Progress reports on:

Finite-β Three-Dimensional MHD Equilibrium Studies

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Resistive Wall Mode/Field Error Penetration Physics

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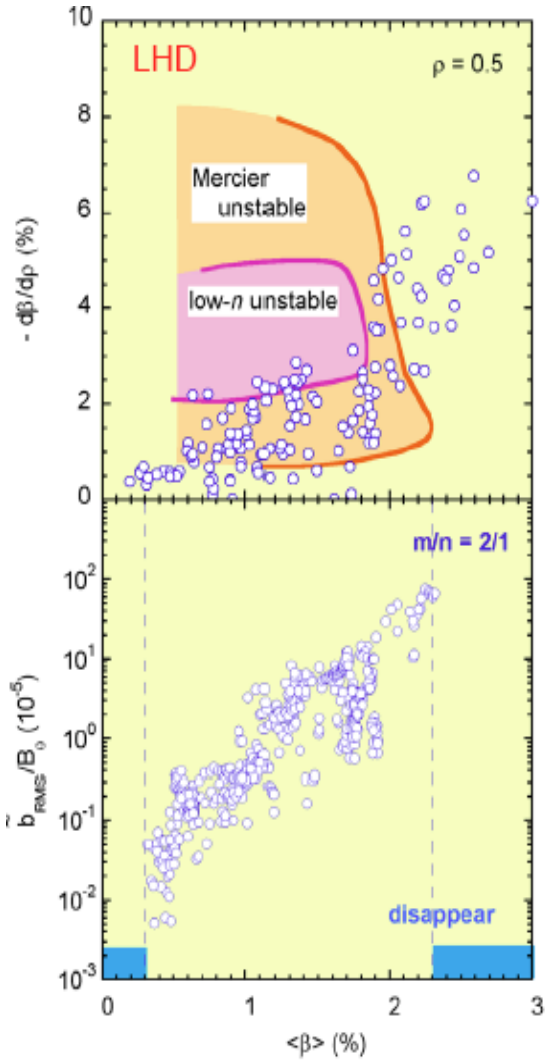
Recent stellarator experiments motivate the need for high β MHD equilibrium studies

- In LHD, low-m/n instabilities are observed --- similar in W7X
  - Onset largely in accord with theory
  - Mode grows, benignly, no disruptions
- In contrast to tokamak, where ideal β-limit is extensively studied --- upper limit invariably leads to disruption
- 3-D MHD equilibrium physics may produce limit on stellarator β.
  - Pressure induced currents degrade magnetic surface integrity?
  - Degradation typically occurs via the production of 'weakly' stochastic edge magnetic fields --- self-consistent transport physics?



NIMROD will be used to study finite-β physics in 3-D geometry --- M. Schlutt et al

- Straight stellarator studies --- pure helically symmetric vacuum states in periodic cylinder geometry
  - Vary vacuum by adding 'weak' 3-D magnetic fields ---> Produces magnetic islands, weak stochasticity.
  - Add heating source --- produce pressure profile
  - Transport properties (parallel and perpendicular) affect the pressure profile formation.
  - Self-consistent pressure induced currents using a resistive MHD model
  - Anisotropic heat transport --- ultimately, integral closure techniques for parallel heat conduction
  - Coupled to analytic theory modeling --- resistive MHD

Simulations start from vacuum solutions to the magnetic field

- Vacuum solutions to the magnetic field in periodic cylinder geometry are given by

$$\vec{B} = \nabla\varphi, \quad \nabla^2\varphi = 0,$$

$$\varphi = B_0[R\zeta + \sum_{m,n} \varepsilon_{mn} \frac{Rm}{n} I_m(\frac{nr}{R}) \sin(m\theta - n\zeta)]$$

  - $z = R\zeta$, I_m = modified Bessel function
  - Shape and spectrum of magnetic fields controlled by choice of ε_{mn}.

