Kinetic effects of energetic particles on resistive MHD instability

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Can a Kinetic - MHD model Explain the Stabilization of the 2/1 in JET

Experimental data from the DIII-D, Asdex, JT-60U and JET experiments show only JET breaks the model of onset of the 2/1 near ideal MHD limit.

- Model: parametric $\Delta'$ near ideal limit (Brennan 2002/3) in modified Rutherford equation for a $\rho_i^*$ dependence of onset (La Haye 2008).

Fit with pole at 1.2 to $\rho_i^* \Delta' r$

(La Haye et al. N. Fusion 2008)

$$\frac{\tau_R}{r} \frac{dw}{dt} = \Delta' r + a_2 \varepsilon^{1/2} (L_q / L_p) \beta_\theta (r/w)(1 - w_m^2 / 3w^2)$$

$$\Delta' r = -(m - k) - k \alpha x [\cot(\alpha x)] , \quad x = \frac{\beta_N}{4l_i}$$

Classic theory: The linear tearing stability index
Can a Kinetic - MHD model Explain the Stabilization of the 2/1 in JET, the 2/1 is stable in JET

\[
\beta_N \quad \text{ITER}^\rho
\]

\[
\rho_{\phi_i}^* (q=2)
\]

Buttery (2007, APS)

Buttery et al (IAEA, 2008)
Puzzle: Why does the JET experiment not show instability like the others?

Likely reason: energetic particles stabilize the 2/1 mode.
- JET ($\beta_{\text{frac}}$) > 30%,
- DIII-D, JT-60U ($\beta_{\text{frac}}$) < 20%

OTHER Possible Causes?

- Accurate $\Delta'$ calculation (Brennan 2002/3/6).
- Accurate equilibrium.
- Other physics, two-fluid effects …?
Recent Results Show Energetic Particle/MHD Coupling Important and Computationally Viable

Historical focus has been on the simplified effects on the 1/1 mode.

Recent Computational Efforts Successful

- Choi, Turnbull, Chan (GA) Show highly accurate prediction of the sawtooth crash in DIII-D (2007).

Our resistive MHD analyses suggest possible energetic particle stabilization of resistive 2/1 modes at high energetic particle beta fractions.
In the Hybrid-Kinetic Approach, Initial value MHD computations are coupled to a $\delta f$ model

In the limit $n_h << n_0$ and $\beta_h \sim \beta_0$ quasi-neutrality, the only modification of the MHD equations is addition of a energetic particle tensor in the momentum equation

$$\rho \frac{dV}{dt} = \rho \left( \frac{\partial V}{\partial t} + V \cdot \nabla V \right) = J \times B - \nabla \cdot p_b - \nabla \cdot p_h$$

where $p_h = p_{h0} + \delta p_h = \begin{pmatrix} p_\perp & 0 & 0 \\ 0 & p_\perp & 0 \\ 0 & 0 & p_\parallel \end{pmatrix} = \int m(v - v_h)^2 \delta f(x,v)dv$.

Is computed from a code advancing the change in the distribution function $\delta f$.
Steady state fields satisfy a scalar pressure force balance

\[ \mathbf{J}_0 \times \mathbf{B}_0 = \nabla p_0 + \nabla p_{h0}, \]

where the assumption is that the equilibrium anisotropic energetic pressure component is 0 and the tensorial \( \mathbf{p}_{h0} \) reduces to a scalar \( p_{h0} \).

The steady state fields satisfy a scalar pressure force balance, which limits the form of the equilibrium energetic particle distribution to isotropic distribution.
The $\delta f$ PIC model

- PIC is a Lagrangian simulation of phase space $f(x, v)$
- PIC evolves the $f(x(t), v(t))$
- $\delta f$ PIC reduces the discrete particle noise associated with conventional PIC
- Vlasov equation
  \[
  \frac{\partial f(z)}{\partial t} + \dot{z} \cdot \frac{\partial f(\dot{z})}{\partial t} = 0
  \]
- Evolution equation for $\delta f$, $\dot{\delta f} = -\delta z \cdot \frac{\partial f_0}{\partial t}$.
- the drift kinetic equations of motion are used as the particle characteristics

\[
\dot{x} = v_{\parallel} \hat{b} + \frac{E \times B}{B^2} + \frac{m^2}{eB^4} (v_{\parallel}^2 + \frac{v_{\perp}^2}{2})(B \times \nabla \frac{B^2}{2}) - \frac{\mu_0 m v_{\parallel}^2}{eB^2} \mathbf{J}_\perp,
\]

\[
m v_{\parallel} = -\hat{b} \cdot (\mu \nabla B - eE).
\]
The slowing down distribution function for energetic particles

The slowing down distribution function

\[ f = \frac{P_0 \exp(\frac{P_\xi}{\psi_n})}{\varepsilon^{3/2} + \varepsilon_c^{3/2}}, \quad P_\xi \propto \psi, \quad \psi_n = C\psi_0 \]

