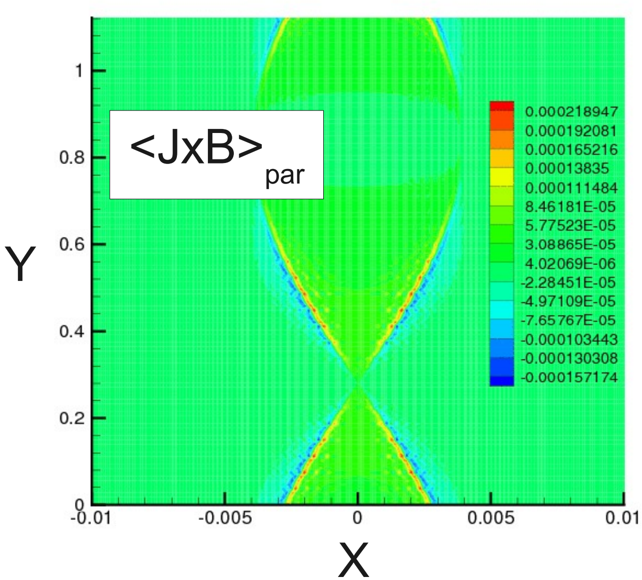
δ remains the same in both cases (~0.1-0.2) as the reduced growth rate compensates for the reduced resistivity in the λ_0=2.7 case

Current spatial resolution: 60x15 poly degree 4 finite element mesh with 3 Fourier modes (120x30 poly degree 4 mesh with 6 Fourier mode is used to check convergence)

Fine scale reconnection structure is not seen in cylindrical geometry

Slab Geometry



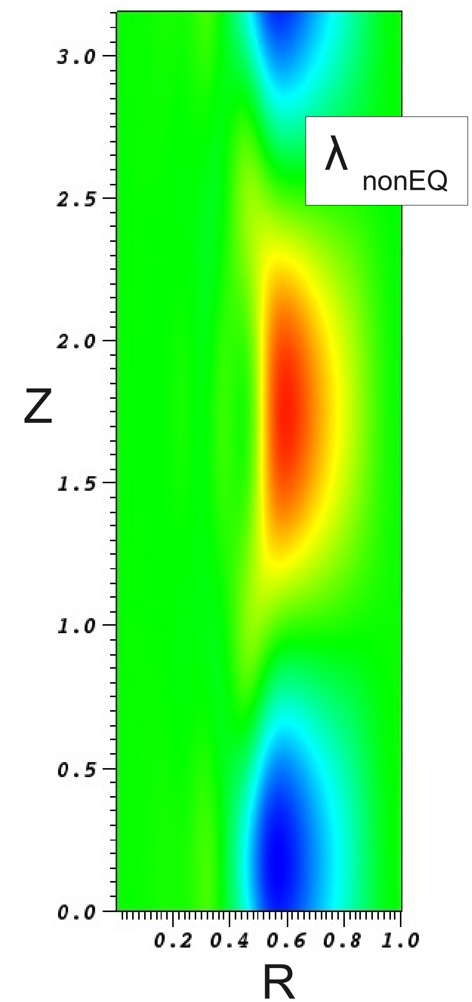


A prerequisite of the fine structure is large m,n ≠ 1 harmonics – this is not seen in the current calculations - m=2 is small.

In the slab case only the largest wavelength mode is unstable, just like the cylindrical cases.

	Slab	Cylinder
L	**0.1667 m**	**~ 1 m**
δ	**2.02×10^{-2} m**	
ρ_s	**0.12 m**	**0.1 m**
Δ'	**1.66 m^{-1}**	**~ 1-5 m^{-1}**
S	**1.33×10^7**	**1×10^5**
β	**0.1**	**0.1**
P_m	**0.1**	**0.1**
λ_0	**0.06**	**2.7**
	$\frac{d\lambda}{dr} = 0$	$\frac{d\lambda}{dr} \neq 0$

