Update: Using NIMROD to Investigate 3-D Magnetic Topology in Straight Stellarator Configurations

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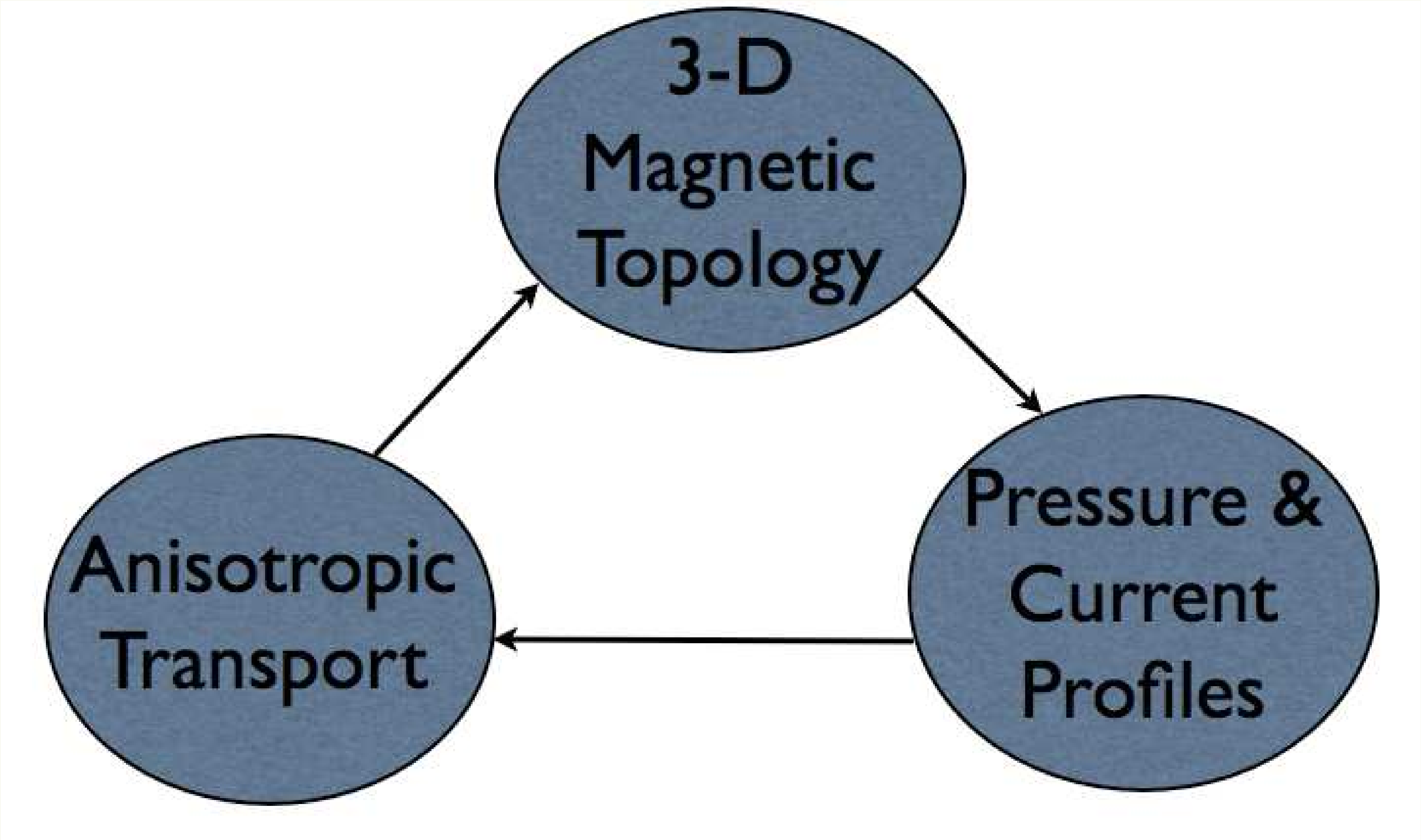
Motivation and Thesis.

- **Recent work indicates that in stellarators, stability may not limit β. Instead β may be limited by equilibrium physics.**[1] [2]
  - **Pressure-induced currents may degrade magnetic surface integrity.**
  - **"Weakly stochastic" edge magnetic fields are produced, possibly as a result of self-consistent transport physics.**
- **GOAL: Study β-limiting phenomena with a two-pronged approach:**
  - **Analytically: 3-D MHD equilibrium island width calculations, accounting for the effect of finite parallel heat conductivity.**
  - **Numerically: Use NIMROD to study 3-D magnetic field structure of stellarator configurations while varying $\frac{\kappa_{\|}}{\kappa_{\perp}}$.**

[1] M. Hirsch, et al., Plasma Phys. Control. Fusion, **50**, 1(2008)
[2] M. Sato, et al., 2008 IAEA Proceedings

This investigation considers self-consistent 3-D equilibrium with anisotropic transport effects.



