Continuum Drift Kinetic Computations in NIMROD

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Status of integral closures in NIMROD.

- Integral forms for electron and ion parallel heat flow (CEL and higher-order moment approach) and parallel ion stress (CEL approach) implemented in NIMROD.
  - computations are costly although method of fitting T and **V** drives greatly speeds up closure calculation (John James thesis),
  - improved parallel scaling using separate groups of processors devoted to closure calculation (scaling talk this afternoon),
  - further improvements include efforts to reuse closure data along field line to minimize number of integrations; also considering mapping techniques.

- Existing implementation uses local, diffusive forms for semi-implicit stabilization.
  - mixed finite element approach used to improve spatial accuracy of semi-implicit operator,
  - continuum closure computation could also be used to improve semi-implicit stabilization of integral closures which are treated explicitly.

Mixed finite-element method (MFEM) treats $\mathbf{q}_{||}$ as fundamental variable.

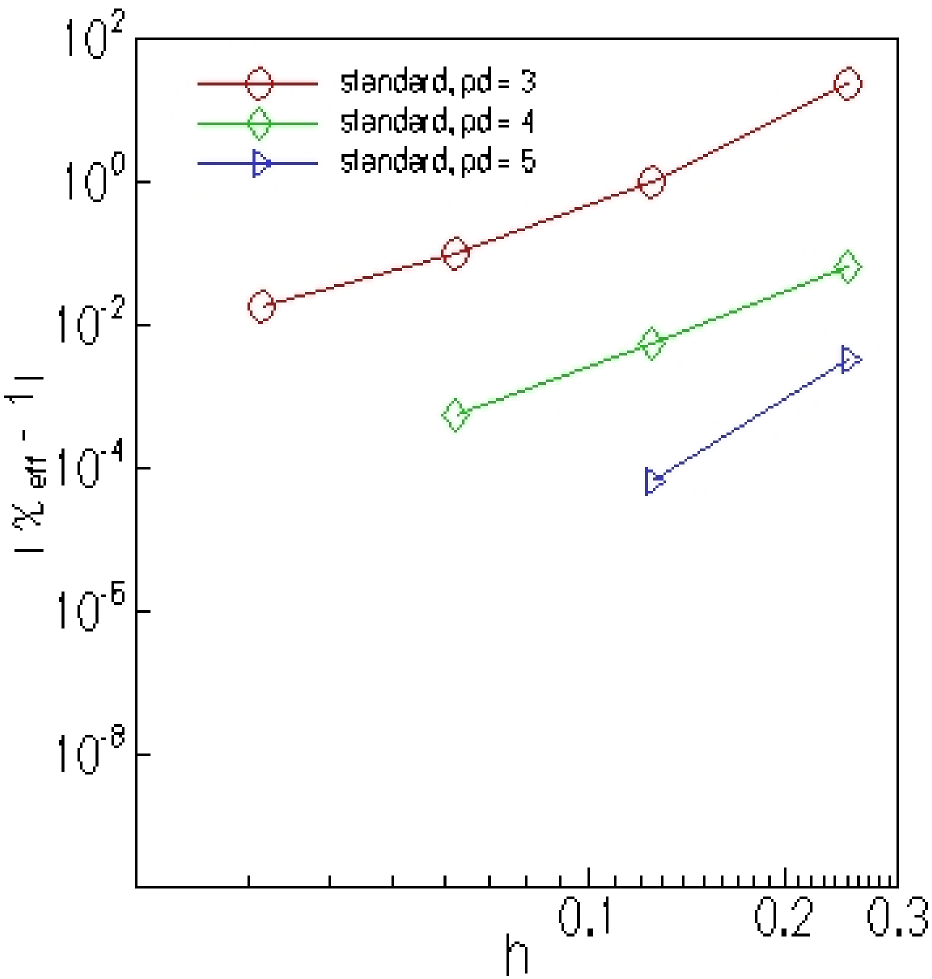
- Solve expanded system for T and auxiliary scalar, $q_{||}$:

$$\frac{3}{2} n \Delta T + \kappa_0 \sqrt{\theta \Delta t}\, \vec{B} \cdot \vec{\nabla} q_{||} = \ldots ,$$

