Implementation and application of moment method

Jeong-Young Ji and Eric D. Held

Department of Physics, Utah State University

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Moment method

- Landau kinetic equation

$$\partial_t f_a + \mathbf{v}\cdot\nabla f_a + \frac{e_a}{m_a}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\partial_{\mathbf{v}} f_a = \sum_b C(f_a, f_b)$$

- Moment expansion: expansion coefficients, m^{lk}'s, are symmetric traceless fluid moments $\mathbf{c}_a = (\mathbf{v}_a - \mathbf{V}_a)/v_{Ta}$

$$f_a(t,\mathbf{x},\mathbf{v}) = f_a^{\mathrm{M}} \sum_{lk} \frac{1}{\sqrt{\sigma_k^l}} \mathsf{m}_a^{lk}(t,\mathbf{x}) \cdot \mathsf{p}^{lk}(\mathbf{c}_a)$$

$$\mathsf{n}_a^{lk} \equiv n_a \mathsf{m}_a^{lk}(t,\mathbf{x}) = \int d\mathbf{v} \frac{1}{\sqrt{\sigma_k^l}} \mathsf{p}_a^{lk} f_a(t,\mathbf{x},\mathbf{v})$$

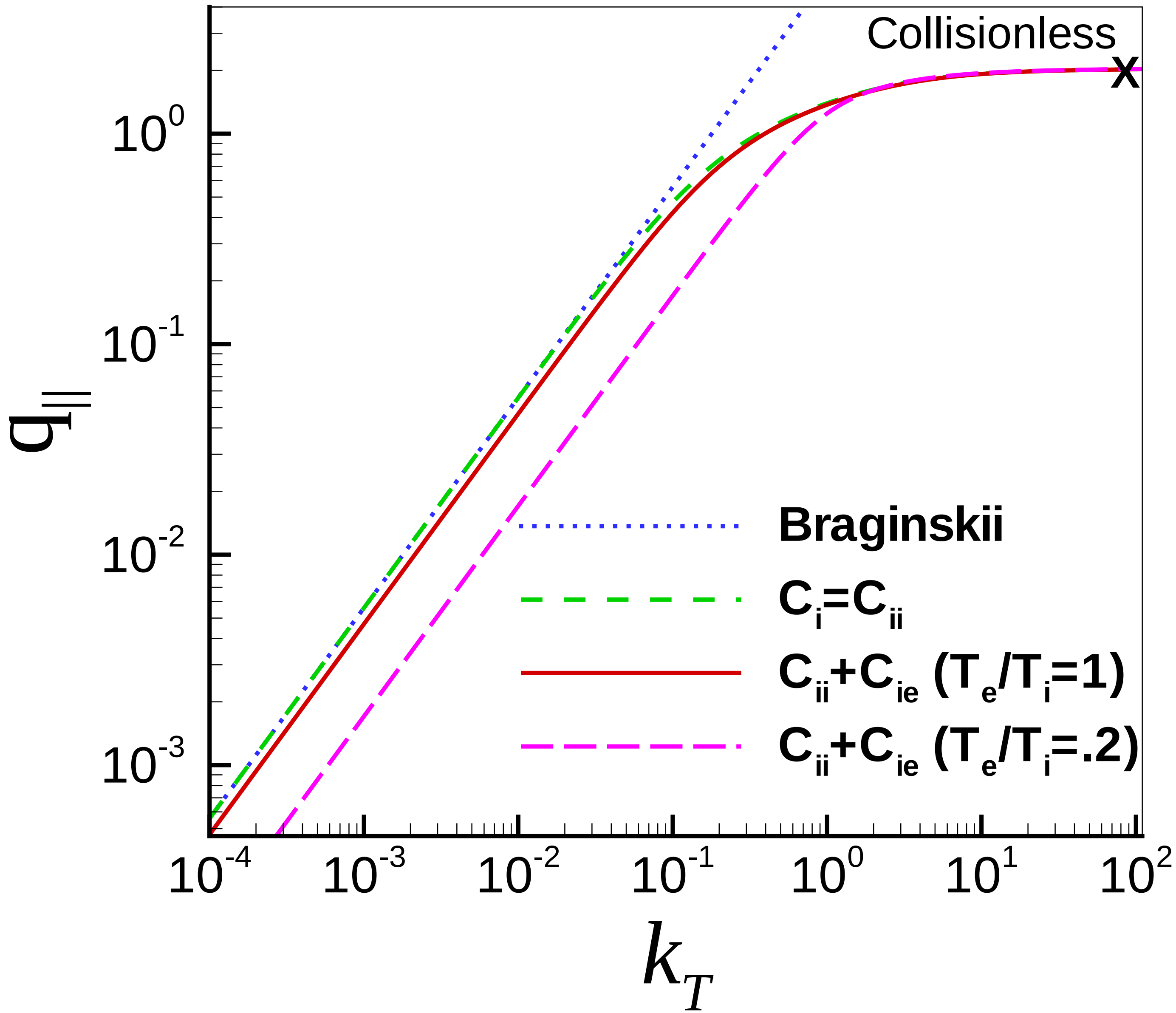
- Moment equations $d_t = \partial_t + \mathbf{V}\cdot\nabla$ and $\mathbf{a} = (q/m)(\mathbf{E}+\mathbf{V}\times\mathbf{B}) - d_t\mathbf{V}$

