Continuum solution to Chapman-Enskog-like drift kinetic equation (CEL-DKE) in NIMROD

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Motivation for continuum solution to closure problem.

- Treat time-dependent problems such as Landau damping or coupling of closures to rapidly evolving instabilities.
- Easily incorporate nonlinearities and particle trapping effects.
- Increase the efficiency of the closure calculation.
- Incorporate accelerations effects.

- Solve lowest-order Chapman-Enskog-like drift kinetic equation:

$$\frac{\partial F}{\partial t} + \vec{v}_{\|} \cdot \vec{\nabla} F + \left(\frac{q}{m} \frac{\vec{v}_{\|}}{v} \cdot \vec{E} \frac{\partial}{\partial v} - \frac{\mu}{B} \frac{\partial B}{\partial t} \frac{\partial}{\partial \mu} \right) F - C(F + f_M) =$$

$$-\frac{2}{3} L_1^{1/2} \left(\frac{d \ln T}{dt} + \vec{\nabla} \cdot \vec{V}_1 \right) f_M + L_1^{3/2} \vec{v}_{\|} \cdot \nabla \ln T + \vec{v}_{\|} \cdot (\vec{\nabla} \cdot \boldsymbol{\Pi} - \vec{R}) f_M$$

$$+2 s^2 P_2\left(\frac{v_{\|}}{v} \right) \left(\boldsymbol{b}\,\boldsymbol{b} - \frac{\boldsymbol{I}}{3} \right) : \vec{\nabla} \vec{V}_1 f_M$$

Advantages of continuum approach.

- Using NIMROD's infrastructure we can

  - test different representations of velocity space basis functions including 2D finite elements,
  - handle time dependence in fully implicit manner,
  - allow for multiple parallelization strategies, and
  - test accuracy against existing, steady-state integral closures.

- Successful formulation of mixed finite-element advance for T bodes well for continuum solution:

$$\frac{3}{2} n \Delta T + \kappa_0 \sqrt{\theta \Delta t}\, \vec{B} \cdot \vec{\nabla} q_{\|} + \ldots = \ldots ,$$

$$\frac{\kappa_0^2 B^2}{\kappa_{\|} - \kappa_{\perp}} q_{\|} + \kappa_0 \sqrt{\theta \Delta t}\, \vec{B} \cdot \vec{\nabla} \Delta T = -\kappa_0 \sqrt{\frac{\Delta t}{\theta}}\, \vec{B} \cdot \vec{\nabla} T^n$$

Simplify.

- Consider solving CEL-DKE by expanding $F = \Sigma F_i(\mathbf{x},t)\,\phi_i(v_{||}, v)$:

$$\frac{\partial F}{\partial t} + \nu_L L(F) + \vec{v}_{||}\cdot\vec{\nabla} F + \left(\frac{q}{m}\frac{\vec{v}_{||}}{v}\cdot\vec{E}\frac{\partial}{\partial v} - \frac{\mu}{B}\frac{\partial B}{\partial t}\frac{\partial}{\partial \mu}\right) F =$$

$$C_{(-L)}(F + f_M) + CEL\,drives$$

- Acceleration and collision terms, $C_{(-L)}(F + f_M)$, couple velocity expansion coefficients in speed variable, v.
- Preliminary implementation ignores acceleration term and uses a moment approach for $C_{(-L)}$ (J-Y Ji's work and J. James' thesis).

Existing implementation solves for coefficients of F expansion on grid in $s=v/v_T$.

- Expanding $F = \Sigma F_l(\mathbf{x},v,t) P_l(v_{||}/v)$ yields:

$$(\boldsymbol{I}\frac{\partial}{\partial t}-\nu_L \boldsymbol{L})\vec{F}+\boldsymbol{A}\, v\, \hat{b}\cdot\vec{\nabla}\,\vec{F}-\boldsymbol{B}\, v(\hat{b}\cdot\vec{\nabla}\ln B)\vec{F}=drives,$$

$$where\ \nu_L=\sum_b \frac{\nu_{ab}}{s_a^3}((2-\frac{1}{s_b^2})E(s_b)+\frac{E'(s_b)}{s_b})$$

- Matrices **A**, **B**, and **L** represent free-streaming, |B| and pitch-angle collisional couplings, respectively.
- With Lorentz pitch-angle scattering operator, speed enters as a parameter only.
- Solve equation on grid in *s*.