NIMROD is used to investigate magnetic topology in 3-D straight stellarator geometry.

Physics to be investigated:

- **Magnetic island formation** by perturbing the helically symmetric magnetic field.
- **"Weakly stochastic" cases** by varying the magnitude of symmetry-breaking terms.
- Self-consistent **pressure-induced currents** using a resistive MHD model.
- Realistic, accurate **3-D simulations of temperature evolution** with self-consistent pressure.

The initial vacuum equilibrium magnetic field can be analytically prescribed.

Solving Laplace's equation in a periodic cylinder yields a scalar potential to describe the vacuum magnetic field for a straight stellarator:

$$\vec{B} = \nabla\phi$$

$$\phi = B_0 \left[R\zeta + \sum_m \epsilon_m \frac{Rm}{n} I_m\left(\frac{nr}{R}\right) sin(m\theta - n\zeta) \right]$$

where:
$\epsilon_m = \frac{b_m}{B_0}$ is the relative amplitude of the helical harmonic of the magnetic field.
$I_m(x)$ is the modified bessel function of order m and argument x.

Magnetic field structure and spectrum are controlled by choice of ϵ_m:

$$\iota \simeq \sum_m \frac{\epsilon_m^2}{4} \left(\frac{1}{\rho}\frac{d}{d\rho}\right)^2 I_m^2(n\rho)$$

* Helically symmetric equilibria are a special subclass of solutions
$\Longrightarrow \vec{B} = \vec{B}(\psi, M\theta - N\zeta)$

The boundary condition is a line-tied perfectly conducting shell.

- In general 3-D magnetic configurations, field lines intersect the computational boundary.
- After the analytic vacuum equilibrium is prescribed, a line-tied condition is enforced at the boundary. That is, the normal component of the magnetic field at the wall is NOT updated for t>0:

$$\frac{d}{dt}\vec{B}\cdot\hat{n}|_{bdry} = 0$$

This results in a vacuum magnetic field structure which persists in time, despite perturbations to the magnetic field.

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Straight Stellarator Parameters and Figures of Merit.

All calculations take place in straight stellarator geometry where:

a = minor radius = 0.4
B_0 = Guide field in axial direction = 1 T
T_{bckgrd} = background temperature (at plasma edge) = 1 eV
Kinematic viscosity = 0.01 m^2/s
Electrical Diffusivity $(\eta/\mu_0) = 1m^2/s$
τ_{res} = Resistive diffusion time = 0.16 s
τ_A = Alfven time = 6.4e-7 s
S = Lundquist number = 250,000
V_A = Alfven speed = 6.2e5 m/s
$\kappa_\perp$ = perpendicular thermal diffusivity = 10 m^2/s
$\kappa_\parallel$ = parallel thermal diffusivity = 1.0e8 m^2/s
P_m = magnetic Prantdl number = 0.01

Good flux surfaces are initially formed.

Example: Helically symmetric m=2,n=2 magnetic field. These surfaces are shown for t=0.

$$\vec{B} = \nabla\phi$$

$$\phi = B_0 \left[R\zeta + \sum_2 \epsilon_2 \frac{2R}{2} I_2 \left(\frac{2r}{R} \right) sin(2\theta - 2\zeta) \right]$$

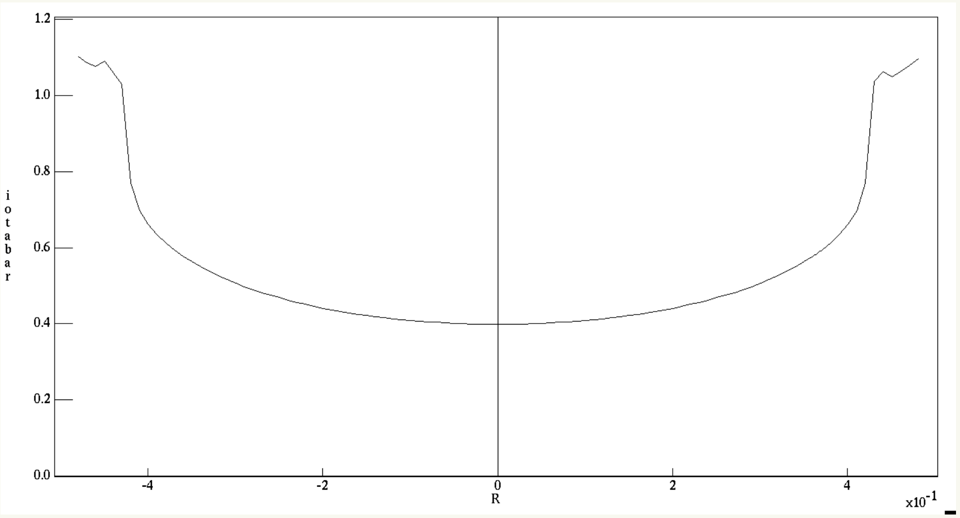

Figure 1: Rotational transform as a function of radius for m=2,n=2 at $\zeta = 0$.

