Dynamic Behavior of Peeling-Ballooning Modes in a Shifted-Circle Tokamak Equilibrium

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RMP Workshop

General Atomics, San Diego
• Edge Localized Modes (ELM’s) limit H-mode plasmas
  ✦ potentially very damaging for diverter and first wall region
    - *ITER expected to be limited by ELM’s*

• Linear, ideal MHD (ELITE) successful in explaining some of the experimental ELM observations
  ✦ ELM onset and growth has been linked to the coupling between ideal kink and ballooning modes (peeling-balloonning)
  ✦ Many of the unresolved questions require going beyond linear ideal MHD

For example:
  - How type 1 ELMs relate to type 3 ELMs? How do RMP’s stabilize ELM’s?
  - How does heat flux go to wall? How localized is the heat flux on the divertor? How does the linear stability correspond to the observed structures? (some filaments, some not, etc.) How does this scale to ITER?
Introduction & Motivation

• An extended MHD code like NIMROD is a natural extension of the ideal studies
  ✦ Can have additional physics:
    - **diffusivities**: resistivity, viscosity, thermal diffusivities
    - **2-fluid physics**: Hall terms, gyroviscosity, electron stress tensor
    - **closure physics**: parallel heat flux, gyrokinetics

• Improve the physical understanding of ELM’s.
  ✦ understand how ELM & RMP effects will scale from present devices to next step fusion devices

• Complication of the problem motivates a nonlinear (NL) study
Introduction & Motivation

- Two step approach to NL dynamics in NIMROD:
  - 1) Detailed linear stability analysis of peeling-ballooning modes
    * Reproduce established linear ideal MHD results (ELITE) in NIMROD
    * Ensure an accurate representation of the physics in the code
  - 11) Phenomenology of nonlinear (NL) peeling-ballooning evolution
    * direct comparison to recent intermediate NL ballooning theory (P. Zhu & C.C. Hegna, to be published in P.oP. 2008)
    * develop analytic NL peeling-ballooning edge-specific theory, compare results from NIMROD
    * establish a framework for accurately representing the physics in NL simulations of ELM’s & RMP’s
General Outline

• Single Linear case examined as precursor to NL studies

• “Ideal-like”/ “Halo” defined in NIMROD

• Technique developed to isolate ballooning and kink drives

• Preliminary nonlinear results guide future analysis

• Summary
Single linear case used as a precursor for nonlinear studies

- \( R_0 = 3\text{m}, a=1\text{m} \)
- \( B_0 = 2\text{T} \)
- \( \beta_{\text{to}} = .005 \)
- \( n = 1.06 \times 10^{20}(\text{m}^{-3}) \)
- Modified TOQ
  - currents in edge region set to 0
  - minimizes numerical errors (no separatrix)
  - pedestal region \(~67-75\text{cm}\) on midplane

TOQ-generated shifted-circle tokamak equilibrium
~S. Kruger & P. Snyder

Equilibrium Flux Surfaces
Equilibrium profiles show potential peeling-ballooning instability

- Steep pressure gradients drive ballooning modes (DCON)
  - Pedestal width twice experimental value, simplify vacuum transition region
- Self-consistent edge currents & \( 2 < q_{\text{edge}} < 5 \) to provide increased kink drive
  - comparable to ballooning drive

\[
\begin{align*}
\beta_t & = 0.005 \\
\frac{< J \cdot B >}{B^2} & = 0.005 \\
\rho & = 0.005
\end{align*}
\]
Detailed “ideal” study in NIMROD

- Carry out ideal and halo in NIMROD with the intent of using the halo placement to “dial in” kink/ballooning drive

  1) kink: halo region just outside pedestal
  2) ballooning: halo region far from pedestal
Halo region defined with an imposed resistivity transition

• Defined in NIMROD by creating a resistivity transition region
  * transitions from low, “ideal” to a large value at a specified $\rho_{\text{Halo}}$

• Transition defined using a tanh function as an $\eta$ multiplier

![Graph showing the transition from low to high resistivity with different $\Delta \eta_{\text{width}}$ values.](image)

- $\rho_{\text{Halo}} = 0.8$
Quantifying “Ideal” in NIMROD requires high spatial resolution

- Start with pure ideal
  - resistivity $S = \infty$ everywhere
    - no halo region

- linear ideal MHD, $n = 12$
- no dissipation in system
- $k_{\text{visc}}, k_{\text{perp}} = 0$

n=12 mode
$V_n$ eigenfunction
Lundquist scans define critical, “ideal-like” value in NIMROD

- systematically decreased $S$ from $\infty$
- critical value defined, below which plasma behaves ideally

$n = 12, k_\nu = 0, k_\perp = 0$

$S_{\text{crit-ideal}} \sim 5 \times 10^7$

Ideal Plasma behavior in NIMROD
Beyond a critical halo-resistivity the physics doesn’t change

