Island suppression simulations for SWIM
(physics results — a work in progress)

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NIMROD team meeting August 28, 2008 San Diego, CA

Previously, I've demonstrated that the addition of an ad hoc source term to the right–hand side of the MHD Ohm's law (to mock up the lowest–order effect of RF (ECCD) interaction with the MHD equations) can significantly affect the saturation amplitude of low–order resistive tearing modes. Here, I begin to quantify the physical mechanisms (Δ' and current profile modification by the RF) involved in this process. As well, I attempt a mathematical description (which still needs some work) of the "cartoon physics" model of RF/MHD interaction presented by Carl Sovenic at the December 2007 SWIM meet-up, hoping to capture physics related to the equilibration of RF effects over flux surfaces.

Goal for our part of SWIM — numerically simulate ECCD stabilization of NTM's

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• Experimental efforts to stabilize neoclassical tearing modes (NTM's) via electron cyclotron current drive (ECCD) have been very successful; R. J. La Haye [Phys. Plasmas **13**, 055501 (2006)] gives a detailed overview and many references

• For ECCD, the RF–induced current is relatively small [of the same order as the current driven by the electric field] ⇒ small expansion parameter, so existing theory gets us a long way

• Add an RF term to the kinetic equation:

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla f_\alpha + \frac{q_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} = C(f_\alpha) + Q(f_\alpha)$$

• $Q(f_\alpha)$ is a gyrophase–averaged quasilinear diffusion operator

$$Q(f_\alpha) \equiv \frac{\partial}{\partial \mathbf{v}} \cdot \mathcal{D} \cdot \frac{\partial}{\partial \mathbf{v}}$$

where the diffusion tensor $\mathcal{D}$ arises from the RF source.

• $C(f_\alpha)$ is the gyrophase–averaged Fokker–Planck Coulomb collision operator

RF terms appear in the fluid equations

- Taking fluid moments in the conventional manner yields

$$\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha \mathbf{v}_\alpha) = 0 \qquad \text{(RF produces no particles)}$$

$$m_\alpha n_\alpha \left(\frac{\partial \mathbf{v}_\alpha}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla)\mathbf{v}_\alpha \right) = n_\alpha q_\alpha (\mathbf{E} + \mathbf{v}_\alpha \times \mathbf{B}) - \nabla p_\alpha - \nabla \cdot \pi_\alpha + \mathbf{R}_\alpha + \mathbf{F}^{rf}_{\alpha 0}$$

$$\mathbf{F}^{rf}_{\alpha 0} \equiv \int m_\alpha \mathbf{v} Q(f_\alpha) d\mathbf{v} \qquad \text{(additional momentum imparted by RF waves)}$$

$$\frac{3}{2} n_\alpha \left(\frac{\partial T_\alpha}{\partial t} + (\mathbf{v}_\alpha \cdot \nabla) T_\alpha \right) + n_\alpha T_\alpha \nabla \cdot \mathbf{v}_\alpha = -\nabla \cdot \mathbf{q}_\alpha - \pi_\alpha : \nabla \mathbf{v}_\alpha + Q_\alpha + S^{rf}_{\alpha 0}$$

$$S^{rf}_{\alpha 0} \equiv \int \frac{1}{2} m_\alpha v^2 Q(f_\alpha) d\mathbf{v} \qquad \text{(additional energy imparted by RF waves)}$$

- Small expansion parameter allows $Q(f_\alpha) \sim Q(f_{M\alpha})$, where $f_{M\alpha}$ is a local Maxwellian (i.e. it has spatially varying velocity, temperature, and density profiles).

Understanding basic physics without doing the whole problem

- At this point, one has to worry about closures, small expansion parameters, etc. Ultimately, we get a set of self–consistent fluid equations for RF–influenced MHD.
- While work proceeds on that front, use simpler models (e.g. looking at resistive, rather than neoclassical, tearing modes) to understand the physics.
- Consider the electron momentum equation ($\Rightarrow$ MHD Ohm's law). The RF–induced momentum contributes an additional term;

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} = \eta \mathbf{J} - \frac{\mathbf{F}_{\mathrm{rf}}}{e}$$