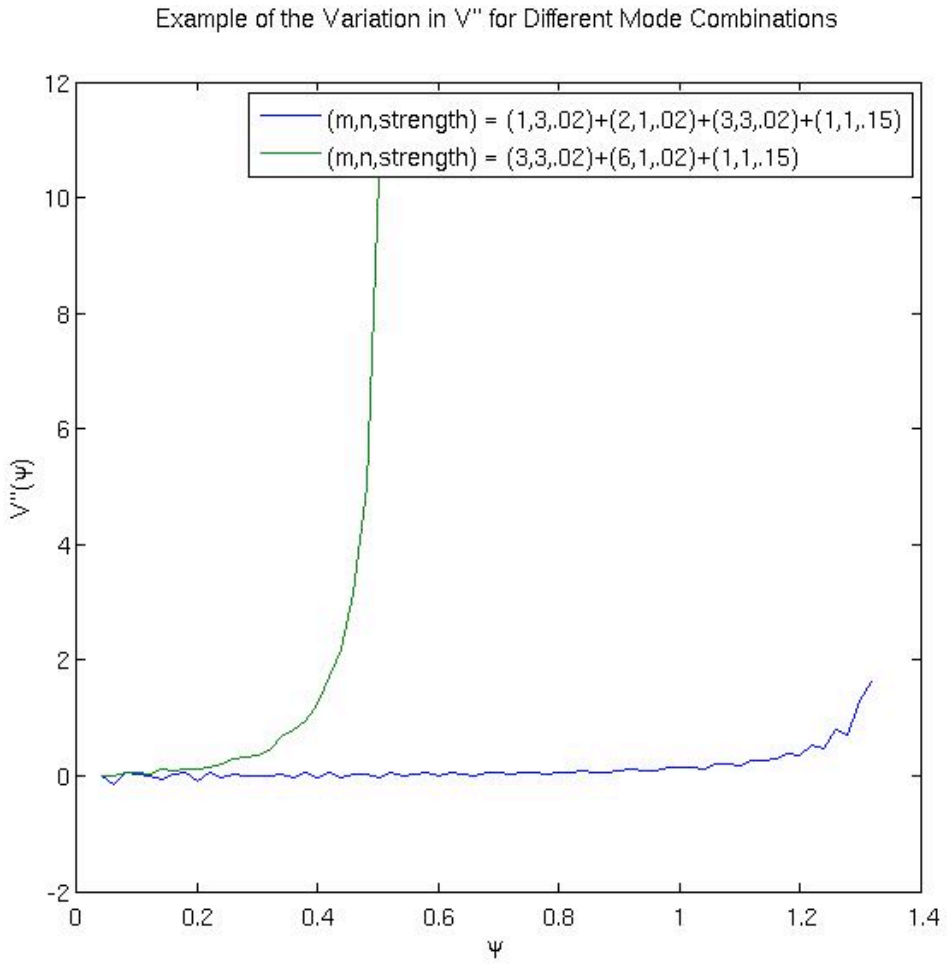
$$\iota \cong \sum 2mn^3 \frac{\varepsilon_{mn}^2}{R^2}[\frac{1}{u}(\frac{I_m I_m'}{u})'] \sim \sum \hat{\iota}_{mn}(\frac{r}{a})^{2m-4}$$

    - m = 1 rotating, helical axis, m = 2 nearly shearless, m =3 has shear but no transform on axis, etc.

By using different combinations of magnetic harmonics, you can construct equilibria with different properties

- Ability to adjust various equilibrium properites

- V’’enters as an Important parameter in resistive stability and isolated magnetic island calculations.



Solutions of the vacuum magnetic field are obtained and loaded into NIMROD

- The analytic vacuum solution is coded in straight cylinder geometry on NIMROD
  - New input variables were created to describe the strength of each helical harmonic. Any number of mode combos can be used.
  - A new subroutine was written (in physics_init.f) to cycle through all blocks, all cells and mode combinations. The routine calculates the field at each location, performs a fft and does a lagr_quad_basis_assign_loc to load the field.
  - At the grid boundary, the normal component of the magnetic field is NOT updated.
  - Implemented in NIMROD --- input vacuum field produced curl-free, divergence free magnetic field. Stable plasma state.

Good flux surfaces are initially formed.

Example: Poincare plots for l=3,m=1 helically symmetric magnetic field. These surfaces are shown for t=0.

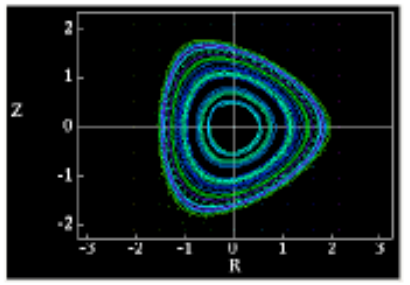


Figure 2: Poincare plot for l=3,m=1 at z = 0.

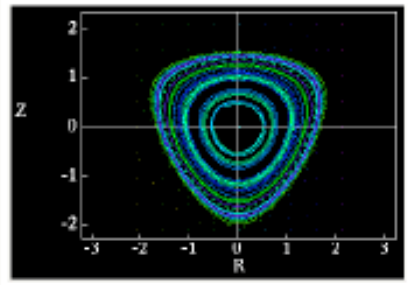


Figure 3: Poincare plot for l=3,m=1 at z = L/4.

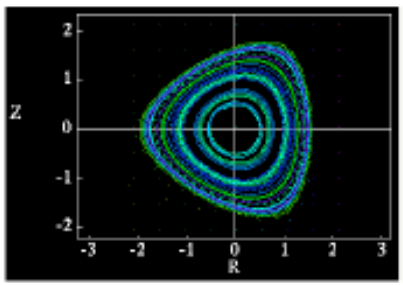


Figure 4: Poincare plot for l=3,m=1 at z = L/2.

