Moment approach to the closure theory for toroidal plasmas

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Closure vs. transport theory

- Closure theory for 5 moment $(n_a, \mathbf{V}_a, T_a)$ equations

$$d_a n_a + n_a \nabla \cdot \mathbf{V}_a = 0 \, (d_a \equiv \partial_t + \mathbf{V}_a \cdot \nabla)$$

$$\frac{3}{2} n_a d_a T_a + n_a T_a \nabla \cdot \mathbf{V}_a + \nabla \cdot \mathbf{h}_a + \nabla \mathbf{V}_a : \boldsymbol{\pi}_a = Q_a$$

$$m_a n_a d_a \mathbf{V}_a - n_a e_a (\mathbf{E} + \mathbf{V}_a \times \mathbf{B}) + \nabla p_a + \nabla \cdot \boldsymbol{\pi}_a = \mathbf{R}_a$$

Solve the kinetic equation or equivalently (n_a:non-Maxwellian moments)

$$D \mathsf{n}_a + \Omega_a \mathbf{b} \times \mathsf{n}_a = C \mathsf{n}_a + \mathsf{G}_a$$

Express $\mathbf{h}_a(\mathsf{n}_a^{11})$, $\boldsymbol{\pi}_a(\mathsf{n}_a^{20})$, Q_a, $\mathbf{R}_a$ in terms of $n_a, \mathbf{V}_a, T_a$

$$\mathbf{R}_\mathrm{e} = -(\alpha)(\mathbf{V}_\mathrm{ei}) - (\beta)(\nabla T_\mathrm{e}), \;\; \mathbf{h}_\mathrm{e} = (\beta)(\mathbf{V}_\mathrm{ei}) - (\kappa)(\nabla T_\mathrm{e})$$

- Transport theory

Solve momentum balance equation

$$ne\mathbf{E}' + ne\mathbf{V}_\mathrm{ei} \times \mathbf{B} = \mathbf{R}_\mathrm{e} \text{ where } \mathbf{E}' = \mathbf{E} + \mathbf{V}_\mathrm{i} \times \mathbf{B} + (ne)^{-1} \nabla p_\mathrm{e}$$

Express fluxes in terms of thermodynamic drives

$$\mathbf{J} = (\sigma)(\mathbf{E}') - (\alpha')(\nabla T_\mathrm{e}), \;\; \mathbf{h}_\mathrm{e} = (\alpha')(\mathbf{E}') - (\kappa')(\nabla T_\mathrm{e})$$

Transport/closure theories

	Braginskii	Neoclassical	transport	Unified closure
Collisionality	high	high: PS	low	general$^{\star 1}$
Magnetic field strength	general	strong	strong	strong
Magnetic geometry	general	nested	nested	general$^{\star 2}$
Collision operator	Landau	Landau	model	Landau$^{\star 3}$

- Solve general moment equations (not the drift kinetic equation)

$\star 1$ No ordering on collisionality (no subsidiary expansion) $\Rightarrow$ Braginskii

$\star 2$ No flux surface average

$\star 3$ Exact full linearized Coulomb collision operators

Moment expansion of a distribution function

- Landau kinetic equation

$$\partial_t f_a + \mathbf{v}\cdot\nabla f_a + \frac{q_a}{m_a}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\partial_{\mathbf{v}} f_a = \sum_b C(f_a, f_b)$$

- Moment expansion: expansion coefficients, m^{lk}'s, are symmetric traceless fluid moments

$$\begin{aligned} f_a(t,\mathbf{x},\mathbf{v}) &= f_a^{(0)} \sum_{lk} \frac{1}{\sqrt{\sigma_k^l}} \mathsf{m}_a^{lk}(t,\mathbf{x}) \cdot \mathsf{p}_a^{lk} \\ \mathsf{n}_a^{lk} \equiv n_a \mathsf{m}_a^{lk}(t,\mathbf{x}) &= \int d\mathbf{v} \frac{1}{\sqrt{\sigma_k^l}} \mathsf{p}_a^{lk} f_a(t,\mathbf{x},\mathbf{v}) \end{aligned}$$

- p^{lk}'s are orthogonal, irreducible, tensorial polynomials and form a complete set

$$\int d\mathbf{v}\mathsf{p}^{jp}\mathsf{p}^{lk}\cdot\mathsf{m}^{lk} f^{\mathrm{M}} = \delta_{jl}\delta_{pk}\sigma_p^j n \mathsf{m}^{jp}$$

Moment equations for electrons and ions $(a,b) = (\mathrm{e},\mathrm{i})$ or (i,e)