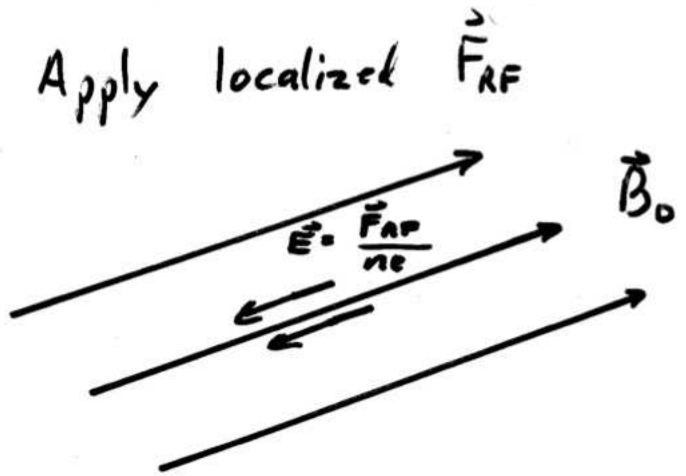
For dimensionless function $f(\mathbf{x}, t)$ and amplitude λ_{rf} (units of inverse length), let the RF term have the form

$$\frac{\mathbf{F}_{\mathrm{rf}}}{e} = \frac{\eta \lambda_{rf} \mathbf{B} f(\mathbf{x}, t)}{\mu_0}$$

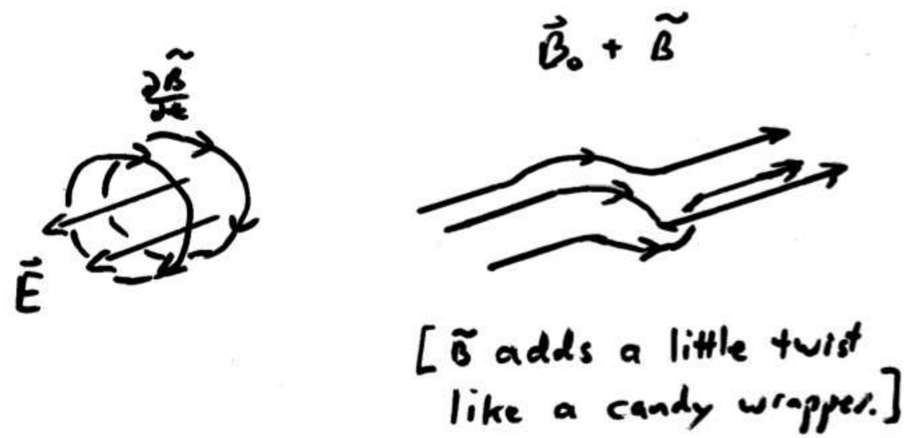
- What is the effect of a spatially localized $f(\mathbf{x}, t)$? Tackle it heuristically, first.