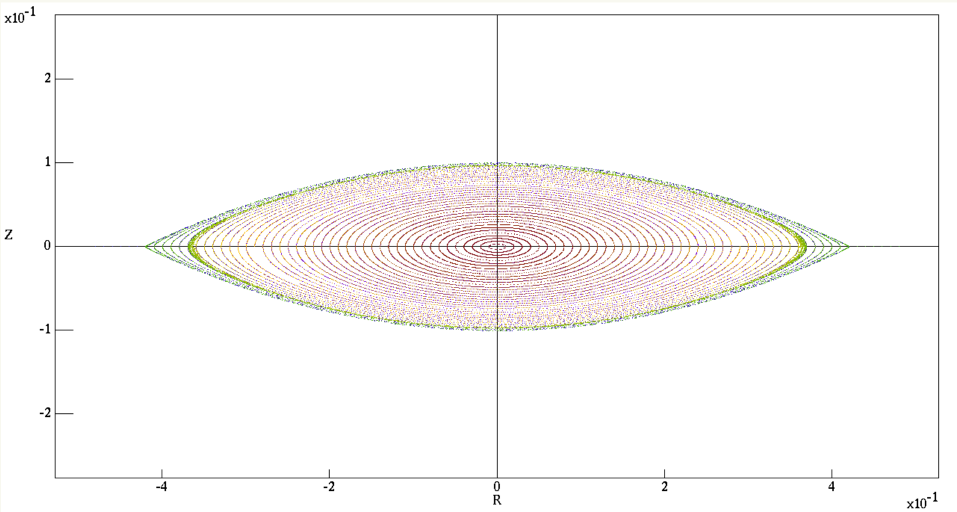

Figure 2: Poincare plot for m=2,n=2 at $\zeta = 0$.

Note: Periodic cylinder is 2π long.

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Good flux surfaces are initially formed, continued.

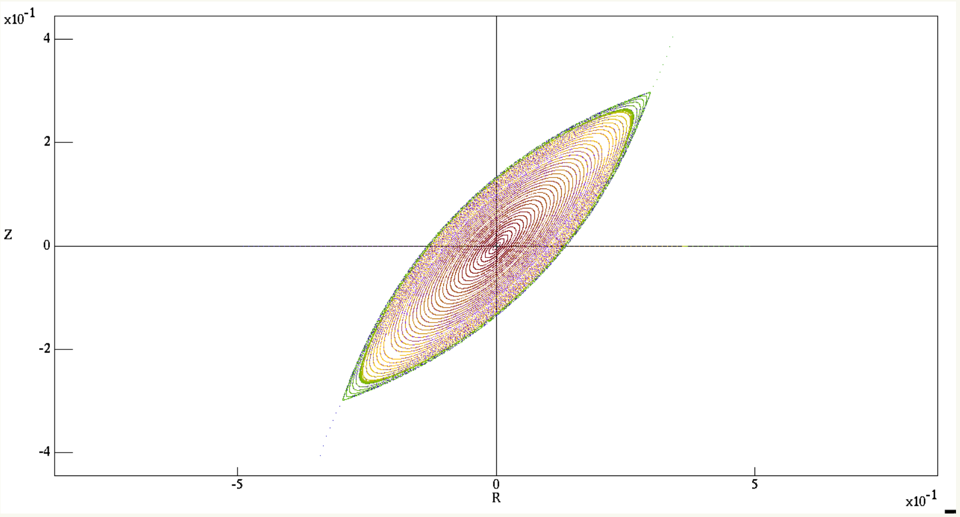

Figure 3: Poincare plot for m=2,n=2 at $\zeta = \pi/4$.

