Two-Fluid 1/1 and Tearing

C. R. Sovinec, J. R. King, and N. A. Murphy
(with acknowledgments to P. Zhu)
University of Wisconsin-Madison

NIMROD Team Meeting
August 27-28, 2008
San Diego, CA
Outline

• Introduction
• 1/1 Kink problem
  – Description
  – Results
• Tearing problem
  – Description
  – Results
• Conclusions
Introduction: with progress on solving the 3D two-fluid B-advance at large $\Delta t$, we are ready to exercise it on production applications.

- The cylindrical kink study started as a nonlinear benchmark.
  - At least 3 groups have results.
  - The problem is helically symmetric; though, NIMROD computes it as fully 3D.
  - A toroidal version of this problem will be 3D.
- The tearing computation is a step toward 3D two-fluid RFP relaxation.
  - It uses a short cylinder to limit the spectrum.
  - Cases reported here are not reversed, also to limit coupling.
  - We will progress to reversed equilibria and more nonlinear coupling.
Cylindrical 1/1 Description: the nonlinear single-helicity evolution of this mode has been studied computationally and analytically.

- Aydemir first compared linear behavior from Hazeltine’s 4-field model with those of more complicated models [PFB 3, 3025].
- His nonlinear computation with the same model predicted increasing $d\ln(E_k)/dt$ in the nonlinear stage [PFB 4, 3469].
- Wang and Bhattacharjee developed an analytical model that predicts temporary finite-time singular behavior in island width [PRL 70, 1627].
- Ottaviani and Porcelli considered large $\Delta'$ reconnection in a 2D slab with a reduced incompressible model [PoP 2, 4104].
- Lukin reproduced Aydemir’s result using incompressible, helically symmetric computations with hyper-resistivity [dissertation, Princeton, 2007].
- Germaschewski will present the same type of fast reconnection behavior from a full model with helical symmetry at APS-DPP ‘08.
The physical parameters used for the (3D) NIMROD computations are similar to those of the published (2D) results.

• The pressure profile is flat, and \( q(r) = 0.98 + 0.51(r/a)^2 \) has the 1/1 resonant surface at \( r = 0.2a \).

• Most of the computations have \( R/a = 1 \); one has \( R/a = 2 \).

• All of the computations use the following parameters:

\[
S = \frac{\tau_r}{\tau_{Hp}} = 10^6 \quad \tau_r = \mu_0 a^2 / \eta \quad \tau_{Hp} = a \sqrt{\mu_0 \rho / B_p} \\
\beta = 5 \times 10^{-3} \quad \delta = d_i / 2 = 0.11 \quad \left( d_\alpha = c \sqrt{\frac{\varepsilon_0 m_\alpha}{n_\alpha q_\alpha}} \right) \quad \rho_s = 1.5 \times 10^{-2} \\
\mu_0 \nu_{iso} / \eta = Pm = 0.1 \quad T_i \equiv 0
\]

• Earlier computations had \( \mu_0 D / \eta = 0.01 \) without hyper-diffusivity, and later computations have \( D = 0 \) and \( \mu_0 D_h / \eta a^2 = 10^{-4} \).

• Most of the computations have \( d_e = 5 \times 10^{-3} \), but one has \( d_e = 0.01 \).

\[
\beta_{pol} = \beta (R/r_s)^2 = 1/8 \text{ at } R/a = 1, \quad 1/2 \text{ at } R/a = 2
\]
Without a pressure gradient (no equil. diamagnetic effects), two-fluid linear results are similar to resistive MHD.

- Linear growth rates for the two models differ by about 15%.
  - \( R/a = 1 \): \( \gamma_{\text{MHD}} \tau_{\text{Hp}} = 1.82 \times 10^{-2} \) and \( \gamma_{\text{MHD}} \tau_{\text{Hp}} = 2.06 \times 10^{-2} \)
  - \( R/a = 1 \): \( \gamma_{\text{MHD}} \tau_{\text{Hp}} = 7.74 \times 10^{-3} \) and \( \gamma_{\text{MHD}} \tau_{\text{Hp}} = 9.30 \times 10^{-3} \)
- Axial flow velocity is more distinguished for the two-fluid model than other fields--still not much.