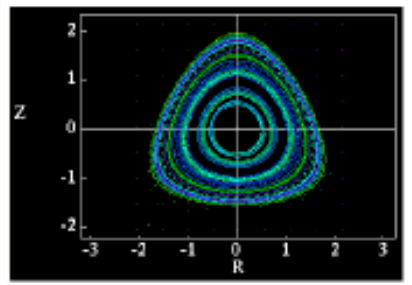


Figure 5: Poincare plot for l=3,m=1 at z=3L/4.

Good flux surfaces are seen to persist in initial NIMROD runs.

Example: Poincare plots for l=3,m=1 helically symmetric magnetic field. This initial vacuum magnetic field is then perturbed with a small shear Alfven wave. These surfaces are shown for t= one resistive time.

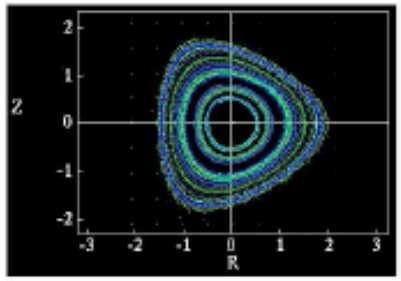


Figure 6: Poincare plot for l=3,m=1 at z = 0.

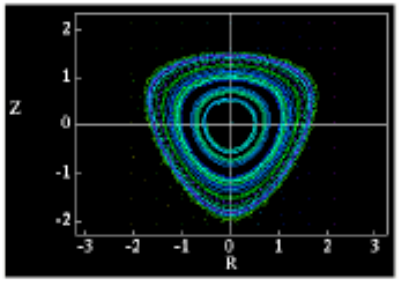


Figure 7: Poincare plot for l=3,m=1 at z = L/4.

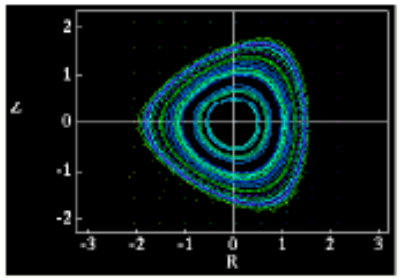


Figure 8: Poincare plot for l=3,m=1 at z = L/2.

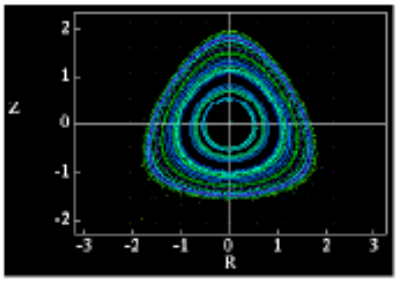


Figure 9: Poincare plot for l=3,m=1 at z=3L/4.

Helically symmetric MHD equilibria can be generated

- 2-D solutions are available --- equilibrium fields are functions of two variables, r and u = mθ – nζ. In vacuum,

$$\vec{B} = B_0[R_0\zeta + \frac{nR_0}{m}\sum_{k=1}\varepsilon_k I_{km}(\frac{knr}{R_0})\sin(ku)]$$

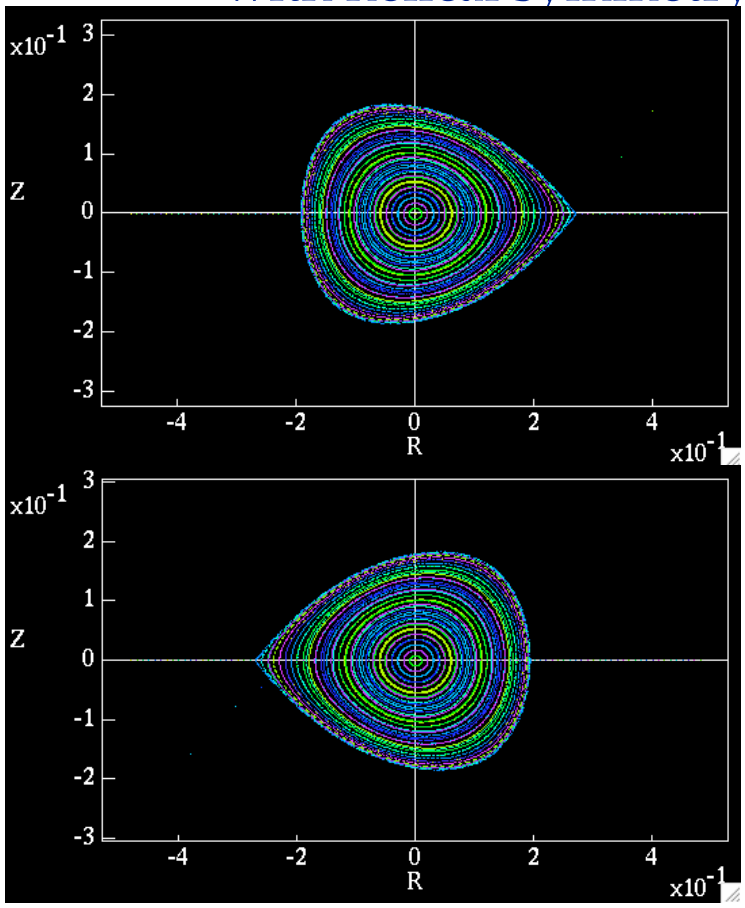
  - Grad-Shafranov equation describes equilibrium

$$\Delta_h\psi = \frac{2nm/R_0}{m^2 + n^2r^2/R_0^2} f(\psi) - f\frac{df}{d\psi} - (m^2 + n^2r^2/R_0^2)\frac{dp}{d\psi},$$

$$\Delta_h = \frac{m^2 + n^2r^2/R_0^2}{r}[\frac{\partial}{\partial r}\frac{r}{m^2 + n^2r^2/R_0^2}\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial^2}{\partial u^2}]$$