$$\sum_{l,k} D_a^{jp,lk}(d_t, \nabla, \nabla\mathbf{V}, \mathbf{a}, \mathbf{b}\times)\mathsf{n}_a^{lk} = \sum_{b,l,k} (A_{ab}^{jp,lk}\mathsf{n}_a^{lk} + B_{ab}^{jp,lk}\mathsf{n}_b^{lk})$$

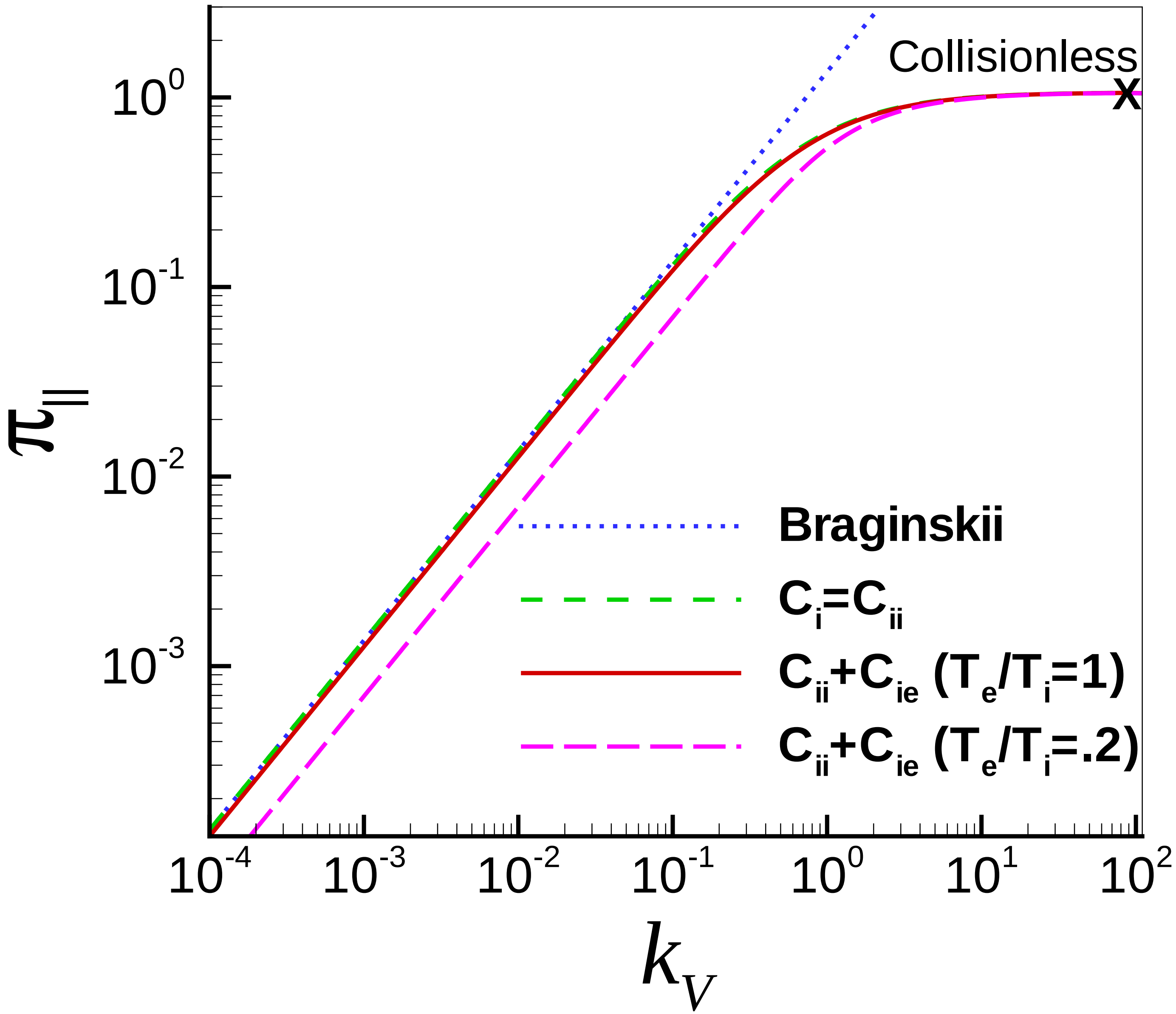
Works on moment method

- Exact linearized Coulomb collision operator in the moment expansion
- Landau collision operators and general moment equations for an electron-ion plasma
- Moment approach to deriving parallel heat flux for general collisionality
- Moment approach to deriving a unified parallel viscous stress in magnetized plasmas
- Classical transport theory with the ion-electron collision operator
- Relaxation of non-Maxwellian moments: analytical solutions of the kinetic equation for uniform plasmas
- Plasma transport theory for a general magnetic geometry
- 21 moment approximation in NIMROD