Coupling to the fluid equations: time discretization and parallelization issues (I).

- Nimrod uses staggered advance.

$$\vec{V} \qquad\qquad (n\,,\vec{B}\,,T\,)$$

- For parallel heat flow closure, can couple fully implicit solves for T and F.

$$\vec{V} \qquad\qquad (n\,,\vec{B}\,,(T\,,F\,))$$

- Must solve simultaneously for coefficients, F_l, on speed grid, s_i, $i=1,\ldots,n$.
- Closure moment $q_{//}$ couples to T equation.
- Leads to large system of equations with “parallelization” performed inside solver.

Coupling to the fluid equations: time discretization and parallelization issues (I).

$$\frac{3}{2} n \Delta T + \Delta t \vec{\nabla} \cdot q_{\|} \hat{b} = \ldots, \qquad T\ equation$$

$$((a_{l,l}/s_i) + \theta \Delta t (\nu_L(s_i)/s_i) L_{l,l}) \Delta F_l +$$

$$\theta \Delta t \hat{b} \cdot \vec{\nabla} (a_{l,l+1} \Delta F_{l+1} + a_{l,l-1} \Delta F_{l-1}) +$$

$$\theta \Delta t (\hat{b} \cdot \vec{\nabla} \ln(B)) [b_{l,l+1} \Delta F_{l+1} + b_{l,l-1} \Delta F_{l-1}] +$$

$$\delta_{l1} a_{l,l} L_1^{3/2} f_M \hat{b} \cdot \Delta \ln T = \ldots, \qquad coupled\ equations\ for\ F_l$$

$$where\ q_{\|} = -T \int d\vec{v}\, v_{\|} L_1^{3/2} P_1(v_{\|}/v) F_1$$

A simple quadrature scheme for *s* dependence.

- For heat flow, need speed integrals of form:

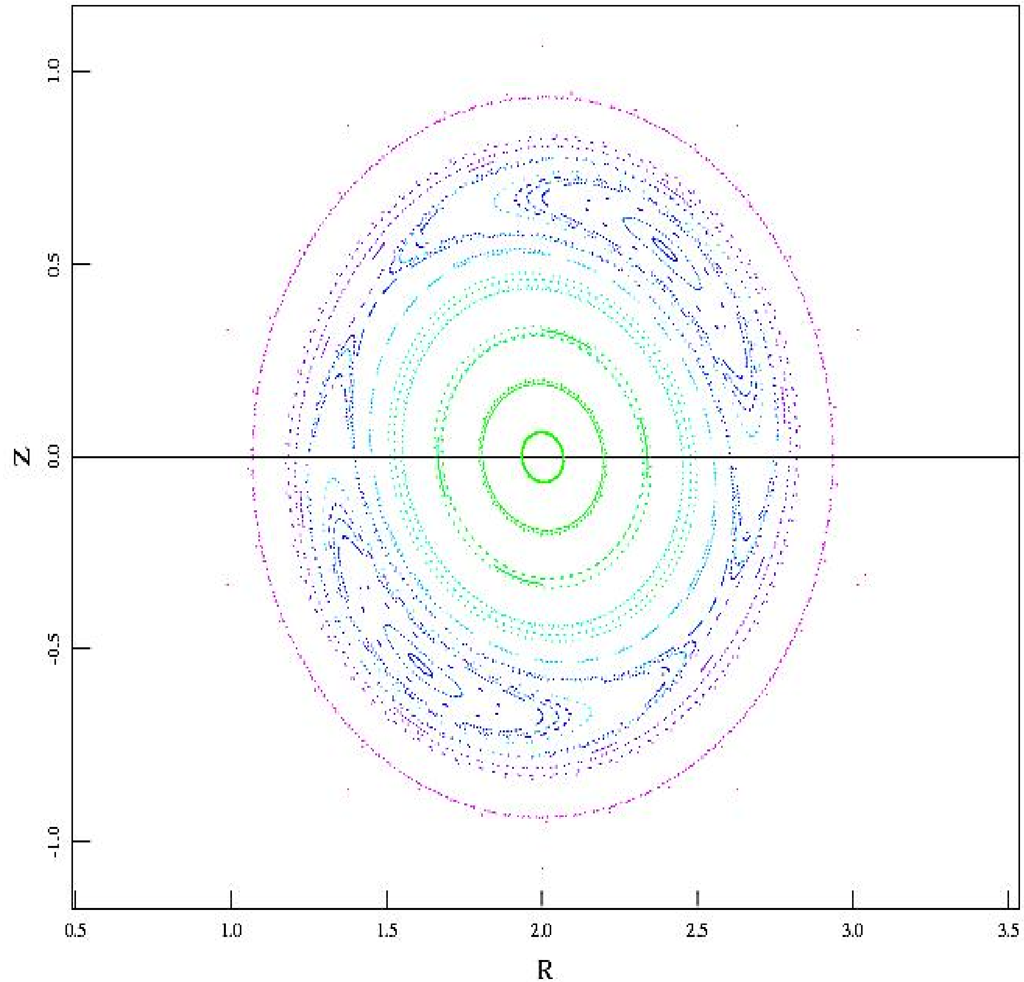
$$\int_0^\infty ds\, s^3 \left(\frac{5}{2} - s^2\right)^2 e^{-s^2} F = \sum_{i=1}^{N} w_i F(s_i^2).$$

