Late Nonlinear Stage of Tokamak Ballooning Instability

The Effects of Hyperdiffusivity

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Can/how does ballooning filament break off from core plasma?

- Question:
  - What physics governs the late/terminal state of a ballooning filament?
- Previous work I: ELM-milestones (2005,2006)
  - DIII-D like equilibrium
  - Large diffusivity for numerical stability
- Previous work II: g-mode (2007)
  - 2D Shearless slab (purely interchange)
  - Low dissipation regime difficult to simulate
- This work: ballooning of ESC equilibrium
  - Toroidal coupling: energy cascades even linearly
  - 3D global structure: has to resolve parallel direction
  - Low dissipation regime: filaments carry high temperature
  - Physically intriguing and computationally challenging
    - What are the optimal physical and numerical dissipations?

A Solution: Hyperdiffusivity [Sovinec, 2008]

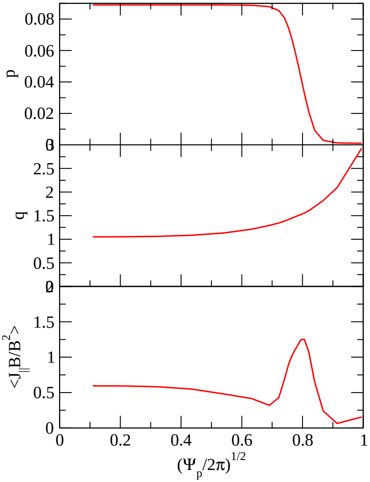
- Numerical difficulties mostly arise from cascading to small structures in density evolution
- Hyperdiffusivity improves numerical stability, while leaving macroscopic structures intact.

$$\partial_t n + \mathbf{u} \cdot \nabla n = -n\nabla \cdot \mathbf{u} - \nabla \cdot (D_h \nabla \nabla^2 n) \qquad (1)$$

- Recently implemented hyperdiffusivity in `nimuw`
  - improves numerical stability of both linear and nonlinear runs;
  - allows physically ideal, dissipationless runs
  - simulations are able to reach later nonlinear stage of tokamak ballooning.

Tokamak equilibrium: Circular boundary and pedestal-like pressure

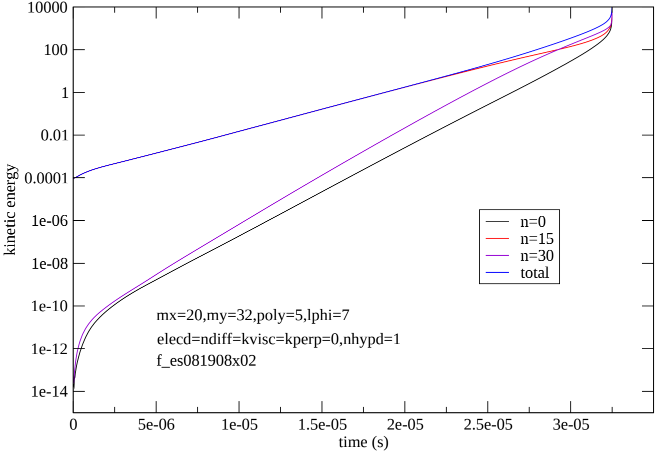
- $R_b = R_0 + a\cos(\theta + \delta\sin\theta)$, $Z_b = Z_0 + \kappa a \sin\theta$
- $Z_0 = 0$, $R_0 = 3$, $a = 1$, $\kappa = 1$, $\delta = 0$
- $q = q_0[1 + (q_a/q_0 - 1)r^4]$
- $r = \sqrt{\Psi/\Psi_a}$, Ψ – toroidal flux
- $q_0 = 1.05$, $q_a = 3.0$, $B_0 = 1.0$, $\tau_A = R_0 q_0/u_{A0} \sim 10^{-6}s$
- $p = p_p + h_p \tanh[(r_p - r)/L_p]$
- $p_p = 0.045$, $h_p = 0.044$, $r_p = 0.7$, $L_p = 0.05$