Effect of ion-electron collision operator on ion heat flux



Effect of ion-electron collision operator on ion stress



21 moment approximation: Heat flux: $\mathsf{n}^{11} = -\sqrt{4/5}\mathbf{q}/v_T T$

$$
\begin{aligned}
\underbrace{d_t \mathsf{n}^{11} - \sqrt{\frac{2}{5}} v_T \nabla \cdot \mathsf{n}^{20} + \sqrt{\frac{7}{5}} v_T \nabla \cdot \mathsf{n}^{21} - \frac{\sqrt{5}}{2} n v_T \nabla \ln T + \Omega \mathbf{b} \wedge \mathsf{n}^{11}} &= \underbrace{\hat{c}^{11}} \\
-\sqrt{\frac{2}{3}} v_T \nabla \mathsf{n}^{02} & \quad \text{H} \\
+\sqrt{\frac{2}{5}} \frac{2}{v_T} \mathbf{a} \cdot \mathsf{n}^{20} + \frac{3}{2} d_t \ln T \mathsf{n}^{11} - \frac{9}{\sqrt{10}} v_T \nabla \ln T \cdot \mathsf{n}^{20} + 2\sqrt{\frac{7}{5}} \nabla \ln T \cdot \mathsf{n}^{21} v_T & \quad \text{T} \\
+\frac{7}{5} \nabla \cdot \mathbf{V} \mathsf{n}^{11} + \frac{7}{5} \underline{\nabla} \mathbf{V} \cdot \mathsf{n}^{11} + \frac{2}{5} \nabla \underline{\mathbf{V}} \cdot \mathsf{n}^{11} & \quad \text{V} \\
-2\sqrt{\frac{2}{3}} v_T (\nabla \ln T) \mathsf{n}^{02} & \quad \text{H,T} \\
-2\sqrt{\frac{3}{5}} \nabla \mathbf{V} : \mathsf{n}^{30} & \quad \text{H,V}
\end{aligned}
$$

H: higher-order moments, T: coupled to $\mathbf{a}$, $d_t T$, ∇T, V: coupled to $\nabla \mathbf{V}$

$$
\hat{c}^{lk} = \int d\mathbf{v} \frac{1}{\sqrt{\sigma_k^l}} \mathsf{p}^{lk} C(f)
$$

n^{12}: energy weighted heat flux

$$
\begin{aligned}
d_t \mathsf{n}^{12} - \frac{2}{\sqrt{5}} v_T \nabla \cdot \mathsf{n}^{21} + \underbrace{\Omega \mathbf{b} \wedge \mathsf{n}^{12}} &= \underbrace{\hat{c}^{12}} \\
+\sqrt{\frac{7}{6}} v_T \nabla \mathsf{n}^{02} - v_T \nabla \mathsf{n}^{03} + \frac{3 v_T}{\sqrt{5}} \nabla \cdot \mathsf{n}^{22} & \qquad \mathsf{H} \\
+\frac{4}{\sqrt{5} v_T} \mathbf{a} \cdot \mathsf{n}^{21} - \sqrt{7} d_t \ln T \mathsf{n}^{11} + \frac{5}{2} d_t \ln T \mathsf{n}^{12} & \qquad \mathsf{T} \\
+\sqrt{\frac{14}{5}} v_T \nabla \ln T \cdot \mathsf{n}^{20} - \frac{13}{\sqrt{5}} v_T \nabla \ln T \cdot \mathsf{n}^{21} & \qquad \mathsf{T} \\
-\frac{2\sqrt{7}}{5} \nabla \cdot \mathbf{V} \mathsf{n}^{11} - \frac{2\sqrt{7}}{5} \underline{\nabla} \mathbf{V} \cdot \mathsf{n}^{11} - \frac{2\sqrt{7}}{5} \nabla \underline{\mathbf{V}} \cdot \mathsf{n}^{11} & \qquad \mathsf{V} \\
+\frac{9}{5} \nabla \cdot \mathbf{V} \mathsf{n}^{12} + \frac{9}{5} \underline{\nabla} \mathbf{V} \cdot \mathsf{n}^{12} + \frac{4}{5} \nabla \underline{\mathbf{V}} \cdot \mathsf{n}^{12} & \qquad \mathsf{V} \\
-\sqrt{\frac{14}{3}} \frac{1}{v_T} \mathbf{a} \mathsf{n}^{02} + 5\sqrt{\frac{7}{6}} v_T \nabla \ln T \mathsf{n}^{02} - 3 v_T \nabla \ln T \mathsf{n}^{03} + \frac{9}{\sqrt{5}} v_T \nabla \ln T \cdot \mathsf{n}^{22} & \qquad \mathsf{H,T} \\
+4\sqrt{\frac{3}{35}} \nabla \mathbf{V} : \mathsf{n}^{30} - 6\sqrt{\frac{6}{35}} \nabla \mathbf{V} : \mathsf{n}^{31} & \qquad \mathsf{H,V}
\end{aligned}
$$