Carl's "cartoon physics" model

Effects of F_{RF} Source: Incomplete Cartoon Physics

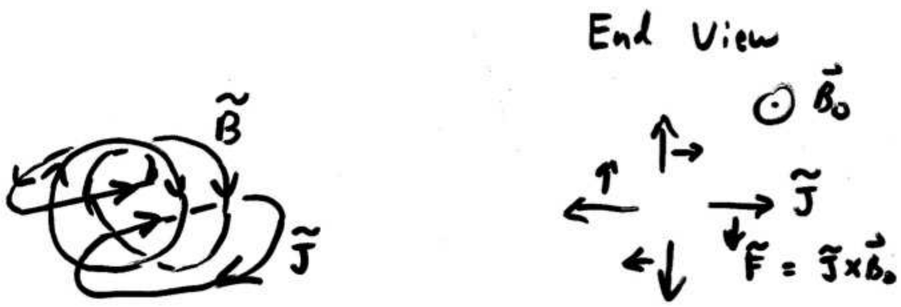


Incomplete Cartoon Physics 2

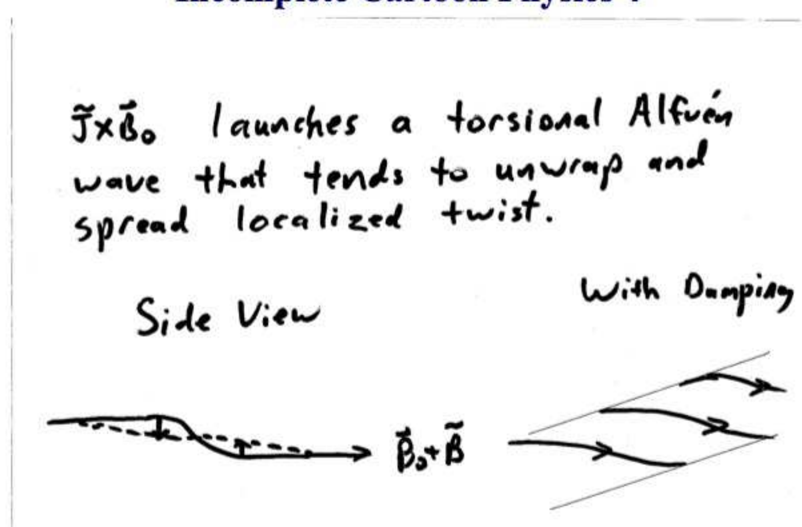


Carl's "cartoon physics" model, continued...

Incomplete Cartoon Physics 3



Incomplete Cartoon Physics 4



Can we capture any extra physics (e.g., equilibration) in an analytic model? — Cylindrical MHD with simple closure & RF

- Consider a "textbook model" of MHD, but add RF effects:

$$\nabla \cdot \mathbf{B} = 0 \tag{1}$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \tag{2}$$

$$\nabla \times \mathbf{B} - \mu_0 \mathbf{J} = 0 \tag{3}$$

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} - \eta \mathbf{J} + \frac{\eta \lambda_{rf} f \mathbf{B}}{\mu_0} = 0 \tag{4}$$

$$\frac{\partial \rho}{\partial t} + \mathbf{u} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{u} = 0 \tag{5}$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p - \mathbf{J} \times \mathbf{B} = 0 \tag{6}$$

$$\frac{\partial p}{\partial t} + \frac{5}{3} p \nabla \cdot \mathbf{u} = 0 \tag{7}$$

- Use bounded cylindrical geometry (radius a), periodic in z with length L.

- A simple equilibrium is $\mathbf{B} = B_0 \hat{z}, \mathbf{u} = 0$, $f = 0$, and constant p, ρ for $r < a$.

What do sudden, spatially localized RF perturbations do?

- Linearize, but assume that first–order perturbations are zero until $f(\mathbf{x},t) = f(\mathbf{x})\eta(t)$ (where $\eta(t)$ is the step function) turns on. Use a Fourier–Bessel series:

$$f(r,\theta,z,t) = f(r,\theta,z)\eta(t) = \sum_{l=1}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} f_{l,m,n} e^{im\theta} e^{2\pi inz/L} J_m\left(\frac{r\lambda_{l,m}}{a}\right)\eta(t)$$

where $\lambda_{l,m}$ is the lth zero of the Bessel function $J_m(x)$.

- The RF source launches shear–Alfven waves (incompressible, transverse):

$$\mathbf{B}_1 = \nabla\psi_1 \times \hat{z} \qquad \mathbf{u}_1 = \nabla\phi \times \hat{z} \qquad p_1 = 0 \qquad \rho_1 = 0$$

$$\psi_1 = \sum_{l=1}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \frac{a^2\lambda_{rf} B_0 f_{l,m,n}}{\gamma_{l,m,n}\tau_R} e^{im\theta} e^{2\pi inz/L} J_m\left(\frac{r\lambda_{l,m}}{a}\right) e^{-\Omega_{l,m,n}t}\sin(\gamma_{l,m,n}t)\eta(t)$$

$$\phi_1 = \sum_{l=1}^{\infty}\sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} \frac{a^2 L\lambda_{rf} i f_{l,m,n}}{2\pi n\gamma_{l,m,n}\tau_R} e^{im\theta} e^{2\pi inz/L} J_m\left(\frac{r\lambda_{l,m}}{a}\right)\eta(t)\cdot \left\{\gamma_{l,m,n}[1 - e^{-\Omega_{l,m,n}t}\cos(\gamma_{l,m,n}t)] - \Omega_{l,m,n} e^{-\Omega_{l,m,n}t}\sin(\gamma_{l,m,n}t)\right\}$$

where

$$\gamma_{l,m,n} = \sqrt{n^2k^2v_A^2 - \Omega_{l,m,n}^2} \qquad k = 2\pi/L$$

$$\Omega_{l,m,n} = \frac{\lambda_{l,m}^2 + n^2k^2a^2}{2\tau_R} \qquad \tau_R = \mu_0 a^2/\eta$$