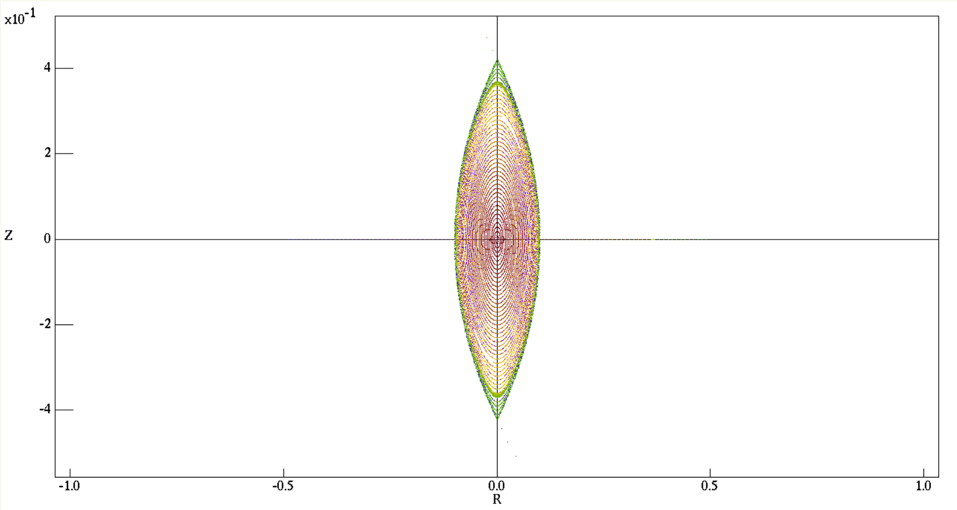

Figure 4: Poincare plot for m=2,n=2 at $\zeta = \pi/2$.

Vacuum solutions are seen to persist in initial NIMROD runs.

Example: The m=2,n=2 helically symmetric vacuum magnetic field is perturbed with a small shear Alfven wave. Lundquist number = 2.4e5.

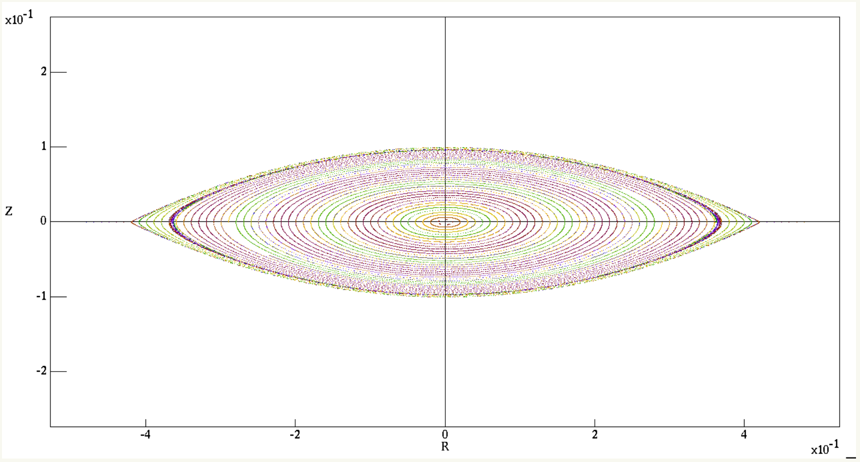

Figure 5: Poincare plot for m=2,n=2, $\zeta = 0$ at **t=0**.

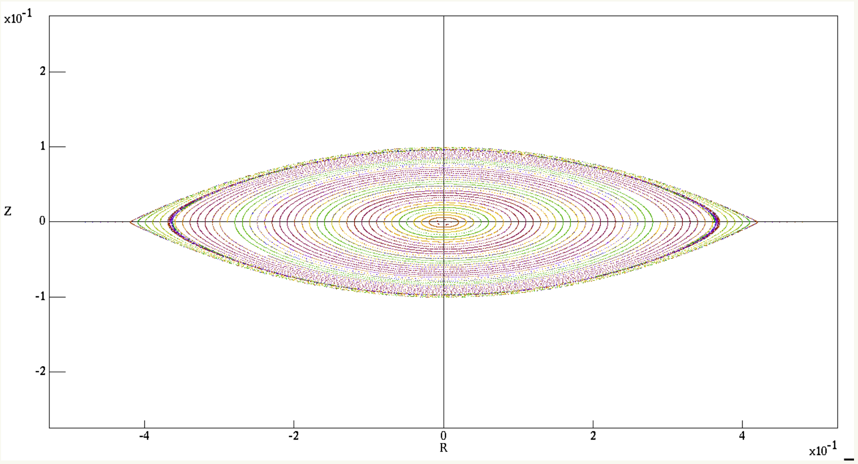

Figure 6: Poincare plot for m=2,n=2, $\zeta = 0$ at **t=6** τ_{Res} .

Vacuum solutions are seen to persist in initial NIMROD runs, continued.