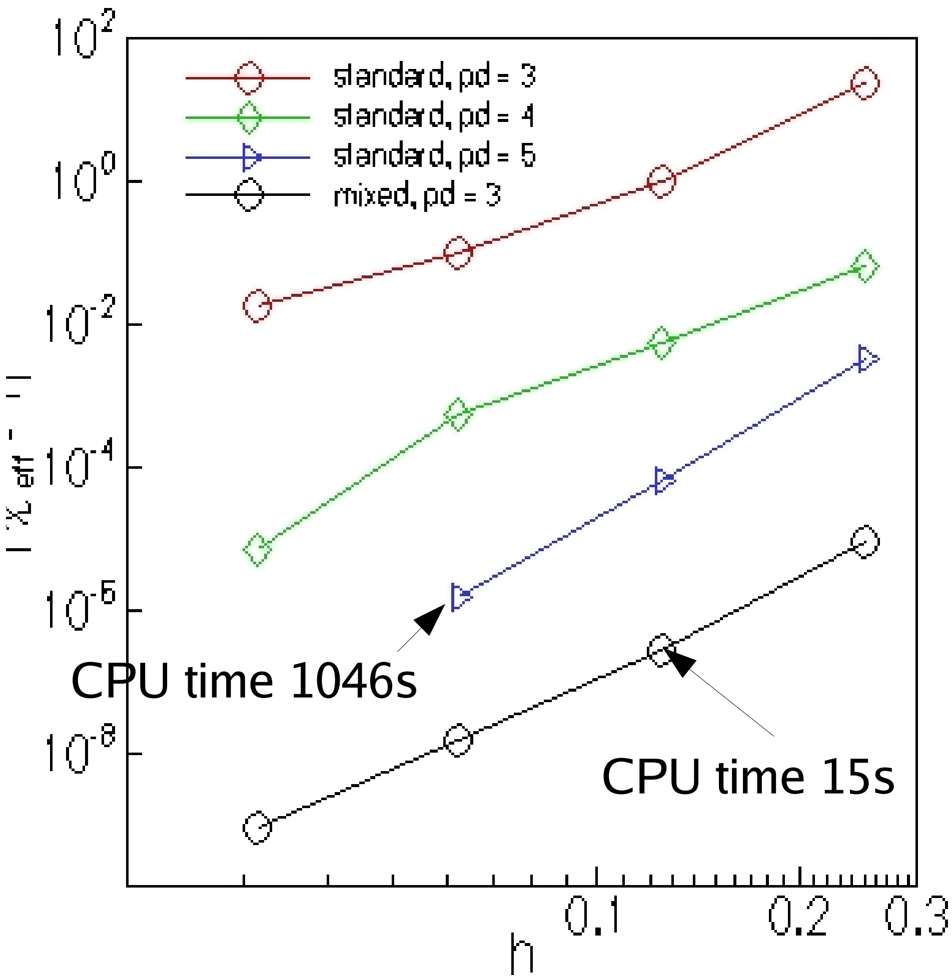
$$\frac{\kappa_0^2 B^2}{\kappa_{||} - \kappa_{\perp}} q_{||} + \kappa_0 \sqrt{\theta \Delta t}\, \vec{B} \cdot \vec{\nabla} \Delta T = -\kappa_0 \sqrt{\frac{\Delta t}{\theta}}\, \vec{B} \cdot \vec{\nabla} T^n$$

- Term “mixed” loosely implies treating flux, $q_{||}$, as expanded quantity.
- Formulation has symmetric treatment of $q_{||}$ and T.
- Constant, κ_0, used for scaling. Constant, θ, is centering parameter.
- Implementation solves either for (ΔT,$q_{||}$) or (T^{n+1},$q_{||}$).

Error reduced considerably in JCP anisotropic test problem (Sovinec *et al.*, JCP 2004).