Most components of RF–induced magnetic field decay at long times, but some grow

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$$\psi_1 = \sum_{l=1}^{\infty} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{a^2 \lambda_{rf} B_0 f_{l,m,n}}{\gamma_{l,m,n} \tau_R} e^{im\theta} e^{2\pi inz/L} J_m\left(\frac{r\lambda_{l,m}}{a}\right) e^{-\Omega_{l,m,n} t} \sin(\gamma_{l,m,n} t) \eta(t)$$

• $\Omega_{l,m,n} \sim \tau_R^{-1}$, so exponential kills off magnetic field perturbations on the resistive timescale here (it should happen much faster than that (Alfven timescale?), so this model for RF still needs some work). However, $n = 0$ components don't decay, they grow:

$$\gamma_{l,m,0} \rightarrow i\Omega_{l,m,0} = \frac{i\lambda_{l,m}^2}{2\tau_R}$$

so that

$$\psi_1 = (n \neq 0 \; pieces) + \sum_{l=1}^{\infty} \sum_{m=-\infty}^{\infty} \frac{a^2 \lambda_{rf} B_0 f_{l,m,0}}{\lambda_{l,m}^2} e^{im\theta} J_m\left(\frac{r\lambda_{l,m}}{a}\right) [1 - e^{-\lambda_{l,m}^2 t/\tau_R}] \eta(t)$$

• At long times, ψ_1 is not a function of z; it has spread itself uniformly along the equilibrium field lines (which point in $\hat{z}$). (Equivalent to flux–surface equilibration in this geometry?)

How NIMROD is modified for physics studies

• In NIMROD, modify MHD Ohm's law by adding a poloidally Gaussian RF spot (I'm using toroidally symmetric RF perturbations for these runs, so it's really a "ring"):

$$\mathbf{E} + \mathbf{u} \times \mathbf{B} = \eta \mathbf{J} - \frac{\eta \lambda_{rf} \mathbf{B}}{\mu_0} \exp\left(- \frac{(R - R_{rf})^2 + (Z - Z_{rf}^2)}{w_{rf}^2} \right) \mathrm{erf}\left(\frac{t - t_{\mathrm{offset}}}{t_{\mathrm{build}}} \right)$$

where t_{build} is faster than a resistive timescale and slower than an Alfven timescale. R_{rf}, Z_{rf}, w_{rf} give location and size of RF hotspot.

• Begin with an equilibrium unstable to $(2, 1)$ resistive tearing mode, and examine effects of RF term on this mode — can markedly decrease the saturation width of magnetic islands

• Two physical mechanisms for stabilization — modification of Δ' by RF fields, or modification of current profile near rational surface by RF fields

• How do we test these effects?