- Linearize about f_M : $\hat{F}_l = F_l / (L_1^{3/2} f_M)$

- Simple quadrature scheme with *N=1* exact for linear functions of s^2 when $s_1^2 = 2.25$ and $w_1 = 2.888889$.

- Use as test case for more sophisticated solution methods.

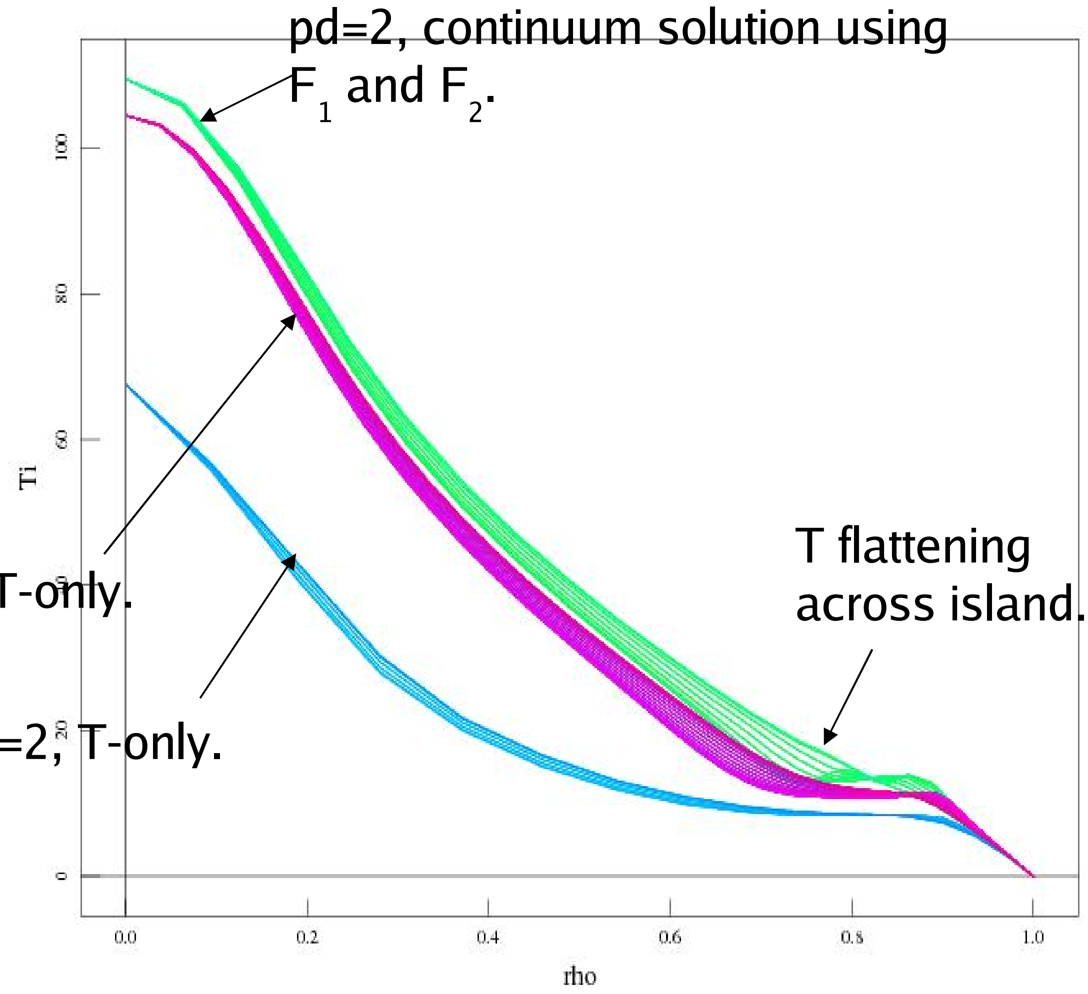
Applied to heat transport problem in cylindrical geometry (Hölzl *et al.*, POP 2008).