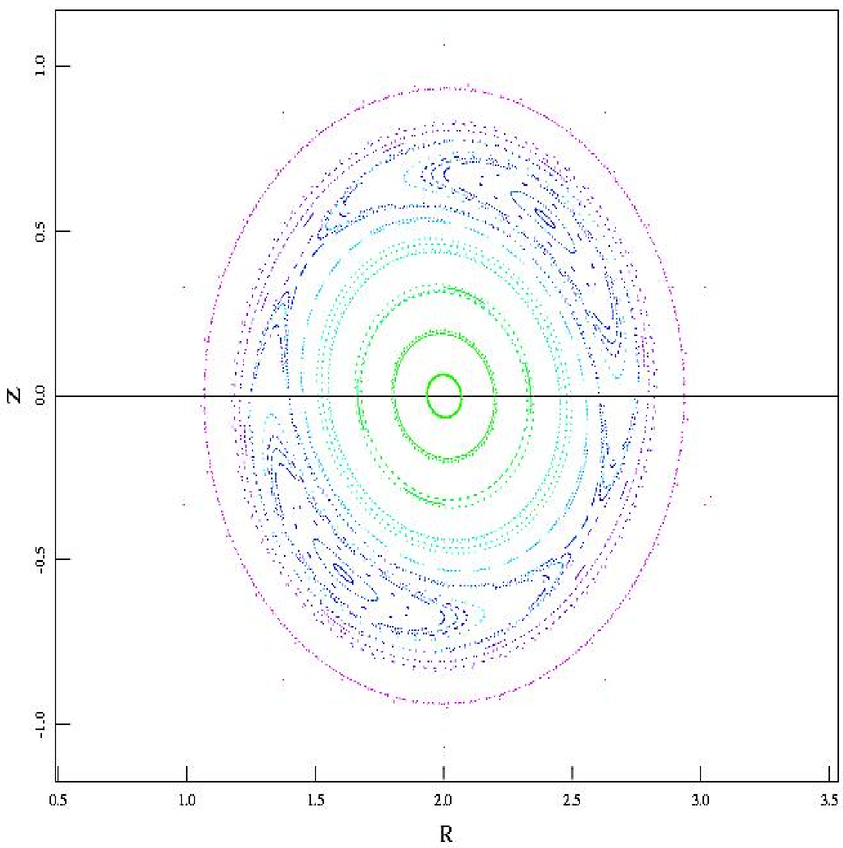
previous result



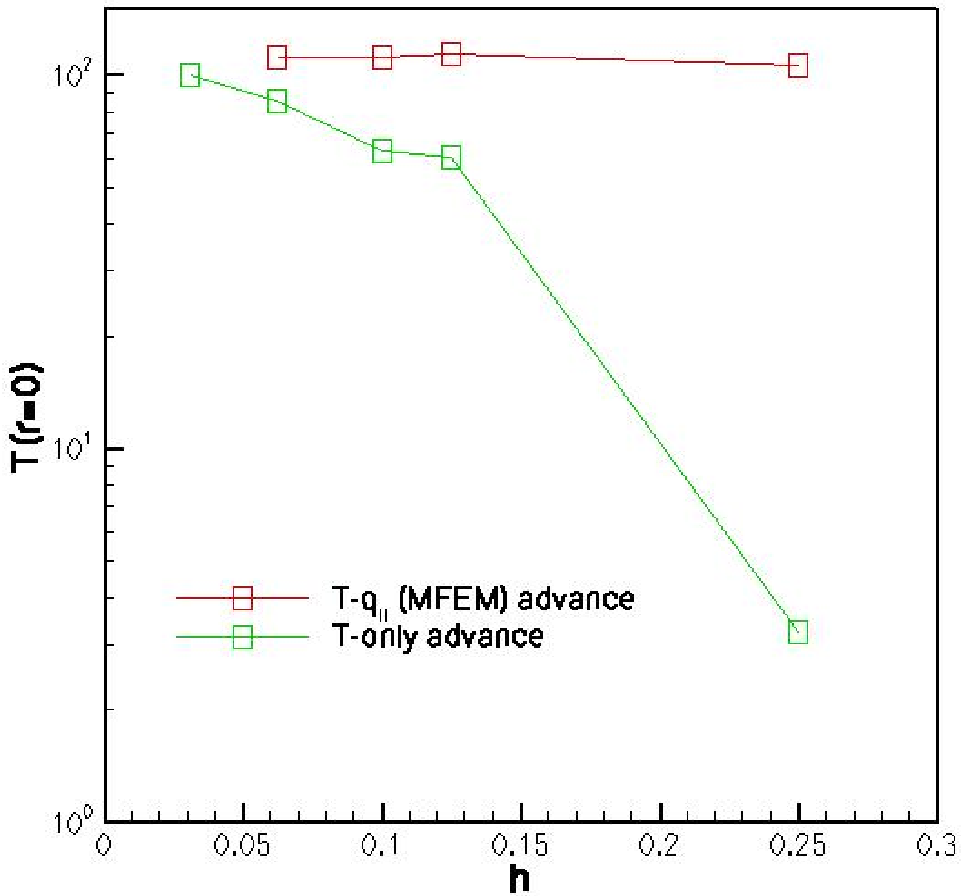
MFEM result (bottom curve)