Modification of Δ' by RF

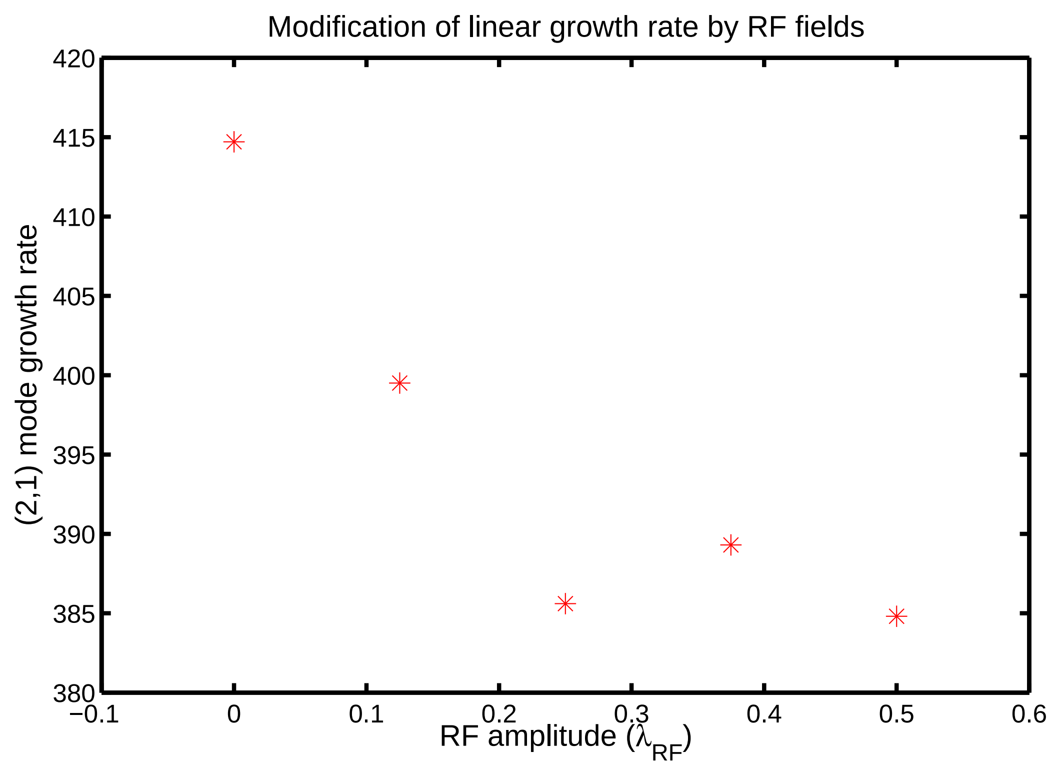
- To examine Δ' modification - allow only axisymmetric modes (lphi=0), and ramp up RF fields (which asymptotically approach a constant on the resistive timescale, such that a new equilibrium (original equilibrium + $n = 0$ RF perturbations) is ultimately obtained).

- When equilibrium is attained, change lphi=2 (using nimset to add modes to the $n = 0$ dumpfile) and find linear growth rate γ of $n = 1$ mode.

- $\gamma \sim (\Delta')^{4/5}$, so Δ' modifications should show up here. Current profile does not flatten during linear stage of mode growth.

- For small RF spot size relative to W_{sat} (the saturated island width in the absence of RF), the effect on the growth rate doesn't initially appear significant ($< 10\%$ variation, no clear trend in this dataset)

- Here, $w_{rf} = 0.037$ and $W_{sat} \sim 0.12$. RF is centered on the rational surface. Magnetic energy at saturation reduced by factor of $2 - 3$ in these runs (smaller islands).



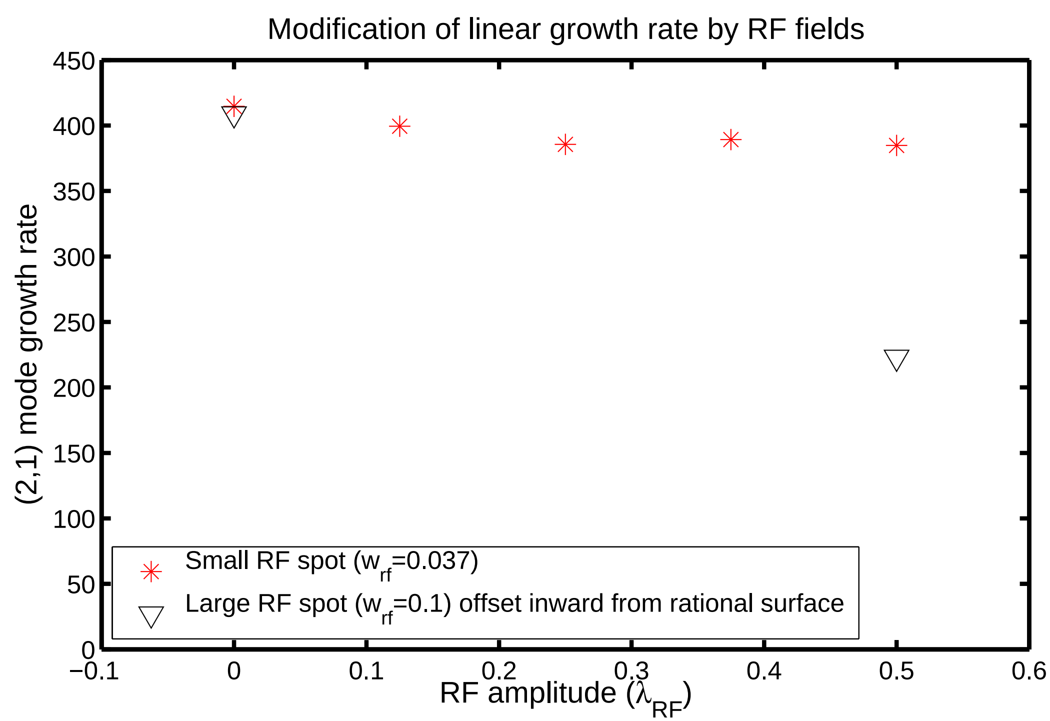
Modification of Δ' by RF — sometimes large!