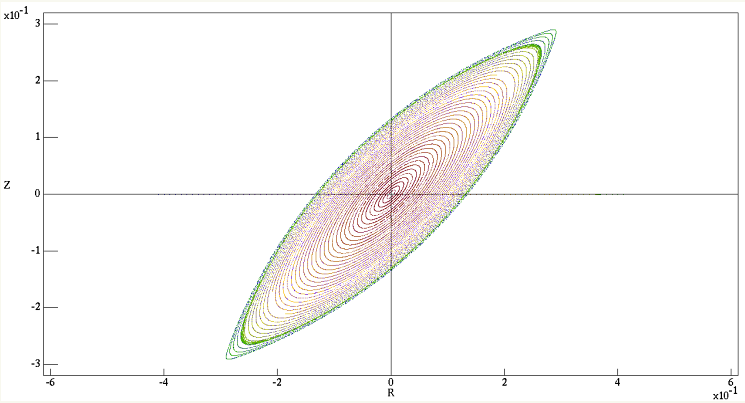

Figure 7: Poincare plot for m=2,n=2,$\zeta = \pi/4$ at **t=0**.

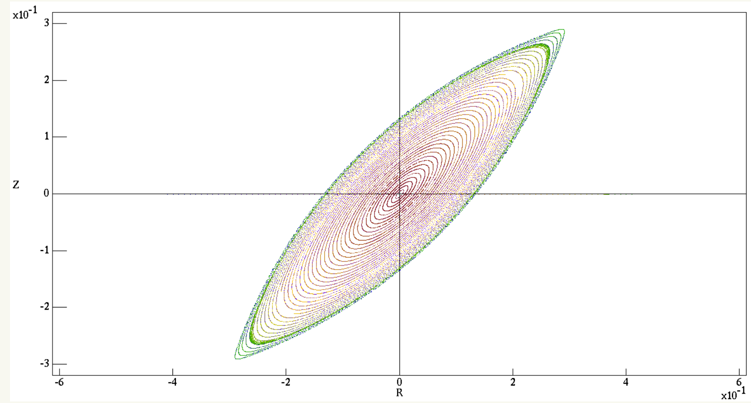

Figure 8: Poincare plot for m=2,n=2,$\zeta = \pi/4$ at **t=6** τ_{Res} .

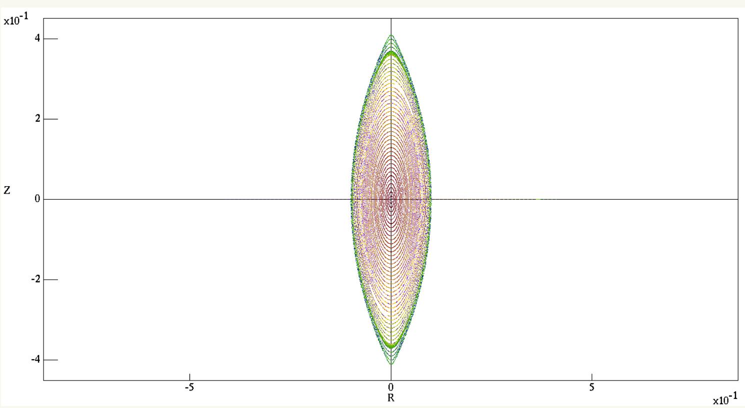

Figure 9: Poincare plot for m=2,n=2,$\zeta = \pi/2$ at **t=0**.

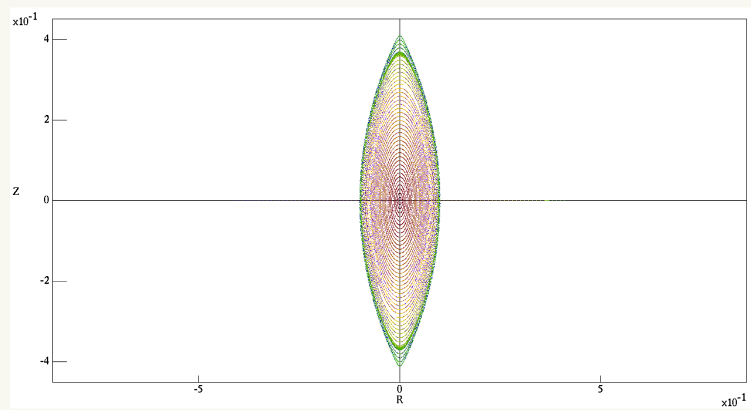

Figure 10: Poincare plot for m=2,n=2,$\zeta = \pi/2$ at **t=6** τ_{Res} .

Magnetic islands are formed by judicious addition of small harmonics.

Example: The m=2,n=2 helically symmetric vacuum magnetic field is perturbed by the addition of a small m=2,n=5 harmonic. Notice the presence of islands at $\iota = 0.5$.