For initial studies, two example 'equilibria' are employed

- Initially, two vacuum configurations are considered --- one with helical symmetry, one perturbed by a 3-D component



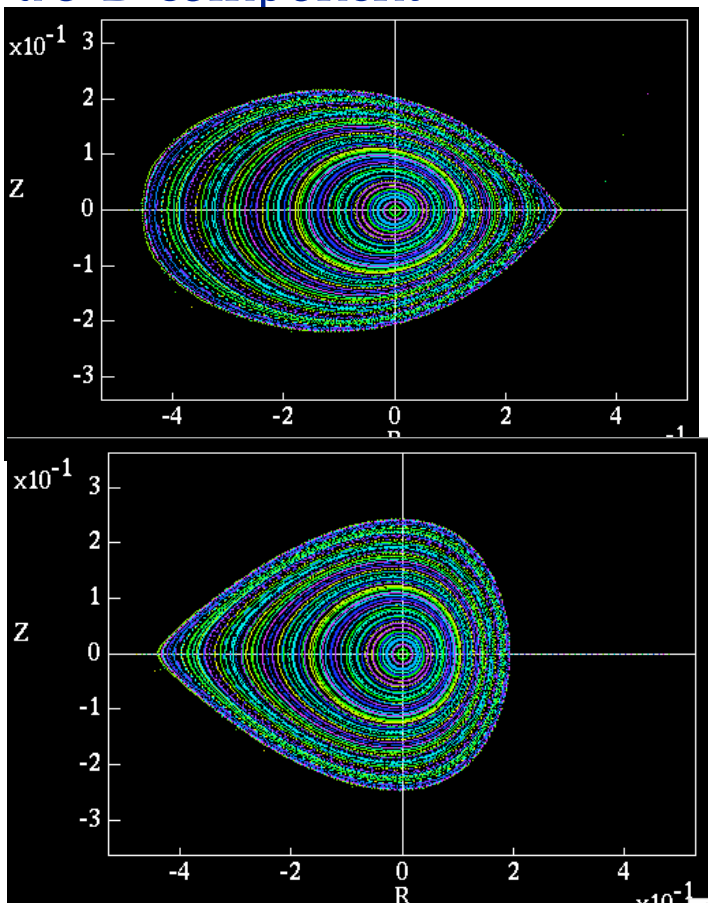
Flux surfaces at t = 0,

<- z = 0 - >

<- symmetric

w/ 3-D field ->

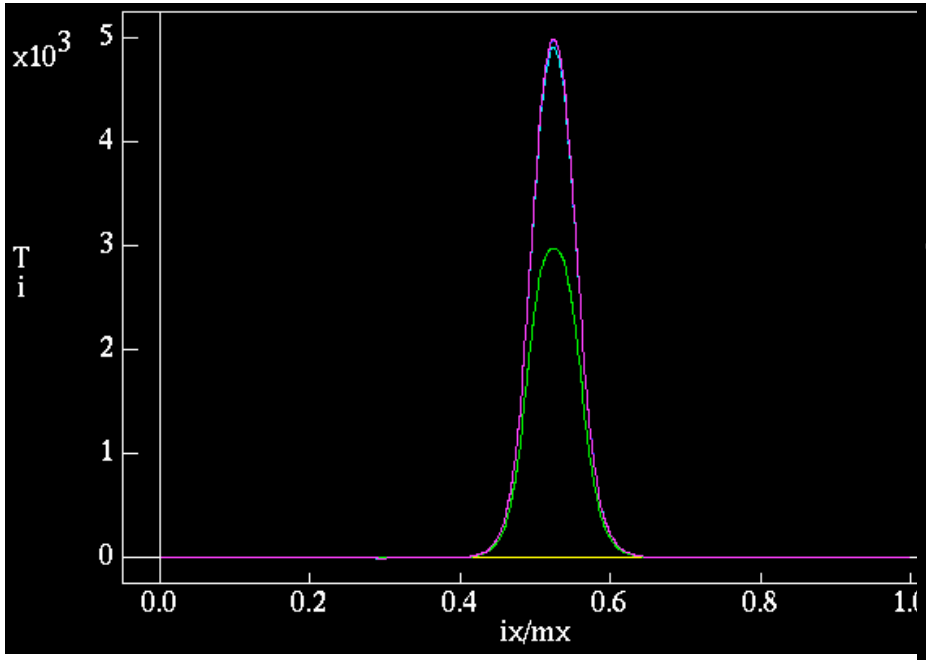
<- z ≠ 0 ->



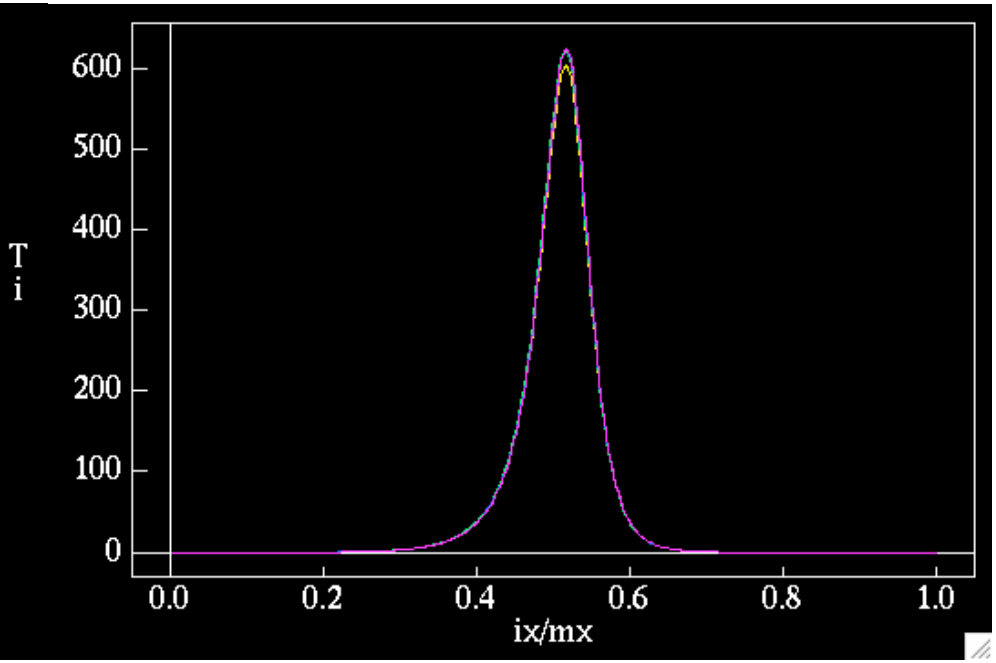
By adding a heating source, finite beta plasmas are generated

- Proscribed heating source and transport coefficients produce a temperature profile