• If the poloidal cross–section of the RF spot is increased so that it exceeds the saturated island size W_{sat}, the Δ' modifications become large.

• In the inverted triangle datapoints, the RF deposition is offset somewhat inward from the rational surface; simulations by Pletzer and Perkins [PoP **6**, 1589 (1999)] suggest that such an offset should *increase* Δ' — further reduction of Δ' appears possible

• Total current in larger RF source is increased by factor of $\sim$ 7 over small RF source — increased current at small RF spot sizes will likely have more pronounced effects (don't have good data on this yet)

• More data gathering in process (expensive datasets, due to need for RF–influenced n=0 equilibria)

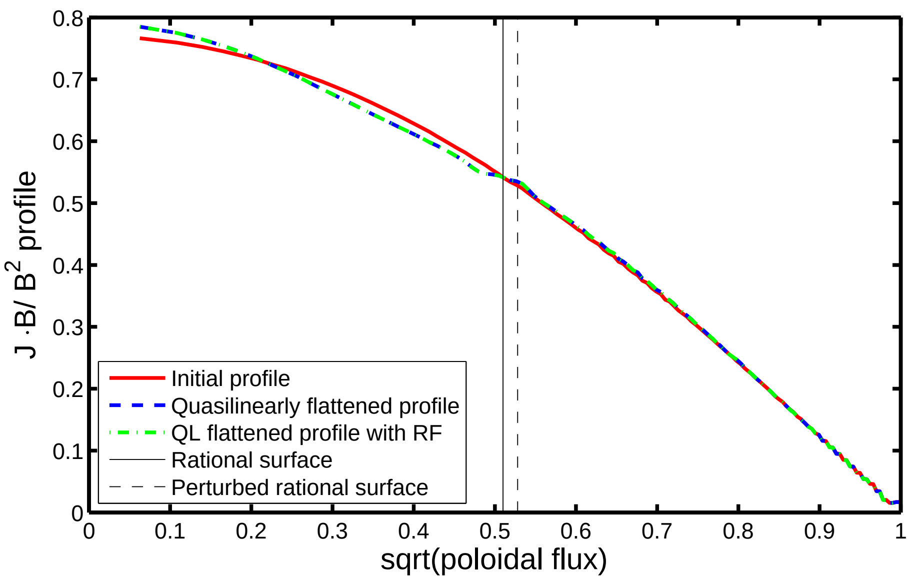


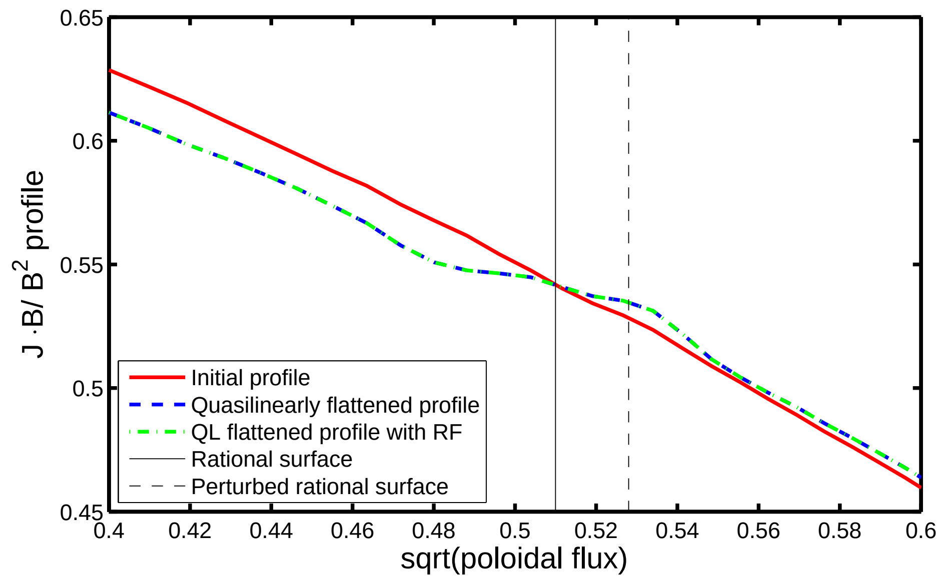
Current profile modification — quasilinear flattening