Ji and Held, PoP (2006, 2008, 2009)

$$\hat{D}_a \mathsf{n}_a + \Omega_a \mathbf{b} \breve{\times} \mathsf{n}_a = (\hat{C}_{aa} + \hat{A}_{ab}) \mathsf{n}_a + \mathsf{G}_a$$

$$\mathsf{n}_a = \begin{bmatrix} \mathsf{n}_a^0 \\ \mathsf{n}_a^1 \\ \mathsf{n}_a^2 \\ \mathsf{n}_a^3 \\ \vdots \end{bmatrix}, \ \mathsf{n}_a^0 = \begin{pmatrix} \mathsf{n}_a^{02} \\ \mathsf{n}_a^{03} \\ \mathsf{n}_a^{04} \\ \vdots \end{pmatrix}, \ \mathsf{n}_a^1 = \begin{pmatrix} \mathsf{n}_a^{11} \\ \mathsf{n}_a^{12} \\ \mathsf{n}_a^{13} \\ \vdots \end{pmatrix}, \ \mathsf{n}_a^2 = \begin{pmatrix} \mathsf{n}_a^{20} \\ \mathsf{n}_a^{21} \\ \mathsf{n}_a^{22} \\ \vdots \end{pmatrix}, \cdots$$

$$\mathsf{G}_a = \begin{bmatrix} 0 \\ \mathsf{G}_a^1 \\ \mathsf{G}_a^2 \\ 0 \\ \vdots \end{bmatrix}, \ \mathsf{G}_a^1 = \begin{pmatrix} \mathsf{G}_{Ta}^{11} + \delta_{a\mathrm{e}} \hat{A}_{\mathrm{ei}}^{11,00} n_\mathrm{e} \\ \delta_{a\mathrm{e}} \hat{A}_{\mathrm{ei}}^{12,00} n_\mathrm{e} \\ \delta_{a\mathrm{e}} \hat{A}_{\mathrm{ei}}^{13,00} n_\mathrm{e} \\ \vdots \end{pmatrix}, \ \mathsf{G}_a^2 = \begin{pmatrix} -\frac{1}{\sqrt{2}} n_a \mathsf{W}_a \\ 0 \\ 0 \\ \vdots \end{pmatrix}$$

$$\mathsf{G}_{Ta}^{11} = \frac{\sqrt{5}}{2} \frac{n_a v_{Ta}}{T_a} \nabla T_a, \ \hat{A}_{\mathrm{ei}}^{1p,00} = -\frac{1}{\tau_{\mathrm{ei}}} \sqrt{\frac{3(p+\frac{1}{2})!}{(p+\frac{3}{2})p!(\frac{1}{2})!}} \frac{\mathbf{V}_{\mathrm{ei}}}{v_{T\mathrm{e}}}$$

$$\mathsf{W}_a = \nabla \mathbf{V}_a + \widetilde{\nabla \mathbf{V}}_a - \frac{2}{3} \mathsf{I} \nabla \cdot \mathbf{V}_a$$

Small gyroradius ordering $\delta = \rho/L \ll 1$

$$\hat{D}_a \mathsf{n}_a + \underbrace{\Omega_a}_{\delta^{-1}} \mathbf{b}\breve{\times}\mathsf{n}_a = \hat{C}_a \mathsf{n}_a + \mathsf{G}_a$$

Temperature, electric potential and flow velocity on the flux surfaces (ψ)

$$\begin{aligned} T &= T_0(\psi) + T_1(\mathbf{x}) \\ \phi &= \phi_0(\psi) + \phi_1(\mathbf{x}) \quad \Rightarrow \quad \mathbf{a}_0 = \frac{q}{m}(-\nabla\phi_0 + \mathbf{V}_1 \times \mathbf{B}) \\ \mathbf{V} &= \mathbf{V}_1(\mathbf{x}) \end{aligned}$$

$$\begin{aligned} \hat{D}_0^{jp,lk} \mathsf{n}^{lk} &= v_{T0}[\hat{\Psi}^{j-}_{pk}\nabla + \hat{\Phi}^{j-}_{pk}\nabla \ln T_0]\mathsf{n}^{j-1,k} + v_{T0}^{-1}\hat{\Theta}^{j-}_{pk}\mathbf{a}_0\mathsf{n}^{j-1,k} \\ &\quad + v_{T0}[\hat{\Psi}^{j+}_{pk}\nabla + \hat{\Phi}^{j+}_{pk}\nabla \ln T_0] \cdot \mathsf{n}^{j+1,k} + v_{T0}^{-1}\hat{\Theta}^{j+}_{pk}\mathbf{a}_0 \cdot \mathsf{n}^{j+1,k} \end{aligned}$$