2/1 island added into zero pressure, cylindrical eq.
Heat source finite for r < 0.2 and zero outside.

Can solve for T coupled to F equations.

- T profiles as function of flux show flattening across 2/1 island.
- All cases used 3 Fourier modes and 10 x 10 grid.
- Predicted T from continuum solution spatially accurate for bicubic (pd=2) finite elements
- Here ν chosen to yield heat flow consistent with Braginskii closure.
- Solution not resolved in velocity variables!



Coupling to the fluid equations: time discretization and parallelization issues (II).

- Can stagger T and F.

$$(\vec{V}, F) \qquad\qquad (n, \vec{B}, T)$$

- Solution for coefficients, F_l, performed on groups of processors with different speeds, s_i, $i=1,\ldots,n$.
- Processors communicate to compute $q_{//}$ which is centered in T advance.
- Small systems of equations with parallelization performed outside solver.
- Limits time step.

Coupling to the fluid equations: time discretization and parallelization issues (II).

(1) Solve coupled equations for F_l, on separate groups of processors for single s_i.

$$((a_{l,l}/s_i)+\theta\,\Delta t(\nu_L(s_i)/s_i)L_{l,l})\,\Delta F_l +$$

$$\theta\,\Delta t\,\hat{b}\cdot\vec{\nabla}(a_{l,l+1}\Delta F_{l+1}+a_{l,l-1}\Delta F_{l-1})+$$

$$\theta\,\Delta t(\hat{b}\cdot\vec{\nabla}\ln(B))[a_{l,l+1}\Delta F_{l+1}+b_{l,l-1}\Delta F_{l-1}]+$$

$$\delta_{l1}\,a_{l,l}L_1^{3/2}f_M\hat{b}\cdot\Delta\ln T=\ldots,$$

(2) Compute parallel heat flow closures.

(3) Advance T.

Generalize velocity space basis to include 2-D finite elements.

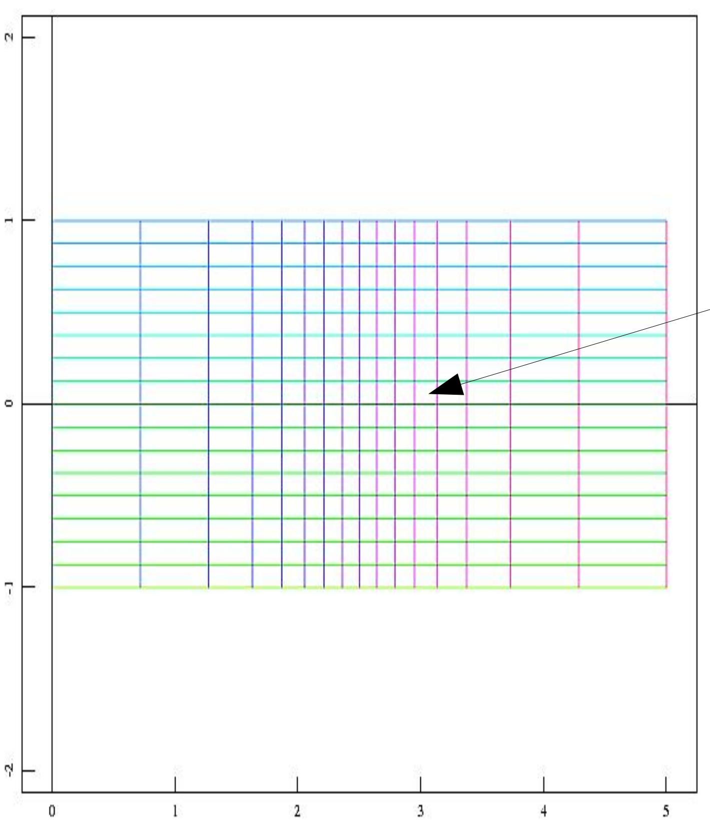
- Expand $F = \Sigma F_i(\mathbf{x},t)\,\phi_i(\xi, \mu)$, where ξ and μ are appropriate velocity variables and insert into CEL-DKE:

$$\frac{\partial F}{\partial t} + \vec{v}_{\parallel} \cdot \vec{\nabla} F + \left(\frac{q}{m}\frac{\vec{v}_{\parallel}}{v} \cdot \vec{E}\frac{\partial}{\partial v} - \frac{\mu}{B}\frac{\partial B}{\partial t}\frac{\partial}{\partial \mu}\right) F - C(F + f_M) = drives$$

- Integrate equation using $\int d\vec{x} \int d\vec{v}\, \phi_{i'}(\xi, \mu)\, \alpha_{j'}(R, Z)\, e^{-in'\phi}$
- Use NIMROD finite element and Gaussian quadrature machinery to compute velocity integrals.
- Consider C^0 or possibly discontinuous basis functions.
- Requires new data types and solver development.

Choose basis functions based on problem.

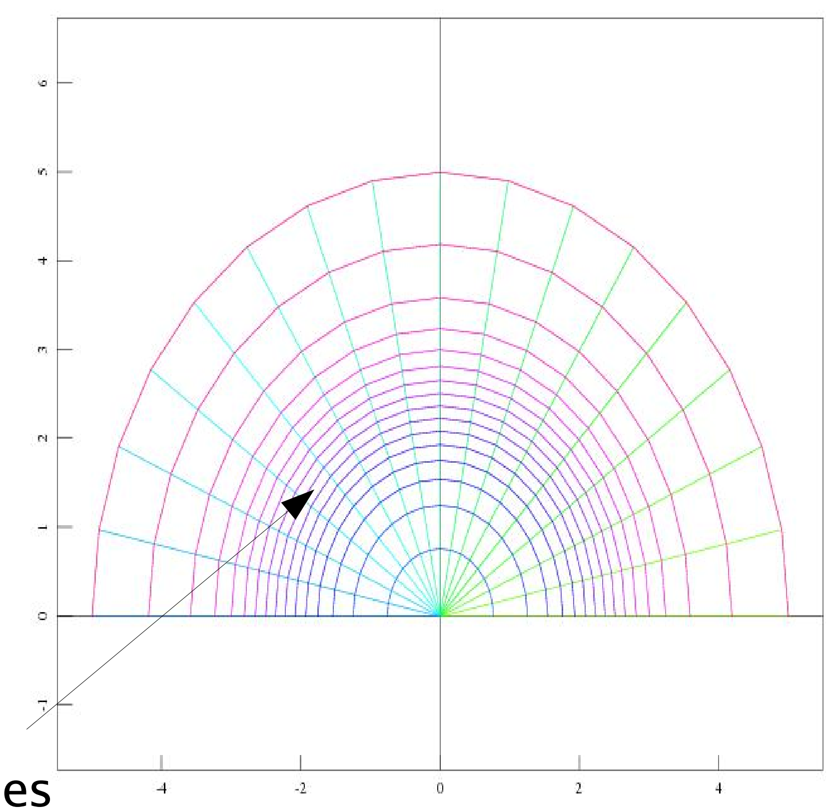
- Legendre functions for pitch-angle with simple grid in speed, *v*, is easiest.
- Allows for packed grids.



(ξ, μ)-space

pack near RF resonance in $v_\perp$

pack in speed and angle for slowing down of hot particles



$(v, \cos^{-1}(\xi))$-space

Future work.

- At present, can solve for coefficients of Legendre expansion, $F_l(\boldsymbol{x},t)$, on grid in s.
- Future work includes:

  - testing convergence of fully implicit advance as Legendre polynomials and grid points in speed are added.

  - implementing staggered advance and comparing with fully implicit solutions.

  - generalizing continuum solution to allow for 2-D finite-element basis functions for velocity dependence.