- In addition to Δ' modification (influencing linear growth rates), the parallel current profile $\mu \equiv (\mathbf{J} \cdot \mathbf{B})/|\mathbf{B}|^2$ can be influenced by RF — quasilinear flattening of the profile occurs as the mode saturates even without RF perturbations, but RF can modify the profile further

- Consider the wide RF spot from the previous slide (the inverted triangle), but now let the mode grow to saturation before turning on RF — then see how μ evolves as RF is added (for this simulation, it doesn't — Δ' modification is the key player here)

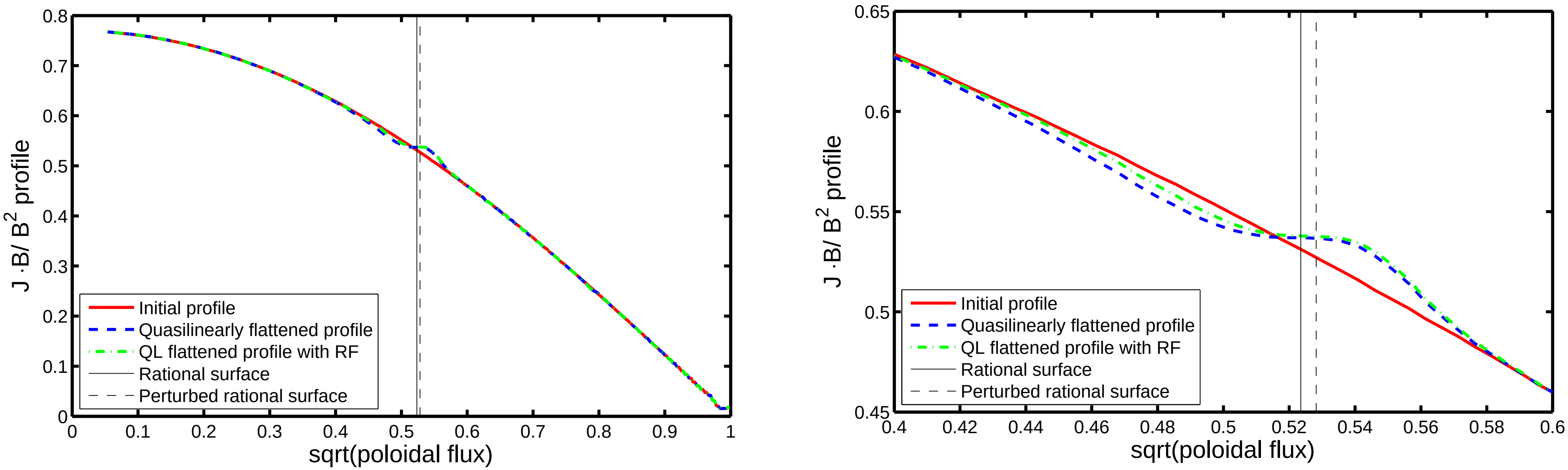
- Rational surface moves slightly outward at saturation — not a large effect for these simulations, but may influence optimum location for RF deposition (are we killing off existing islands, or tuning the RF to prevent them in the first place?)





Current profile modification — quasilinear flattening

- Now consider simulations not dominated by Δ' — small spot size ($w_{rf} = 0.037$), centered on rational surface
- Here, the RF does modify the μ profile inside the rational surface.