Hyperdiffusivity ensures a linear dissipationless ideal ballooning mode to run

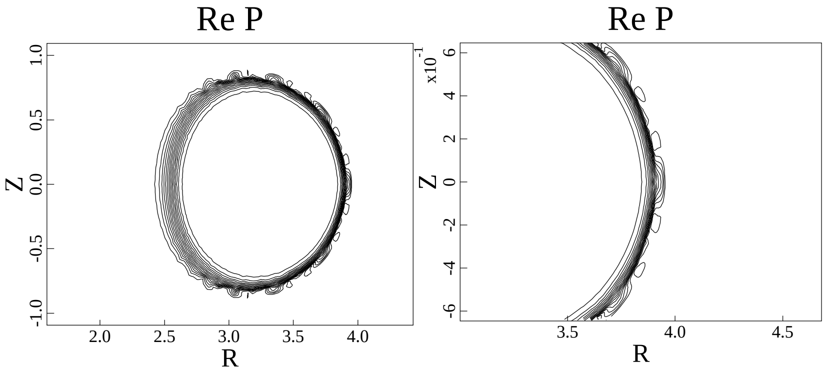
- mx=20, my=32, poly=5, elecd=ndiff=kvisc=kperp=0, nd_hypd=1
- The run was numerically unstable for nd_hypd=0.
- Growth rate insensitive to nd_hypd.

How far can a nonlinear ballooning simulation last (with hyperdiffusivity only)?



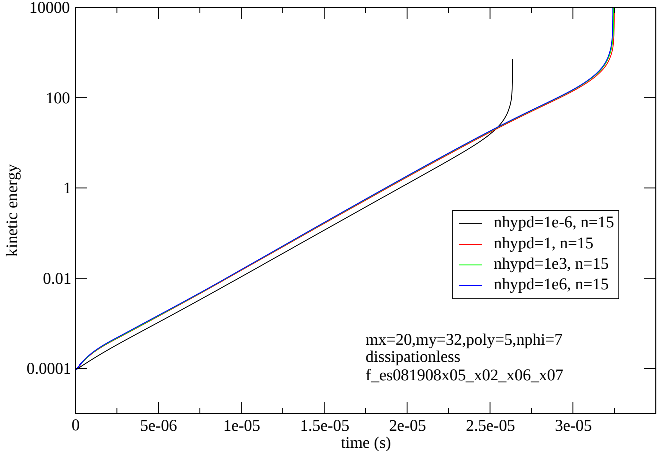
- I.C. n=15 nearly eigen-ballooning-mode
- Clearly nonlinear after $25\mu s$

Distortion of flux surface becomes visible near end of run

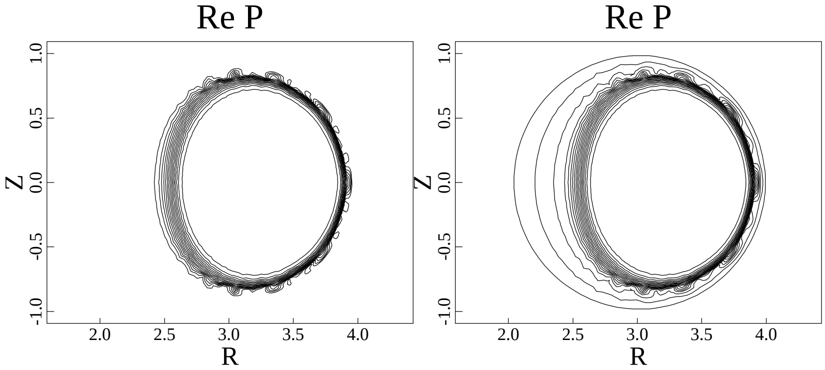


- Left: pressure contour at 30μs; Right: zoom in of Left.
- `elecd=ndiff=kvisc=kperp=0, nhypd=1.`
- Displacement becomes comparable of pedestal width.
- divB error large before numerical instability; code stops due to small dt.

Effects of hyperdiffusivity are mostly numerical



Distortion of flux surface appears better resolved with larger hyperdiffusivity



- Above: Pressure contours at 30μs.
- Left: `nhypd=1e3`; Right: `nhypd=1e6`.
- `elecd=ndiff=kvisc=kperp=0`.

Summary

- Continued study in late nonlinear stage of ballooning
- Recently developed hyperdiffusivity have been applied to the simulation.
- Initial results indicate improvement in numerical stability, extending the nonlinear stage of ballooning simulation in the low dissipation regime.
- Relevant physical dissipation regime to be identified.