Applied to heat transport problem in cylindrical geometry (Hölzl *et al.*, POP 2008).

2/1 island stuck into zero pressure, cylindrical eq.
Heat source finite for r < 0.2 and zero outside



Core T accurate in MFEM case with only 4 grid points and bi-quadratic polynomials.



Motivate continuum solution to closure problem.

- Using NIMROD's infrastructure can easily

  - test different representations of velocity space basis functions including 2D finite elements,

  - handle time dependence in fully implicit manner,

  - incorporate |**B**| and accelerations effects, and

  - allows for multiple parallelization strategies.

- This work will aid implementation of higher-order moment equations as well.

Apply MFEM approach in continuum solution.

- Solve for fully implicit $q_{||}$ closure from lowest order Chapman-Enskog-like drift kinetic equation (CEL-DKE):

$$\frac{\partial F}{\partial t} + v_{||} \hat{b} \cdot \vec{\nabla} F - C(F + f_M) = thermodynamic\ drives.$$

- Expanding F = Σ F_i (**x**,v,t) P_i ($v_{||}$/ v) yields:

$$\frac{\partial \vec{F}}{\partial t} + \boldsymbol{A} v \hat{b} \cdot \vec{\nabla} \vec{F} - \boldsymbol{B} v (\hat{b} \cdot \vec{\nabla} \ln B) \vec{F} - \boldsymbol{C} \vec{F} = drives\ ,$$

- Matrices **A**, **B**, and **C** represent free-streaming, |B| and collisonal couplings, respectively.

Fully implicit solution for coefficients, F_n, couples to T advance through $q_{||}$.

- Ignoring nonlinearities, treating T drive only and evaluating remaining speed dependence at $v = v_T$ yields coupled system:

$$\frac{3}{2} n \Delta T + \Delta t \vec{\nabla} \cdot q_{||} \hat{b} = \dots,$$

T drive in F_1 equation

$$(a_{i,i} + \theta \Delta t \nu_0 c_{i,i}) \Delta \hat{F}_i / v_T +$$

$$\theta \Delta t \hat{b} \cdot \vec{\nabla} [a_{i,i+1} \Delta \hat{F}_{i+1} + a_{i,i-1} \Delta \hat{F}_{i-1} + \delta_{i1} a_{i,i} \Delta T] +$$