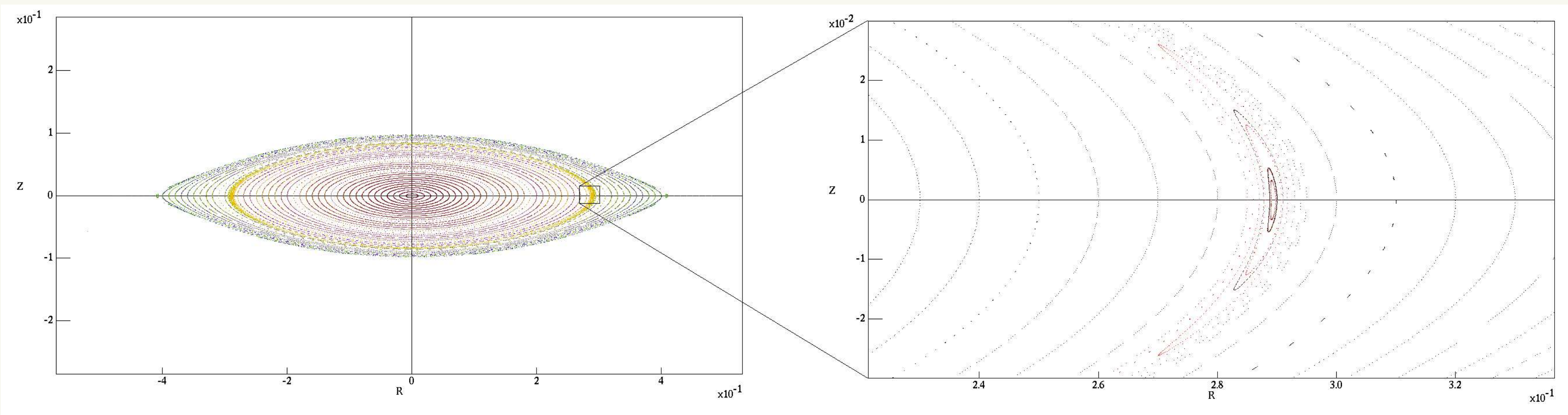

Figure 11: Poincare plot for m=2,n=2 with m=2,n=5 perturbation.

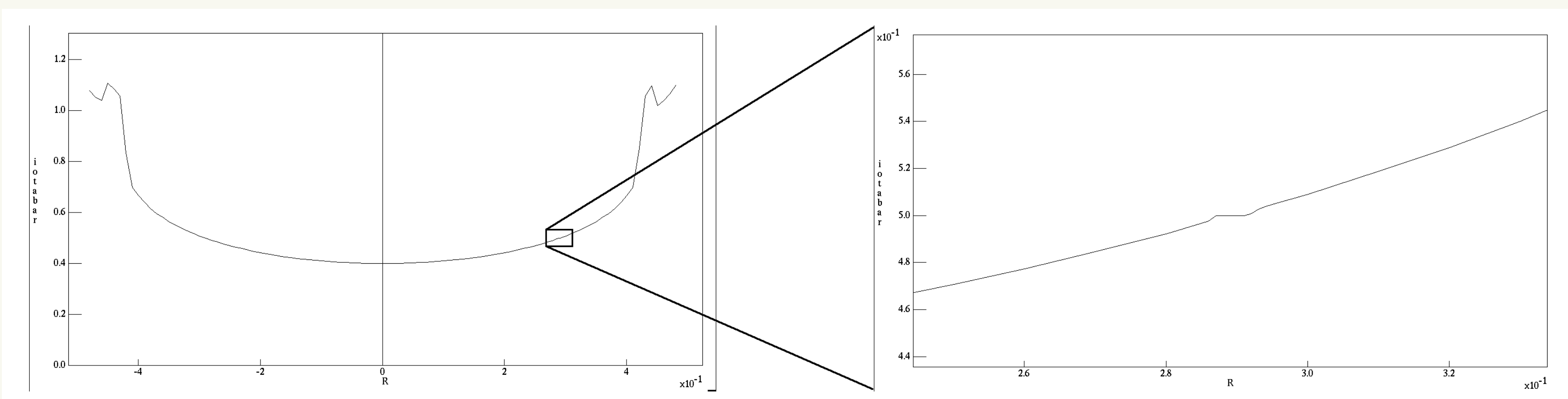

Figure 12: Rotational transform in the island region.

Preliminary heating results: Finite β modifies the magnetic field structure.

Results shown here for the m=2,n=2 helically symmetric case:

- Uniform heating concentrated in a cylinder of radius 2.5 cm, centered on the magnetic axis.
- Times shown range from t=0 to t=1.17e-4 s.