Symmetric vacuum



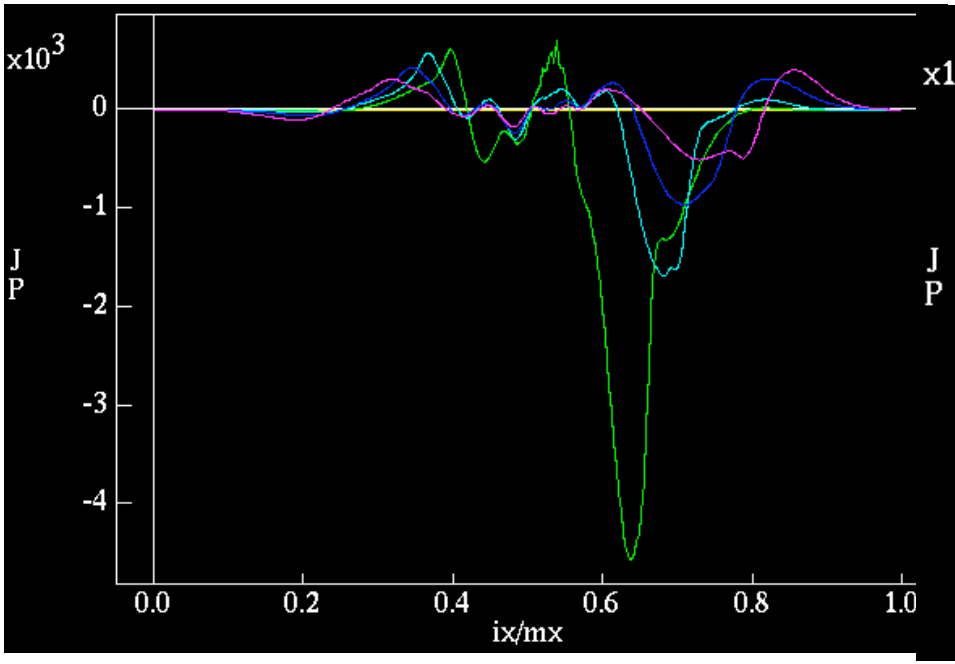
3-D vacuum



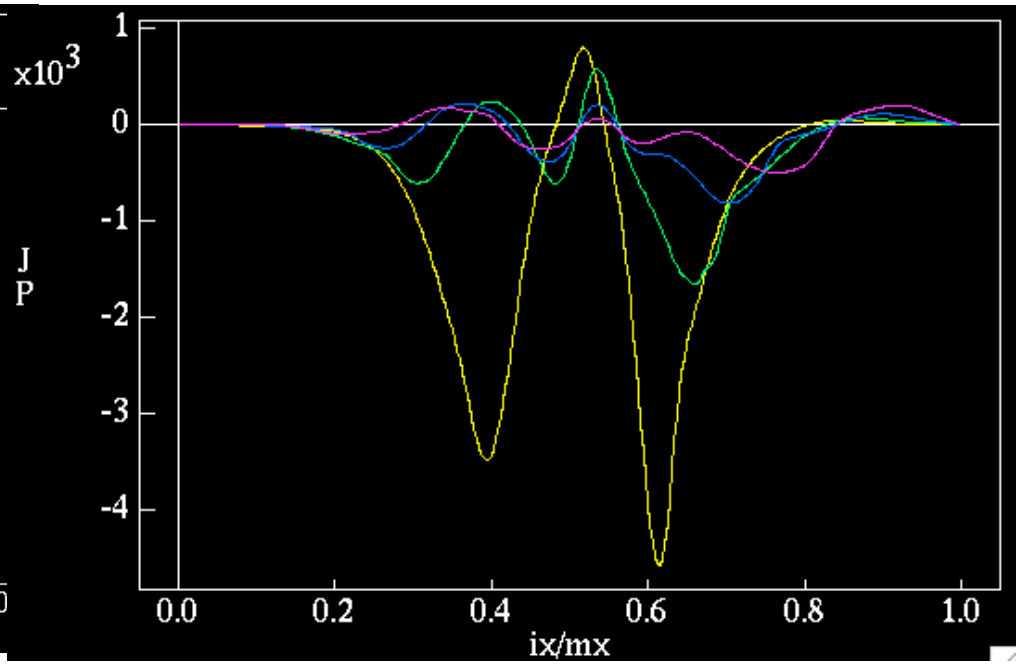
Self-consistent currents are calculated

- In equilibrium, currents are to diagmagnetic and Pfirsch-Schluter