- similar to $S_{\text{crit-ideal}}$, we want to define $S_{\text{crit-Halo}}$
- increase vacuum resistivity until no effect is produced
- $\eta_{\text{crit-Halo}} \sim 1-10$ (Ω•m)
- $S_{\text{crit-Halo}} \sim 0.5$
- $S_{\text{crit-ideal}} / S_{\text{crit-Halo}} \sim 10^8$

\[\rho_{\text{ped}} = 0.75\]
\[\rho_{\text{vac}} = 0.84\]
Introduction of a halo region doesn’t affect $S_{\text{crit-ideal}}$

$\rho_{\text{ped}} = 0.75$
$\rho_{\text{vac}} = 0.84$

$S_{\text{in}} \sim 5 \times 10^7$
$S_{\text{in}} \sim 5 \times 10^8$
$S_{\text{in}} \sim 5 \times 10^9$

$\eta_{\text{out}} \sim 10^{-1}$

$\eta_{\text{out}} \sim 10^{-2}$

$\gamma \tau_a$

$n$
Lundquist/Resistivity ratio is not a good characterization parameter

\[ \frac{\rho_{\text{ped}}}{\rho_{\text{vac}}} = 0.75 \]

\[ \frac{\rho_{\text{ped}}}{\rho_{\text{vac}}} = 0.84 \]

\[ \eta_{\text{out}} / \eta_{\text{in}} \sim 10^8 \]

\[ S_{\text{in}} / S_{\text{out}} \sim 5 \times 10^7 \]

\[ S_{\text{in}} / S_{\text{out}} \sim 5 \times 10^8 \]

\[ S_{\text{in}} / S_{\text{out}} \sim 5 \times 10^9 \]

**Note:** the two criteria, \( S_{\text{in}} \) and \( S_{\text{out}} \), must be simultaneously and separately satisfied

\[ \gamma \tau_a \]

\[ \eta_{\text{out}} / \eta_{\text{in}} \]

\[ n \]

\[ \rho_{\text{ped}} = 0.75 \]

\[ \rho_{\text{vac}} = 0.84 \]
Kink & ballooning drives are adjusted within a single equilibrium

- Developed a technique where relative rates of ballooning / kink drive are changed by adjusting the location of the halo region relative to the plasma pedestal region
  - *actual NIMROD calculations*

**No Halo: Ballooning Dominant Spectra**

<table>
<thead>
<tr>
<th>n</th>
<th>γ_{mag}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
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</tbody>
</table>

**Halo: Increases Low-n Kink Drive**

<table>
<thead>
<tr>
<th>n</th>
<th>γ_{mag}</th>
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<tbody>
<tr>
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<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>
Halo location relative to the q rational surfaces affects instability drives.

- adjusting the halo location “dials in” kink, ballooning, & peeling-ballooning behavior
- using q profile identify mode rational surfaces

Pedestal Region

sqrpsi

1.0
0.8
0.6
0.4
0.2
0.0

0.76
0.74
0.72
0.70
0.68
0.66
0.64
0.62
0.60

6000
4000
2000
0

11/4 surface
8/3 surface

q
P

2.6
2.8
3.0
2.4
2.2
2.0

0.68
0.70
0.72
0.74
0.76

sqrpsi
Halo region scanned relative to pedestal & mode rational surfaces

\[ \eta \text{ multiplier} \]

\[ \begin{align*}
\text{\( \rho_{\text{ped}} \)} & \quad \text{\( \rho (8/3) \)} \\
\text{\( \rho (11/4) \)} & \quad \text{\( \rho_{\text{vac}} = 0.751 \)}
\end{align*} \]
Low-n modes are sensitive to location of halo transition

- When $\rho_{\text{ped}} < \rho_{\text{Halo}} < \rho_{\text{rat}} = 8/3$
  n=3 kink mode is driven

- compared to ideal spectrum:
  more clearly ballooning dominant

- convergence challenging
Eigenfunctions have typical peeling-ballooning structure

- $n=12$ Halo-free mode structure, ballooning
- $n=3 \ \rho_{\text{vac}}=0.751$ mode structure, peeling-ballooning
In addition to the linear, began preliminary NL calculations in NIMROD.

Purely a demonstration of technique:

- $S_{\text{crit}}$: $S_{\text{in}} \sim 5 \times 10^5$
- not resistivity independent halo: $\eta_{\text{out}} \sim 10^{-2}$ ($\Omega \cdot \text{m}$)
  - $S_{\text{out}} \sim 5 \times 10^2$
- $\rho_{\text{vac}} = 0.84$
- calculation grid points not packed

Used to:

- guide future studies
- use results to design analysis tools
  - develop method to estimate transition between NL stages
  - determine growth regime to compare with analytic studies
- (P. Zhu & C.C. Hegna, to be published in P.oP. 2008)
Linear $n=9$ eigenmode used to excite NL growth