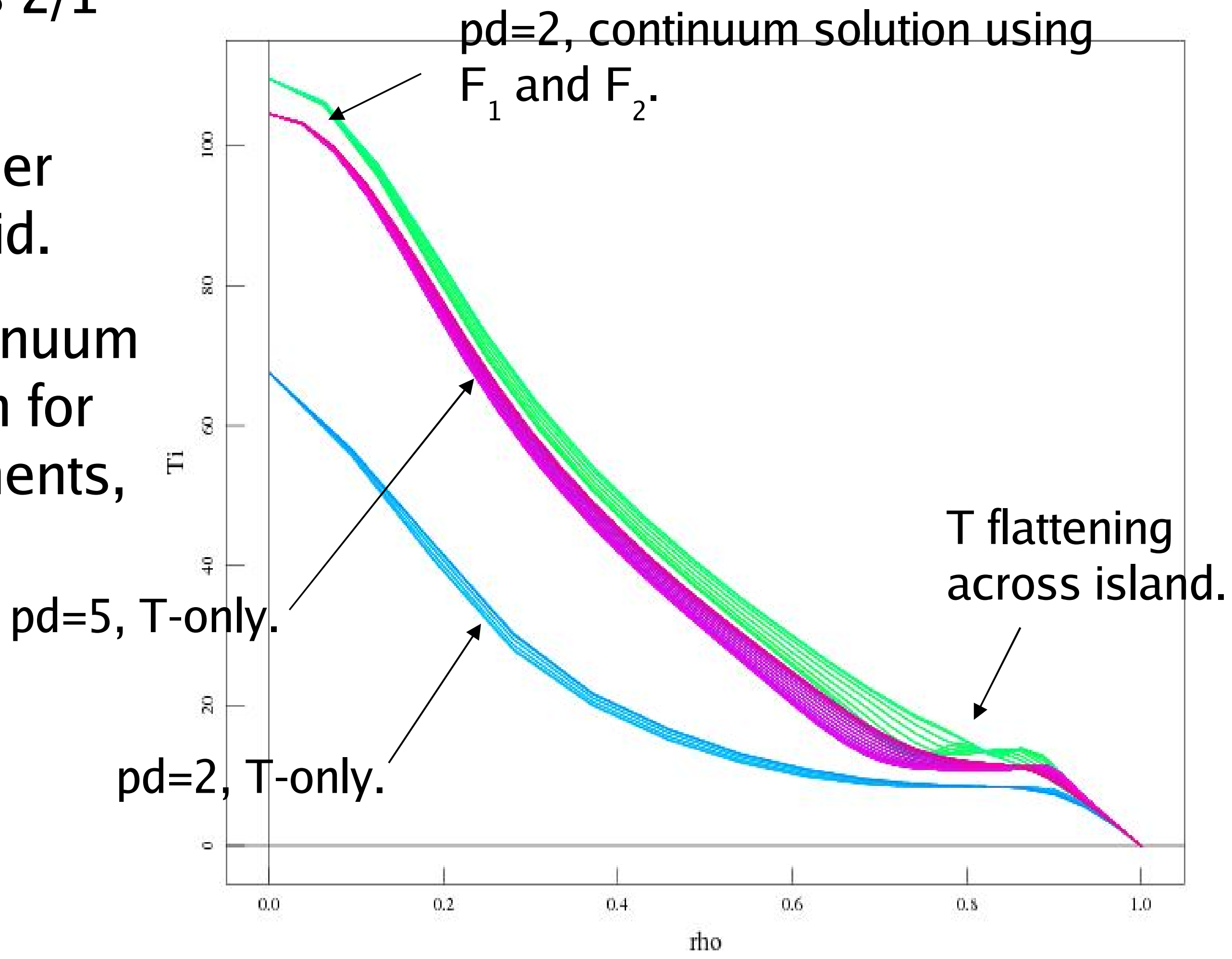
$$\theta \Delta t (\hat{b} \cdot \vec{\nabla} \ln(B)) [b_{i,i+1} \Delta \hat{F}_{i+1} + b_{i,i-1} \Delta \hat{F}_{i-1}] = \dots,$$

Linearize about a Maxwellian

$$q_{||} = -T \int d\vec{v}\, v_{||} L_1^{3/2} P_1(v_{||}/v) F_1, \; with\, \hat{F}_i = F_i / (L_1^{3/2} f_M).$$

Apply continuum solution to Hölzl problem.

- T profiles as function of flux show flattening across 2/1 island.
- All cases used 3 Fourier modes and 10 x 10 grid.
- Predicted T from continuum solution accurate even for lower-order finite elements, pd = 2.



Solve separated equations for expansion coefficients of *F* and stagger from *T*.