Expand $\mathsf{n}_a = \mathsf{n}_a^{(0)} + \mathsf{n}_a^{(1)} + \mathsf{n}_a^{(2)} + \cdots, \quad \mathsf{G}_a = \mathsf{G}_a^{(0)} + \mathsf{G}_a^{(1)}$

- δ^{-1}: $\Omega\mathbf{b}\breve{\times}\mathsf{n}^{(0)} = 0$
- δ^{0}: $\hat{D}_0\mathsf{n}^{(0)} + \Omega\mathbf{b}\breve{\times}\mathsf{n}^{(1)} = \hat{C}\mathsf{n}^{(0)} + \mathsf{G}^{(0)}$
- δ^{1}: $\hat{D}_0\mathsf{n}^{(1)} + \hat{D}_1\mathsf{n}^{(0)} + \Omega\mathbf{b}\breve{\times}\mathsf{n}^{(2)} = \hat{C}\mathsf{n}^{(1)} + \mathsf{G}^{(1)}$

Solutions of $\mathsf{b}\breve{\times}\mathsf{n}^{lk(0)} = 0$

$$\mathsf{b}_{\|}\mathbf{V} = \mathbf{bb}\cdot\mathbf{V} \equiv \mathbf{V}_{\|},\ \mathsf{b}_{\times}\mathbf{V} \equiv \mathbf{b}\times\mathbf{V} = \mathbf{V}_{\times},\ \mathsf{b}_{\perp}\mathbf{V} \equiv (\mathsf{I}-\mathbf{bb})\cdot\mathbf{V} = \mathbf{V}_{\perp}$$

$$\mathsf{b}_{\times\perp}\mathsf{W} \equiv \mathsf{W}_{\times\perp},\ \mathsf{K}^{-1} \equiv \tfrac{1}{2}\mathsf{b}_{\times\perp} + 2\mathsf{b}_{\|\times}$$

- Vector moments: $\mathbf{b}\times\mathsf{n}^{1k(0)} = 0 \quad\Rightarrow\quad \mathsf{n}_{\perp}^{1k(0)} = 0$
- Rank 2 tensor moments: $\mathbf{b}\breve{\times}\mathsf{n}^{2k(0)} = \mathbf{b}\times\mathsf{n}^{2k(0)} - \mathsf{n}^{2k(0)}\times\mathbf{b} = 0$

$$\mathsf{K}^{-1}(\mathbf{b}\times\mathsf{n}^{2k(0)} - \mathsf{n}^{2k(0)}\times\mathbf{b}) = \mathsf{n}_{\mathrm{CGL}}^{2k(0)} - \mathsf{n}^{2k(0)} = 0$$

$$\mathsf{n}^{2k(0)} = \mathsf{n}_{\mathrm{CGL}}^{2k(0)} \equiv n_{zz}^{2k(0)}(-\tfrac{1}{2}\mathbf{e}_1\mathbf{e}_1 - \tfrac{1}{2}\mathbf{e}_2\mathbf{e}_2 + \mathbf{bb}) = \tfrac{3}{2}n_{zz}^{2k(0)}(\mathbf{bb} - \tfrac{1}{3}\mathsf{I})$$

- General-rank tensor moments $(l \geq 3)$: solve coupled equations for $\mathsf{n}_{\|\cdots\times\cdots\perp\cdots}^{lk(0)}$
  $\mathsf{n}_{\|\cdots\times\cdots\perp\cdots}^{lk(0)} = 0$ for odd number of $\perp$ or $\times$

$$\mathsf{n}^{lk(0)} = \frac{(2l-1)!!}{l!}n_{\|}^{lk(0)}\mathsf{P}^{l}(\mathbf{b})$$

$n_{\|}^{lk(0)} \equiv n_{zz\ldots z}^{lk(0)}$ can be found from the δ^0-order parallel moment equations

δ^0-order equations $\hat{D}_0 \mathsf{n}^{(0)} + \Omega \mathbf{b} \breve{\times} \mathsf{n}^{(1)} = \hat{C} \mathsf{n}^{(0)} + \mathsf{G}^{(0)}$