- 22 modes included: $n=0$-21
- initialized with linear $n=9$

- nonlinear beating expected to produce $n=0$ & $n=18$ mode growth at twice linear $n=9$ rate.
Summary

• Currently developing/documenting detailed linear peeling-ballooning analysis in NIMROD
 ✦ Defined critical Lundquist values for defining an “ideal-like” plasma and halo region in NIMROD
    - \( S_{\text{crit-ideal}} \sim 5 \times 10^7; S_{\text{crit-halo}} \sim 0.5 \)
    - Ratio of these values are greater than in experiment
  ✴ Demonstrated a technique that varies the linear spectral properties of a single equilibrium
    - scans show extreme spectral sensitivity to halo location
      *(especially when \( \rho_{\text{Halo}} \sim \rho_{\text{qmn}} \))
    - edge ballooning & kink effects can be “dialed in” by using a sharp resistivity transition region located at relevant flux positions
Summary

- Preliminary NL results show qualitatively needed resolution and expected energy growth rates for a single NL filament growth
  - promising for future analysis
    - compare with current NL ballooning theory
      - (P. Zhu & C.C. Hegna, to be published PoP 2008)
    - compare and contrast nonlinear peeling vs ballooning components
    - start analytic representation of nonlinear peeling-ballooning modes to be compared with calculated results
Mode convergence occurs at high resolution? NEEDED?

- Determining needed resolution
  - good packing for laptop execution
  - not realistic for future NL studies but a good reference point
Benchmarking exercise with ELITE is ongoing.
MAST Experiments Have Filament Growth in Bad Curvature Region During ELMs

- Filament structure of instability shows ballooning-like structure
- Nonlinear ballooning growth may explain precursor and collapse phases
  - ELM’s timescale in MAST: ~20-200µs

Temperature(a) and density(b) profiles obtained at times relative to the Dα intensity(c) and line averaged density(d). During ELM phase in MAST. (Kirk et al. 2007)
Future Work

• Finish the linear parameter scan over full range of modes
• Examine linear spectral behavior as a function of vacuum position, pedestal height, plasma resistivities, etc
• Benchmark linear analysis with established codes such as GATO and ELITE
• Extend linear results to full nonlinear analysis of peeling-balloonning modes in NIMROD
  ✦ develop phenomenological understanding of nonlinear evolution and compare ballooning dominant results with current ballooning theory (P. Zhu)
  ✦ start analytic representation of nonlinear peeling-balloonning modes to be compared with calculated results
NIMROD Simulations Show Localized Filaments in the Nonlinear Phases of ELMs

- Transition phase (left) and late nonlinear phase (right)
- Mode growth timescale during transition phase ~ 20\(\mu\)s
- Nonlinear Ballooning description may describe ELM dynamics.

*Bonita Squires, Sherwood Mtg, March 30 - April 2, 2008, Boulder CO*
Plasma edge defined by x\text{vac} in NIMROD

- Plasma edge and vacuum region defined by x\text{vac}
  - Pedestal location at 0.755
  - Pedestal width

\[
\frac{1}{1} - 1
\]

- x\text{vac} transition 0.78 (S=10^5 inside & 10^2 outside)
  - Transition width leads to overlap with pedestal region
Scott’s preliminary spectral results show kink and ballooning instabilities

- The low-n growth associated with kink modes
- The large-n behavior ballooning
- Reproduce these results & observe kink and ballooning interactions for varying physics params
  - pedestal width
  - relative Lundquist #’s
  - edge location
  - “removing” kink modes

S. Kruger, April 20, 2008
Edge Coordinating Committee Meeting on V&V
San Diego, CA
Trouble along the way

- Specifically had trouble with $n = 1$
  - strange saw-tooth growth
  - overlap region between pedestal and vacuum may be the problem
  - may be real physics
    - two modes (resistive & ideal) may simultaneously exist
    - Scott also saw this sawtoothing
    - perhaps nimrod bounces between two solutions
  - Moving the vacuum region out seems to eliminate the issue...
  - Increasing the Lundquist number increases problem (Even with $n$ up to 2 & 3....$ELMBf$)
Xvac moved out to eliminate sawtoothing???

- xvac location was moved out systematically to look at effect
  - n=1 problem gone for larger xvac, but issue remains with S=constant
- Also, entire spectrum is effected by resistive edge effects

"Xvac = 0.78 - Scott's equilibrium"
Increasing S (constant) increases problem

- For \( S = 10^6 \) the sawtooothing is seen up to \( n = 3 \) mode.
- Not entirely sure if it is the exact same behavior.
- Ping and Chris believe this is converged growth, I am not sure.

Magnetic Energy vs. \( t \)

\[
\begin{align*}
\text{Magnetic Energy vs. } t & \\
0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0 & 0.95 & 0.96 & 0.97 & 0.98 & 0.99 & 1.00 \\
0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0 & 0.95 & 0.96 & 0.97 & 0.98 & 0.99 & 1.00 \\
\end{align*}
\]