- First solve for ΔF (actually $\Delta f_i = f_i^{k+1} - f_i^{k}$) using $T^{k+1/2}$:

$$\left[1+\theta\,\Delta t\left(\frac{\nu_L}{2}+\gamma_i v\,\hat{b}\cdot\vec{\nabla}\right)\right]\Delta f_i=\Delta t\left(\frac{\nu_L}{2}+\gamma_i v\,\hat{b}\cdot\vec{\nabla}\right)f_i^k$$

$$+\Delta t\,W_{i1}^{-1}L_1^{3/2}v\,(\hat{b}\cdot\vec{\nabla}\ln T^{k+1/2})\,f_{Max}^{k+1/2}$$

- Then solve for $\Delta T = T^{k+3/2} - T^{k+1/2}$ using centered f^{k+1}:

$$\left[1+\theta\,\Delta t\,\vec{\nabla}\cdot\kappa_\perp\vec{\nabla}\right]\Delta T=\theta\,\Delta t\,\vec{\nabla}\cdot\kappa_\perp\vec{\nabla}\,T^{k+1/2}$$

$$-\Delta t\,\vec{\nabla}\cdot q_{\|}^{k+1}\,,$$

where $q_{\|}^{k+1}=-T\int d\vec{v}\,v_{\|}\,L_1^{3/2}\,P_1\sum_i W_{1i}\,f_i^{k+1}\,,$

Solving on speed grid expands system. Can keep nonlinearities.

- Solve coupled systems at particular v.

$$\frac{3}{2} n \Delta T + \Delta t \vec{\nabla} \cdot q_{\|} \hat{b} = \ldots,$$

T drive in F_1 equation

$$(a_{i,i} + \theta \Delta t \nu (v_j / v_T) c_{i,i}) \Delta F_i / v_j +$$

$$\theta \Delta t \hat{b} \cdot \vec{\nabla} [a_{i,i+1} \Delta F_{i+1} + a_{i,i-1} \Delta F_{i-1} + \delta_{i1} a_{i,i} \Delta T] +$$

$$\theta \Delta t (\hat{b} \cdot \vec{\nabla} \ln(B)) [b_{i,i+1} \Delta F_{i+1} + b_{i,i-1} \Delta F_{i-1}] = g(v_j),$$

$$q_{\|} = -T 2 \pi \int_{-1}^{1} d\xi \sum_j w_j v_{\|} L_1^{3/2} P_1 \left(v_{\|} / v \right) F_1 (x, t, v_j).$$

Generalize velocity space basis to include 2-D finite elements.

- Solve for fully implicit $q_{||}$ closure from lowest order Chapman-Enskog-like drift kinetic equation (CEL-DKE):

$$\frac{\partial F}{\partial t} + v_{||}\hat{b}\cdot\vec{\nabla} F - C(F + f_M) = thermodynamic\, drives.$$

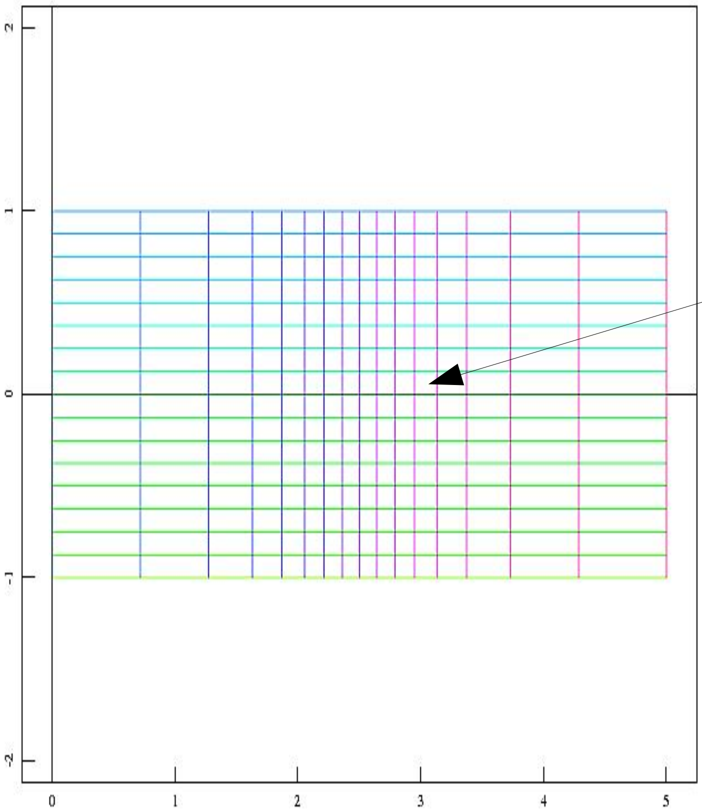
- Expand F = Σ $F_i \phi_i(\xi, \mu)$, where ξ is pitch-angle variable and μ is magnetic moment and compute integrals of form