effects. Transients apparent in the simulation

Symmetric vacuum



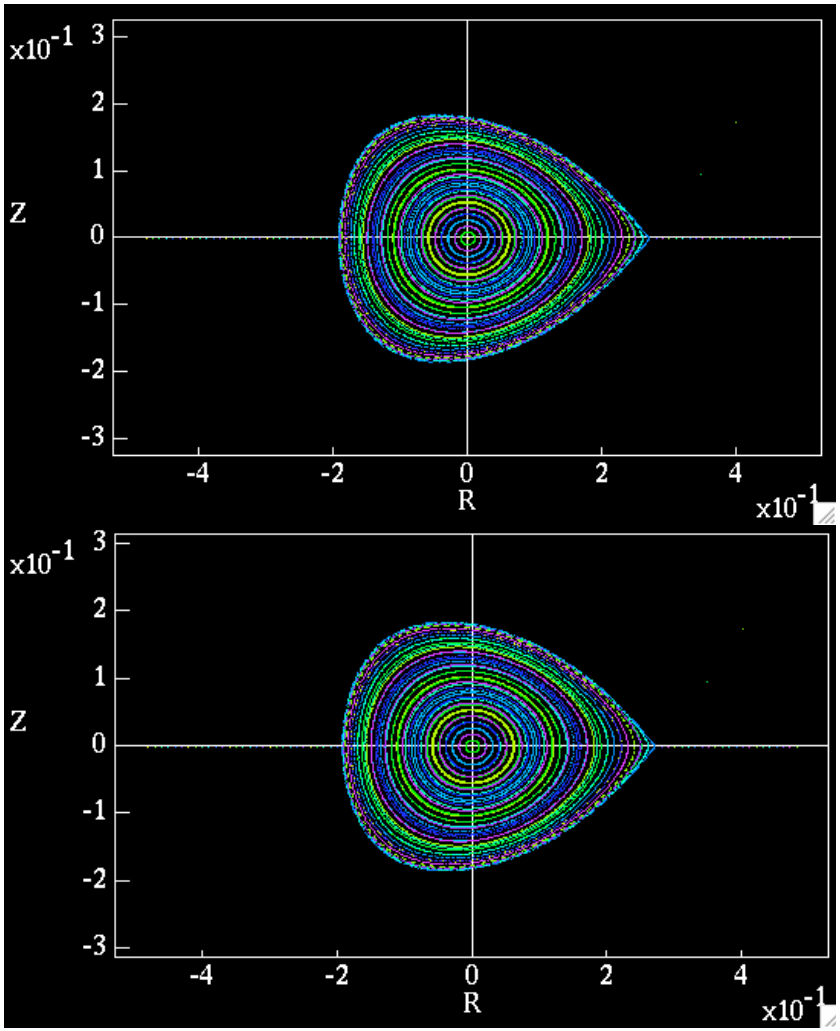
3-D vacuum



Initial simulations are low-β; not a discernable effect on the equilibrium flux surfaces