The linearized evolution equation for \( \delta f \) becomes

\[ \delta \dot{f} = f_0 \left\{ \frac{mg}{e\psi_n B^3} [(v_\parallel^2 + \frac{v_\perp^2}{2})] \delta \mathbf{B} \cdot \nabla \mathbf{B} - \mu_0 v_\parallel \mathbf{J} \cdot \delta \mathbf{E} \right\} \]

\[ + \frac{\delta \mathbf{v} \cdot (\nabla \psi P - \rho_\parallel \nabla g)}{\psi_n} + \frac{3}{2} \frac{e\varepsilon^{1/2}}{\varepsilon^{3/2} + \varepsilon_c^{3/2}} \mathbf{v}_D \cdot \delta \mathbf{E} \}, \]

where

\[ \mathbf{v}_D = \frac{mg}{eB^3} (v_\parallel^2 + \frac{v_\perp^2}{2})(\mathbf{B} \times \nabla \mathbf{B}) + \frac{\mu_0 m v_\parallel^2}{eB^2} \mathbf{J} \parallel, \]

\[ \delta \mathbf{v} = \frac{\delta \mathbf{E} \times \mathbf{B}}{B^2} + v_\parallel \frac{\delta \mathbf{B}}{B} \cdot \delta \mathbf{E}. \]
Equilibrium pressure and safety factor profiles as a function of $\psi$ in the D shape


$Pr$ (the ratio of the viscosity to electric diffusivity) = 100

$f \sim \exp(\psi / C)$

$q_{\text{min}} \approx 1.5, \quad q_{95} \approx 4.4$
Anisotropic pressure of energetic particles produces real frequency, the 2/1 mode rotates

- $\beta_{frac} = 12.5\%$
- $S = 2.7 \times 10^6$
- $\frac{\beta N}{4 l_i} = 0.9$
- $\gamma \tau_A = 4.0 \times 10^{-4}$
- $\omega \tau_A = 0.8 \times 10^{-4}$

2/1 modes with real frequencies observed

- Similar to ideal 1/1 mode (Kim 04, 08)

- $\beta_{frac} = 12.5\%$
- $\frac{\beta N}{4 l_i} = 0.41$
- $\gamma \tau_A = 9.5 \times 10^{-3}$
- $\omega \tau_A = 1.7 \times 10^{-3}$
The eigen function of $V_r$, the $n=1$ spatial projection of $\delta f$ in phase space, Trapped cone

Trapped particle region of phase space

- Geometric effects: “stripes”
Linear Growth rates (of the resistive 2/1 mode) as a function of S for MHD only cases, $\text{Exp}(-4\psi)$
Contours show linear Growth rates (of the resistive 2/1 mode) as a function of S for MHD only cases, \( \text{Exp}(-4\psi) \)

\[ p \propto \exp(-4\psi) \]
Growth rates and real frequencies (of the resistive 2/1 mode) as a function of $S$ (linear cases) with energetic particles.

$$p \propto \exp(-4\psi), \quad \frac{\beta_N}{4l_i} = 0.83$$

$\bigcirc$ : marginal cases

Poloidal FFT
Growth rates (of the resistive 2/1 mode) as a function of $S$ (linear cases), MHD only, $\exp(-4\psi)$


$S$ (the ratio of the resistive time to Alfvén time), $Pr$ (the ratio of the viscosity to electric diffusivity) = 100
Growth rates and real frequencies (of the resistive 2/1 mode) as a function of $S$ (linear cases)

$p \propto \exp(-4\psi)$

$\beta/4l_i = 0.75$

$\beta/4l_i = 0.82$

$\beta/4l_i = 0.90$

$\bigcirc$ : marginal cases
Growth rates for series of equilibria ($\beta_N / 4l_i$)

(stability diagram sketch)
Conclusion and Discussion

Coupled Energetic Particles and resistive MHD

• Linear resistive MHD analyses suggest energetic particle stabilization of 2/1 modes at high energetic particle fractions. -->no onset 2/1 mode, JET
  • The growth rate as a function of S are damped for higher particle fractions, accompanied by an increasing real frequency.
  • The growth rates significantly reduce with $\beta$ due to mode resonance of the trapped particles and “barely passing” particles. (Similar to Kim08.)
  • An energetic particle effect driven in the bulk of the plasma, not a direct effect in the tearing layer. Thus, strongly affects the resistive mode.
  • Near the ideal limit, still, damping effects (2/1 ideal mode has damping effects?).
    • $\exp(-6\psi), \exp(-2\psi)$ cases (different pressure peaking), Nonlinear calculations.
  • Analytic, pseudo-analytic analysis.
Anisotropic pressure of energetic particles produces real frequency, the ideal 1/1 kink rotates

The initial effect is damping but not at larger fractions.

Converts to “Fishbone” modes: 1/1 modes with real frequencies observed in beam heated experiments