$$\int d\vec{x} \int d\vec{v}\, \phi_j \left(\phi_i \frac{\partial F_i}{\partial t} + \phi_i \hat{b}\cdot\vec{\nabla} F_i - F_i \boldsymbol{C}(\phi_i) = drives, \right)$$

- Use NIMROD finite element and Gaussian quadrature machinery to compute velocity integrals.

Choose from various basis functions for velocity variables.

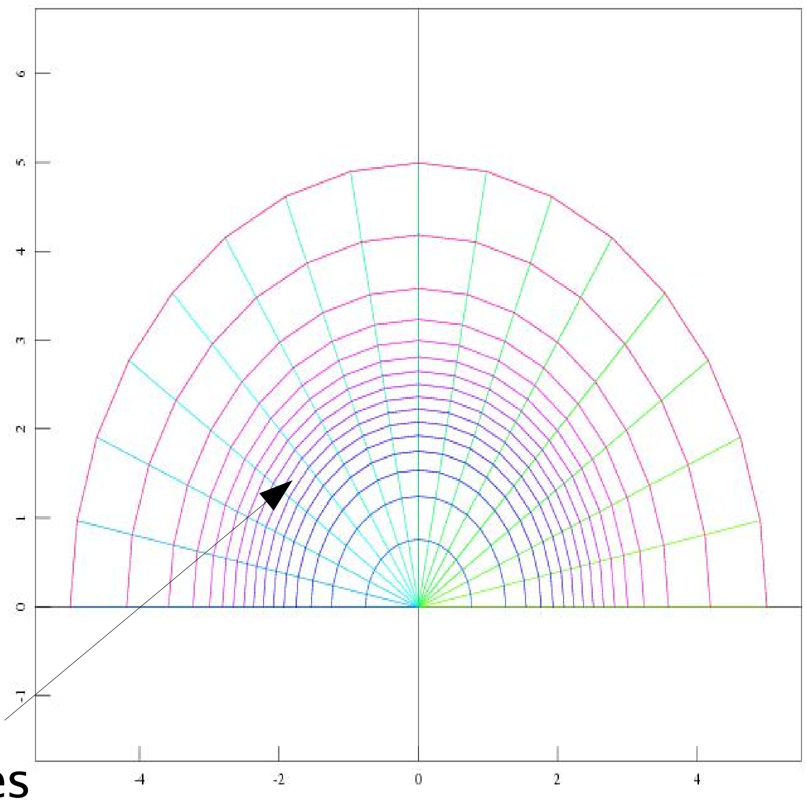
- Legendre functions for pitch-angle with simple grid in speed, *v*, is easiest.
- Can also use 2D finite elements with packed grids.



(ξ, μ)-space

pack near RF resonance in $v_\perp$

pack in speed and angle for slowing down of hot particles



(v, $\cos^{-1}(\xi)$)-space

Future work.

- Test convergence as Legendre polynomials and grid points in speed are added.
- Test staggered advance against simultaneous, fully implicit solution for T and F.
- Apply 3-D iterative solves in continuum solution of CEL-DKE and/or higher order moment equations.
- Include flow and collisional (beyond Lorentz pitch-angle scattering) drives.
- At present, can compute full matrix that arises from spatial coupling and solve for $F_i(\boldsymbol{x},t)$ on grid in v using SuperLU.
- Use recent improvements to 3-D preconditioning for non-symmetric systems.