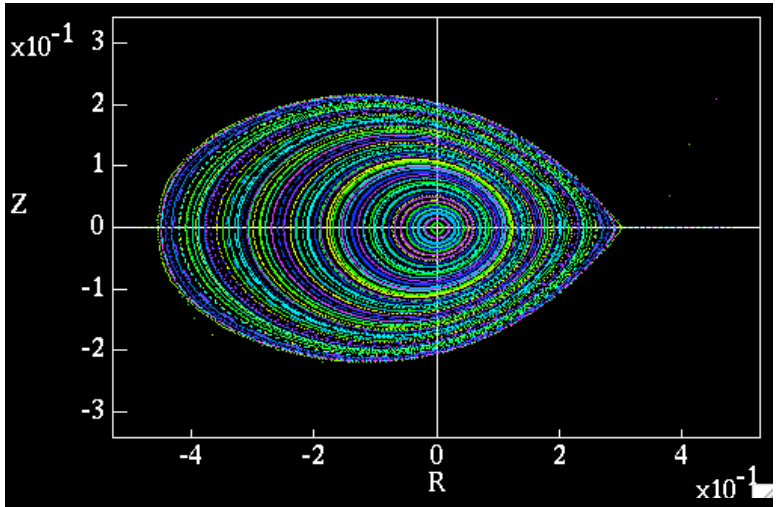
2-D

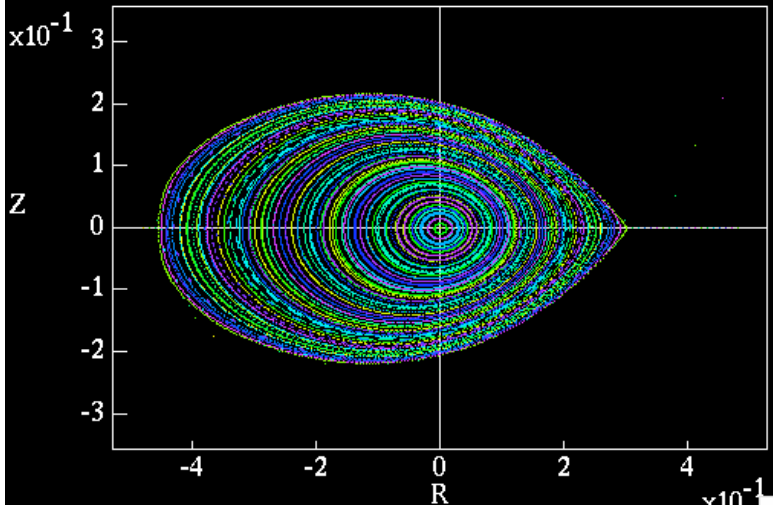
3-D



t = 0

t = 800





Future Plans

- Consistency of computed heated symmetric solutions with Grad-Shafranov theory.
- Lower dissipation
- Come up with a ‘standard’ heated symmetric base case which then can be perturbed.
  - Contstruct sequences of equilibria which vary rotational transform, gradient, V’’ strength, etc.
- Temperature dependent transport coefficients.
- Resolution issues. Incorporate mixed finite elements.
- Multiple helicity cases with heating. Understand the role of transport physics on pressure induced islands/stochasticity, etc.
- Incorporate Held’s Chapman-Enskog-like closures.

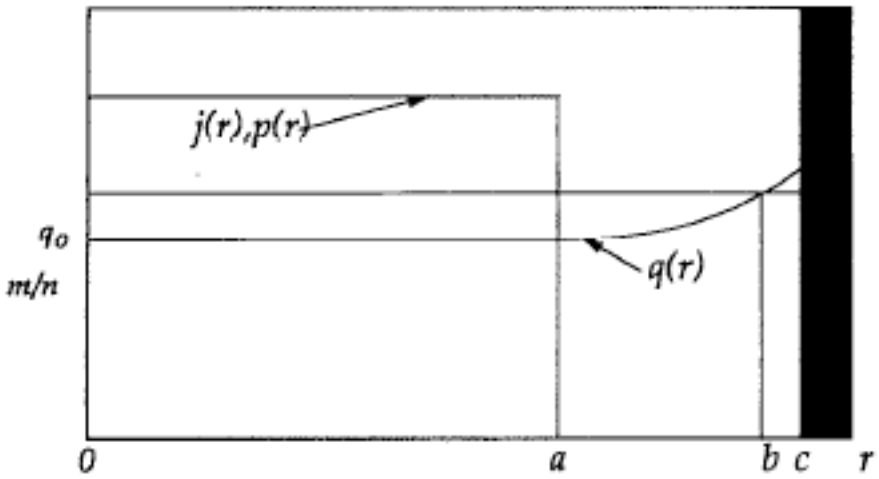
Initial Studies of RWM, field error problems in a periodic cylinder --- A. Montgomery et al