Cylindrical Geometry

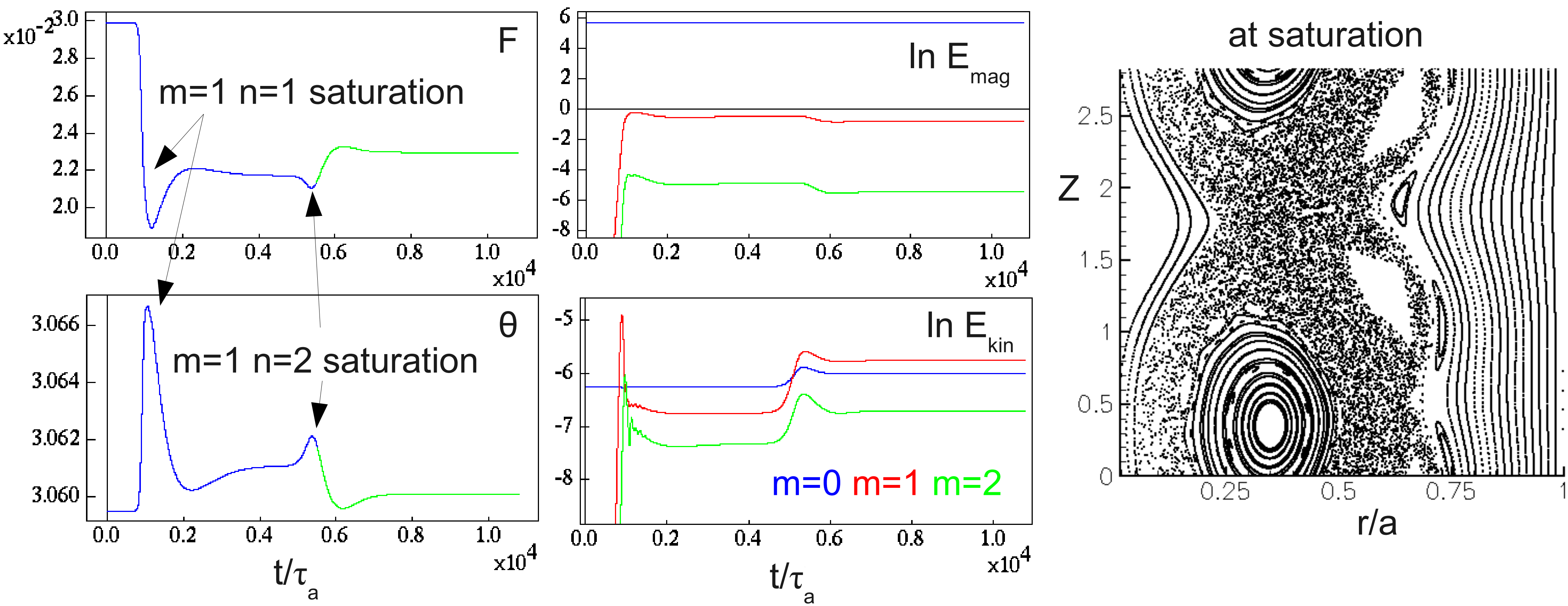


Multihelicity λ_0=3.7 cases provide insight into mode coupling

At λ_0=3.7 both the n=1 and n=2 modes are unstable.

Nonlinear two fluid cases are not resolved as of now, but work will continue in this direction in the future.

single fluid results only - $S=5x10^3$ β=0.1 P_m=0.1



Conclusions – linear and nonlinear

- Two fluid physics affects the phases of the eigenfunctions creating a Hall dynamo
- Two fluid effects and curvature cause a mode rotation
- The magnitude of the linear Hall dynamo scales with ρ_s
- At λ_0=3.3, S=5x10^3 the magnitude of the saturated Hall dynamo scales with ρ_s
- At large ρ_s (=0.5), with λ_0=3.3, S=5x10^3 the change in parallel current and other axistmmetric quantities is much smaller than the corresponding single fluid limit (convergence needs to be checked)
- At λ_0=2.7, S=10^5, the magnitude of the saturated Hall dynamo is the same regardless of ρ_s, and on the same order as the MHD dynamo
- At λ_0=2.7, the saturated fluctuation induced electric fields drive the same perturbation to the mean parallel current, causing saturation, regardless of the ion gyroradius (ρ_s 0.02 – 0.5 scanned)
- Fine scale reconnection structure is seen at λ_0=2.7-3.3 for any ρ_s

Future work

- Multihelicity cases with λ_0=3.7 or possibly 3.3