Axial flow velocity with resistive MHD.

Axial flow velocity with the two-fluid model.
Numerical parameters for the nonlinear computations have been varied to achieve good resolution.

• Mesh packing extends beyond the 1/1 surface to resolve nonlinear distortion.
• The nonlinear computations have $6 \leq \text{poly\_degree} \leq 8$.
• Toroidal resolution has been checked by varying $n_{\text{max}}$ from 5 to 85. With the largest $n_{\text{max}}$, $\Delta z < d_i$.
• Unlike typical NIMROD behavior, severely under-resolved cases do not crash when enough quadrature points and $D_h$ are used.
  • Severely under-resolved cases tend to behave like MHD.
  • With insufficient toroidal resolution, smooth blobs appear in the poloidal plane due to Gibbs phenomena in the toroidal direction.
• $\Delta t$ is initially based on linear convergence ($\sim \tau_A$) and later limited for flow and solver to $\sim 0.05$-$0.1$ $\tau_A$. 
Nonlinear 1/1 results: at least some of the expected two-fluid effects are observed in resolved computations.

- Reconnection transitions to x-point in all two-fluid cases; \( d_e = 0.01 \) shown.

Poincare section: \( 2\text{fl} \) at peak magnetic energy.

Poincare section: \( 2\text{fl} \) at peak kinetic energy.

Poincare section: \( 2\text{fl} \) reconnection completing.

Poincare section: \( \text{MHD} \) after peak magnetic energy.
**Nonlinear 1/1 results:** current density becomes localized in the two-fluid model.

Parallel current density peaks azimuthally at the x-point in the $d_e=0.01$ two-fluid computation.

Parallel current density peaks maintains a Sweet-Parker-like current sheet throughout for the MHD case.
Nonlinear 1/1 results: at these parameters (and full model without hyper-resistivity) kinetic energy does not show the strong increase in ‘growth rate.’

Slope of $\log(E_k)$ shows no increased growth for $R/a=1$ and $d_e=0.01$ or $d_e=0.005$. (Latter is shown.)

Slope of $\log(E_k)$ shows slightly increased growth for $R/a=2$, $d_e=0.005$.

• Besides checking resolution, we are considering the importance of physical parameters like poloidal-beta.
Tearing Problem Description: to address two-fluid relaxation and dynamo, we are investigating the nonlinear evolution of a pinch tearing mode.

• Cylindrical pinch computations will have multiple helicities active. [Previous slab case is strictly 2D.]

• We consider a non-reversed paramagnetic pinch profile with uniform pressure at $\beta=0.1$.

• Use $R/a=0.62$ to limit the number of unstable modes.

• $S=10^5$, $Pm=0.1$

• 3 conditions:
  MHD $\gamma\tau_A=2.37\times10^{-4}$
  2fl, $\rho_s=0.029$, $d_e=0.017$, $\gamma\tau_A=2.76\times10^{-4}$
  2fl, $\rho_s=0.10$, $d_e=0.057$, $\gamma\tau_A=3.91\times10^{-4}$
**Tearing results:** as in the 2D slab computation, evolution to a saturated island is expected.

Evolution of logarithms of magnetic and kinetic energies show saturation for MHD.

The smaller $\rho_s$ two-fluid computation also saturates. The $m=0$ flow increases with this model.
Possibly associated with the small growth rate, there is no obvious localized distortion from 2-fluid physics.

Number density (left) and current density (right) in the saturated state of the small $\rho_s$ computation show sinusoidal helical distortions near the resonance.

- With the small growth rate, the hybrid skin depth is larger than $\rho_s$. 
The larger $\rho_s$ case is more problematic and crashes during the nonlinear phase.

Energy spikes as the time-step crashes to very small value.

- The number density becomes noisy.
- We are testing the hyper-diffusivity in addition to checking resolution.
Conclusions

• With new preconditioning and hyper-diffusivity, we are returning to 3D two-fluid applications.

• The 1/1 application clearly shows the development of two-fluid reconnection geometry.

• Physical parameters are being varied to investigate growth rate of kinetic energy.

• The cylindrical tearing computation saturates for the moderately two-fluid regime.

• Progress here is encouraging for multi-helicity computations with a reversal surface.