$\mathsf{n}^{20} = \pi/\sqrt{2}T$: viscous stress (traceless pressure tensor)

$$
\begin{aligned}
d_t \mathsf{n}^{20} - \sqrt{\frac{2}{5}} v_T \nabla \mathsf{n}^{11} + \underbrace{\sqrt{2} n \overline{\nabla \mathbf{V}} + \Omega \mathbf{b} \wedge \mathsf{n}^{20}} &= \underbrace{\hat{c}^{20}} \\
+ \sqrt{\frac{3}{2}} v_T \nabla \cdot \mathsf{n}^{30} & \qquad \mathsf{H} \\
+ d_t \ln T \mathsf{n}^{20} - \frac{3}{\sqrt{10}} v_T \nabla \ln T \mathsf{n}^{11} & \qquad \mathsf{T} \\
+ \nabla \cdot \mathbf{V} \mathsf{n}^{20} + 2 \underline{\nabla} \mathbf{V} \cdot \mathsf{n}^{20} & \qquad \mathsf{V} \\
+ \frac{3}{2} \sqrt{\frac{3}{2}} \nabla \ln T \cdot \mathsf{n}^{30} v_T & \qquad \mathsf{H,T}
\end{aligned}
$$

n^{21}: energy weighted stress

$$
\begin{aligned}
d_t \mathsf{n}^{21} + \sqrt{\frac{7}{5}} v_T \nabla \mathsf{n}^{11} - \frac{2}{\sqrt{5}} v_T \nabla \mathsf{n}^{12} + \underbrace{\Omega \mathbf{b} \wedge \mathsf{n}^{21}} &= \underbrace{\hat{c}^{21}} \\
-\sqrt{\frac{3}{7}} v_T \nabla \cdot \mathsf{n}^{30} + 3\sqrt{\frac{3}{14}} v_T \nabla \cdot \mathsf{n}^{31} & \qquad \mathsf{H} \\
-\sqrt{\frac{7}{5}} \frac{2}{v_T} \mathbf{a} \mathsf{n}^{11} + \frac{7}{2}\sqrt{\frac{7}{5}} v_T \nabla \ln T \mathsf{n}^{11} - \sqrt{5} v_T \nabla \ln T \mathsf{n}^{12} & \qquad \mathsf{T} \\
+2 d_t \ln T \mathsf{n}^{21} - \sqrt{\frac{7}{2}} d_t \ln T \mathsf{n}^{20} & \qquad \mathsf{T} \\
-\sqrt{\frac{2}{7}} \nabla \cdot \mathbf{V} \mathsf{n}^{20} - 2\sqrt{\frac{2}{7}} \underline{\nabla} \mathbf{V} \cdot \mathsf{n}^{20} - 2\sqrt{\frac{2}{7}} \nabla \underline{\mathbf{V}} \cdot \mathsf{n}^{20} & \qquad \mathsf{V} \\
+\frac{9}{7} \nabla \cdot \mathbf{V} \mathsf{n}^{21} + \frac{18}{7} \underline{\nabla} \mathbf{V} \cdot \mathsf{n}^{21} + \frac{4}{7} \nabla \underline{\mathbf{V}} \cdot \mathsf{n}^{21} & \qquad \mathsf{V} \\
+\sqrt{\frac{3}{7}} \frac{2}{v_T} \mathbf{a} \cdot \mathsf{n}^{30} - 6\sqrt{\frac{3}{7}} v_T \nabla \ln T \cdot \mathsf{n}^{30} + \frac{15}{2}\sqrt{\frac{3}{14}} v_T \nabla \ln T \cdot \mathsf{n}^{31} & \qquad \mathsf{H,T} \\
-2\sqrt{\frac{14}{15}} \nabla \mathbf{V} \mathsf{n}^{02} - 2\sqrt{\frac{6}{7}} \nabla \mathbf{V} : \mathsf{n}^{40} & \qquad \mathsf{H,V}
\end{aligned}
$$

Have a nice meeting

and

enjoy the NIMROD open