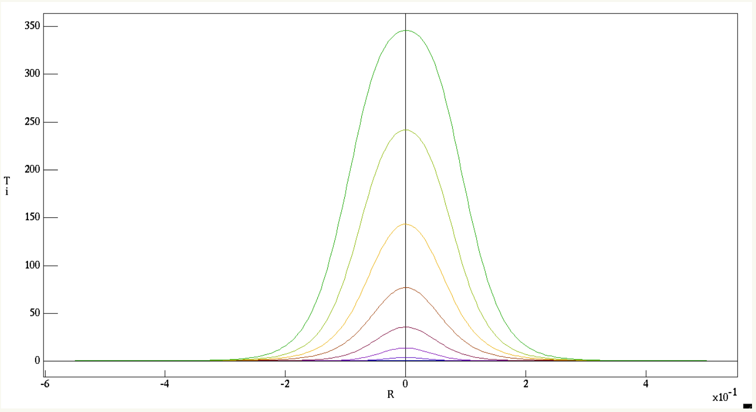

Figure 13: Temperature evolution (eV).

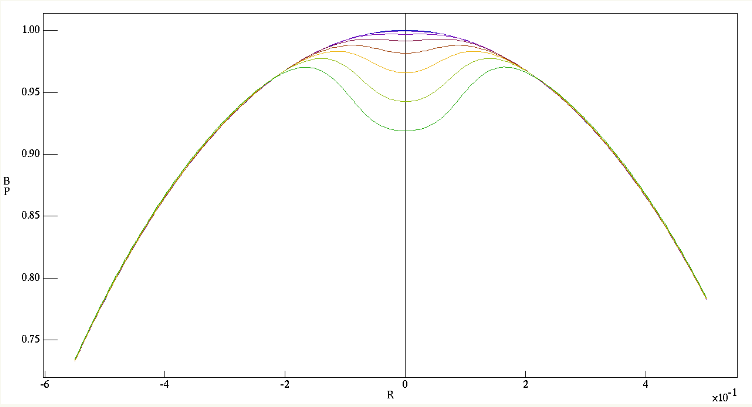

Figure 14: Evolution of ζ-directed magnetic field (T).

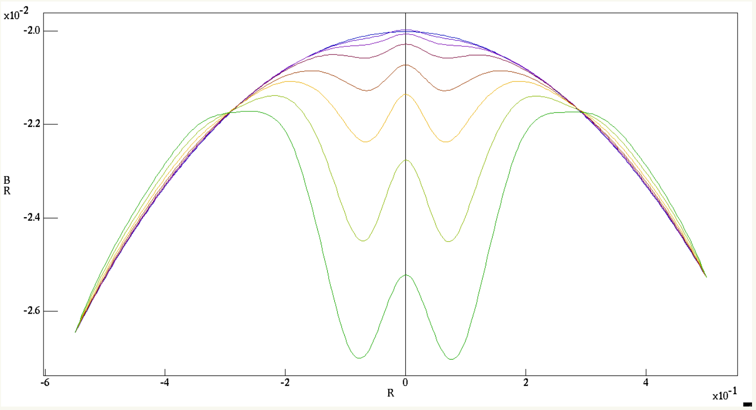

Figure 15: Evolution of R-directed magnetic field (T).

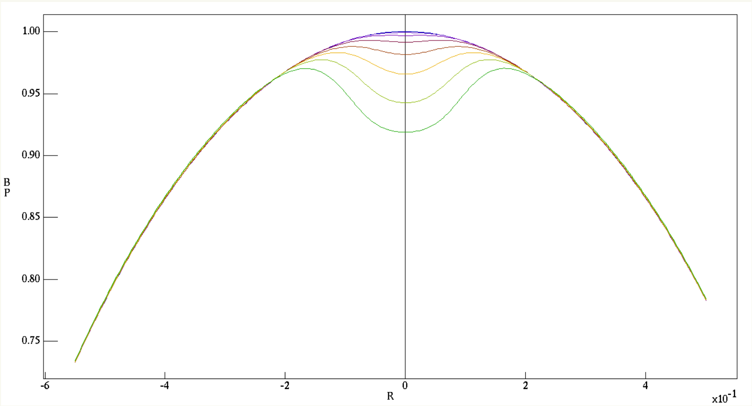

Figure 16: Evolution of Z-directed magnetic field (T).

Preliminary heating results, continued.

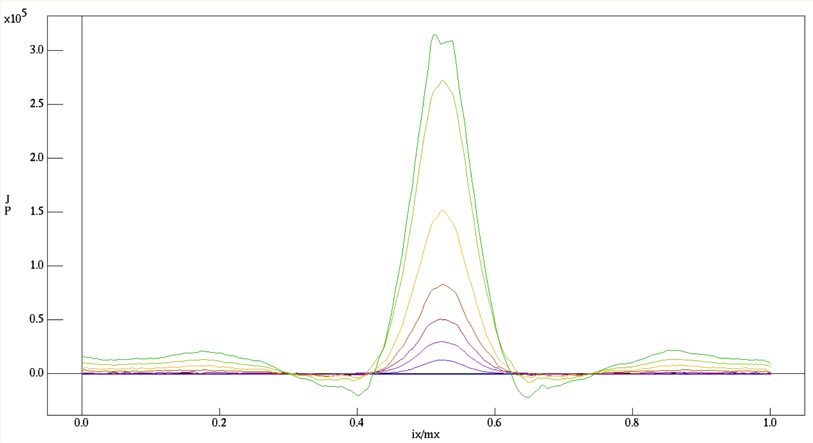

Figure 17: Evolution of ζ-directed current (A).