- Motivation --- further the understanding of high-β tokamak physics in the presence of resistive walls, and field errors.
- Initially, a simple starter problem
- Eventually, high beta tokamak cases with plasma rotation, resistive wall and field error effects, non-zero resistivity (kink and tearing mode physics)

A starter problem has been identified

- Starter problem --- articulated in J. Finn, PoP (1995).
  - Reduced resistive MHD, periodic cylinder
  - flat current and pressure profile for r ≤ a, diffuse plasma for a < r ≤ c).
  - Thin resistive wall at r = c
  - Rigid rotation

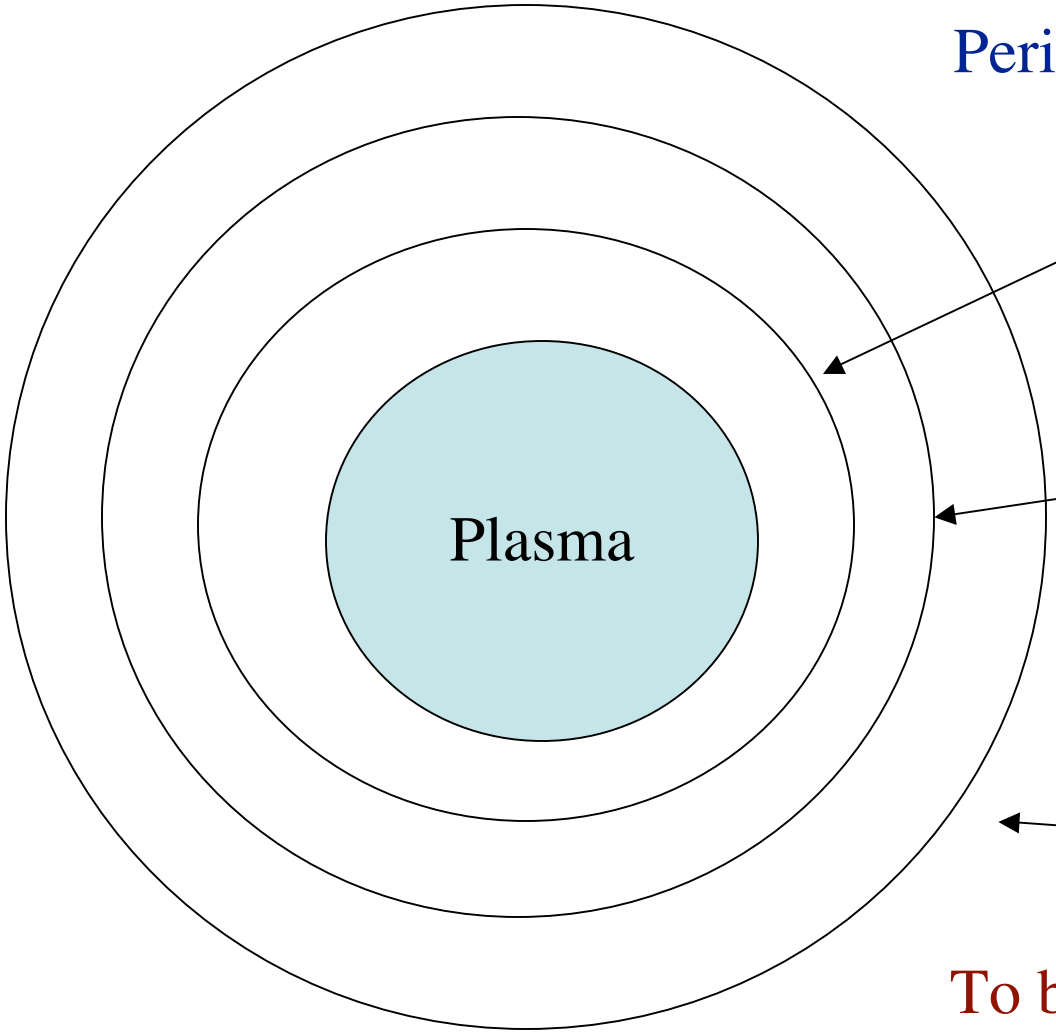
  --> Results obtained analytically



Transition between ideal to resistive mode is emphasized

- Results for a cylinder:
  - The existence of a RWM that cannot be stabilized by rigid plasma rotation in a cylinder depend criticially on the lack of mode rational surfaces in the plasma.
  - Modes with rational surfaces in the plasmas are stabilized just below the ideal threshold by becoming resistive tearing modes. (Connection between ideal and resistive plasma modes.)
  - Tearing modes can be stabilized in the presence of a resistive wall and rotation
- Speculation that a similar effect occurs in a toroidal plasma--- yet to be explored

Starter problem geometry



Periodic cylinder geometry

Thin resistive wall

Error field

$$\delta\vec{J} \sim I(t)\delta(r-d)e^{im\theta-in\zeta}$$

Conducting shell

To be implemented by Andi
with Scott K.'s and A. Cole's help