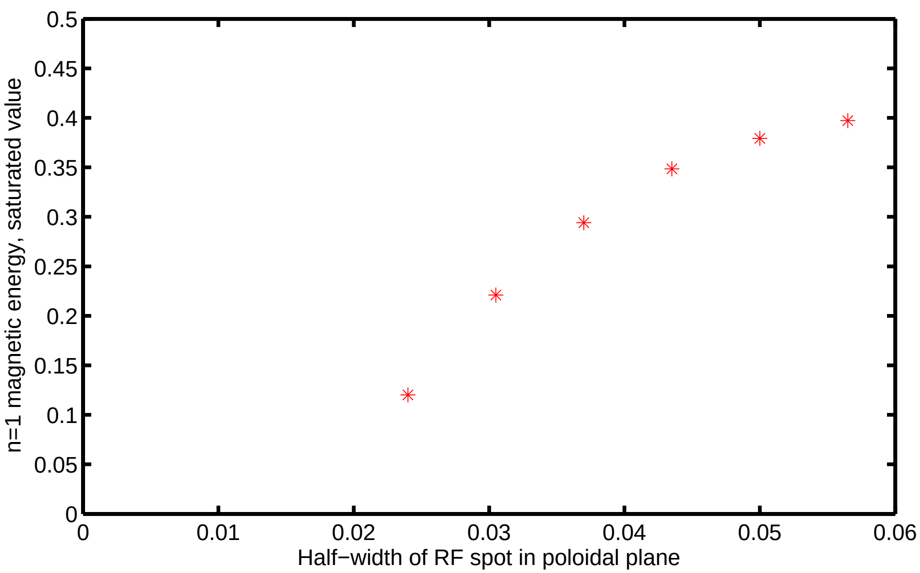


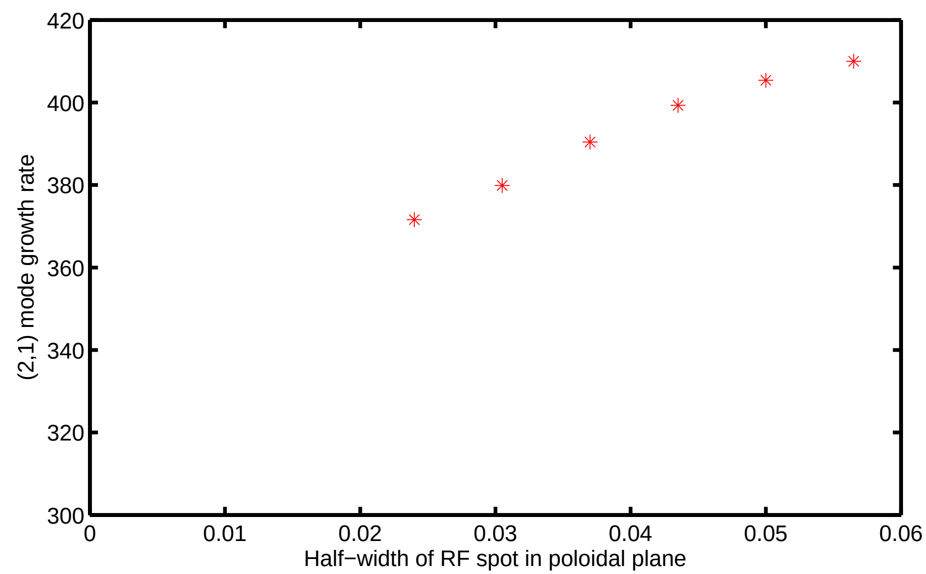
For fixed current, growth rates and island widths decrease with the RF spot size

• Integration of the Gaussian function over the poloidal plane yields a total current $\propto \lambda_{rf} w_{rf}^2$.

• By preserving the total RF–induced current, explore how RF spots of various widths centered at the rational surface affect mode growth rates and saturation amplitudes.

• Previously, showed that increasing total current in a fixed spot size reduced Δ' somewhat; here, preserving total current and increasing spot size increases Δ' and the saturated island width (agreeing with Hegna/Callen's 1997 PoP paper). $\Rightarrow$ localized RF is probably better.





Upcoming work

- Final debugging of diagnostic for magnetic island widths (with Scott Kruger) — will enable comparisons with Rutherford equation

- Further interfacing of NIMROD with GENRAY (ray tracing code) to replace ad hoc RF term with more accurate physics — complicated details given by Scott in yesterday's talk

- Larger–amplitude RF current simulations (to see if Δ' or μ modifications become more prominent). Can we prevent the mode from growing altogether? Can we predict which of the two effects will dominate in a given situation?

- Move to toroidally asymmetric RF perturbations, add plasma rotation and phase RF deposition, etc.