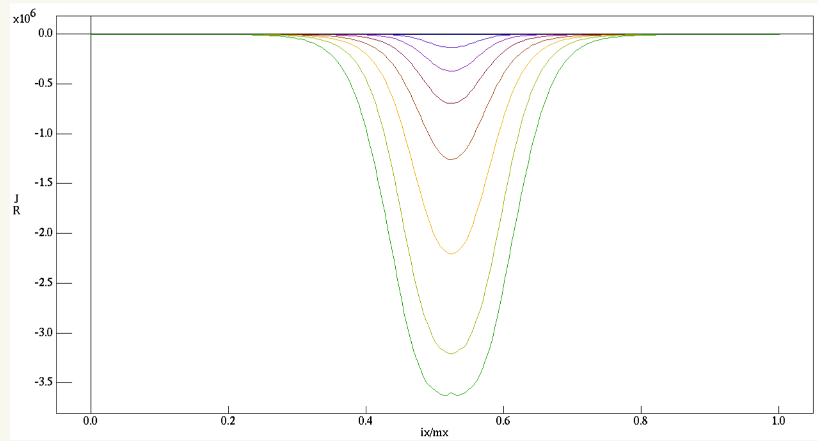

Figure 18: Evolution of R-directed current (A).

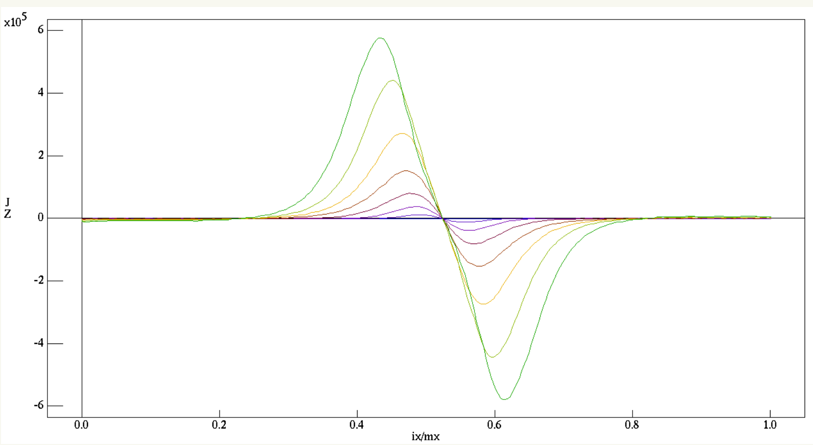

Figure 19: Evolution of Z-directed current (A).

Future work.

- Temperature evolution will be studied in two cases, helically symmetric (2-D) field and the broken symmetry (3-D) case. Physics questions to be probed:
  - Is there an instability-induced β limit? Or is there gradual magnetic surface degradation?
  - Adjust the strength of the symmetry-breaking terms to obtain differing degrees of stochasticity.
  - The ratio $\frac{\kappa_{\|}}{\kappa_{\perp}}$ will be modified and the effect on β will be observed. Is the behavior related to standard stability metrics, e.g. Mercier criterion?
  - Investigate the physics of equlibrium island width as related to differing ratios of $\frac{\kappa_{\|}}{\kappa_{\perp}}$.
- Compare contrast different heat transfer models:
  - Local diffusive model for parallel heat transfer, $\vec{q} = \bar{\bar{\chi}} : \nabla T$. NIMROD has many possibilities to investigate this effect, ranging from constant thermal conductivities to termperature-dependent conductivities.
  - Integral closure model for parallel heat transfer. [3]
- There may be the possibility to add toroidal curvature effects.

[3] E. Held, et al., Phys. Plasmas, **11**, 2419(2004)

Summary.

- A 3-D magnetic field structure in a straight stellarator configuration has been modeled using NIMROD.
  - In vacuum, good flux surfaces are formed by analytically prescribing the magnetic field structure.
  - These vacuum solutions persist in time, even when perturbed.
  - In vacuum, magnetic islands are formed by judicious addition of small harmonics.
  - When this vacuum system is heated, the magnetic field structure changes as a response to this finite pressure.