- Parallel moments

$$D_\| n_\|^{(0)} = C n_\|^{(0)} + \underbrace{G_\|^{(0)}}_{=0}$$

$$n_\|^{(0)} = 0 \qquad \Rightarrow \qquad \mathsf{n}^{(0)} = 0$$

- Perpendicular moments

$$\mathsf{G}_a^{11(0)} = \sqrt{\frac{5}{4}} \frac{n_{a0} v_{Ta0}}{T_{a0}} \nabla T_{a0} \quad \mathsf{G}_a^{lk(0)} = 0 \ (l, k \neq 1, 1)$$

$$\mathsf{n}_{a\perp}^{11(1)} = -\sqrt{\frac{5}{4}} \frac{1}{\Omega_a} \frac{n_{a0} v_{Ta0}}{T_{a0}} \mathbf{b} \times \nabla T_{a0} \quad \Rightarrow \quad \mathbf{h}_{a\perp}^{(1)} = \frac{5}{2} \frac{n_{a0} T_{a0}}{m_a \Omega_a} \mathbf{b} \times \nabla T_{a0}$$

$$\mathsf{n}^{lk(1)} = \frac{(2l-1)!}{l!} n_\|^{lk(1)} \mathsf{P}^l(\mathbf{b}) \quad (l, k \neq 1, 1)$$

$n_\|^{lk(1)} \equiv n_{zz\ldots z}^{lk(1)}$ can be found from the δ^1-order parallel moment equations

Parallel δ^1 order equations $[\hat{D}_0 \mathsf{n}^{(1)} + \Omega \mathsf{b} \breve{\times} \mathsf{n}^{(2)} = \hat{C} \mathsf{n}^{(1)} + \mathsf{G}^{(1)}]_\parallel$

$$\hat{\Psi}^{l-}_{pk}\left(\overline{\nabla \mathsf{n}^{l-1,k}}\right)_\parallel \quad \Rightarrow \quad \bar{\Psi}^{l-}_{pk}\left(\partial_\ell \bar{n}_\parallel^{l-1,k} + \frac{l-1}{2}\partial_\ell \ln B \bar{n}_\parallel^{l-1,k}\right)$$

$$\hat{\Psi}^{l+}_{pk}\left(\nabla \cdot \mathsf{n}^{l+1}\right)_\parallel \quad \Rightarrow \quad \bar{\Psi}^{l+}_{pk}\left(\partial_\ell \bar{n}_\parallel^{l+1} - \frac{l+2}{2}\partial_\ell \ln B \bar{n}_\parallel^{l+1}\right)$$

$$\bar{G}^{1p(1)}_{a\parallel} = \delta_{p1}\frac{\sqrt{5}}{2}\frac{n_a v_{Ta0}}{T_{a0}}\frac{\partial T_{a1}}{\partial \ell} - \delta_{ae}\frac{1}{\tau_{\mathrm{ei}}}\sqrt{\frac{3(p+\frac{1}{2})!}{(p+\frac{3}{2})p!(\frac{1}{2})!}}\frac{V_{\mathrm{ei}\parallel}}{v_{T\mathrm{e}}}$$

$$\bar{G}^{20(1)}_{a\parallel} = -\sqrt{3} n_a \mathbf{bb} : \overline{\nabla \mathbf{V}_{a1}}$$

$$\bar{G}^{02(1)}_{a\parallel\nabla} = -v_{Ta0}\bar{\Psi}^{0+}_{21}\nabla \cdot \mathsf{n}^{11(1)}_{a\perp} = -\sqrt{\frac{10}{3}}\frac{n_{a0}}{q_a}\nabla \cdot \frac{\mathbf{B}\times\nabla T_{a0}}{B^2}$$

$$\bar{G}^{20(1)}_{a\parallel\nabla} = -v_{Ta0}\bar{\Psi}^{2-}_{01}\mathbf{bb} : \overline{\nabla \mathsf{n}^{11(1)}_{a\perp}} = \frac{2}{\sqrt{3}}\frac{n_{a0}}{q_a}(\boldsymbol{\kappa} \cdot \frac{\mathbf{B}\times\nabla T_{a0}}{B^2} + \frac{1}{3}\nabla \cdot \frac{\mathbf{B}\times\nabla T_{a0}}{B^2})$$

$$\bar{G}^{21(1)}_{a\parallel\nabla} = -v_{Ta0}\bar{\Psi}^{2-}_{11}\mathbf{bb} : \overline{\nabla \mathsf{n}^{11(1)}_{a\perp}} = -\sqrt{\frac{14}{3}}\frac{n_{a0}}{q_a}(\boldsymbol{\kappa} \cdot \frac{\mathbf{B}\times\nabla T_{a0}}{B^2} + \frac{1}{3}\nabla \cdot \frac{\mathbf{B}\times\nabla T_{a0}}{B^2})$$

$$\bar{G}^{02(1)}_{a\parallel A} = -v_{Ta0}^{-1}\bar{\Theta}^{0+}_{21}\mathbf{a}_0 \cdot \mathsf{n}^{11(1)}_a = -\sqrt{\frac{10}{3}}\frac{n_{a0}}{T_{a0}}\mathbf{V}_{a1\perp}\cdot\nabla T_{a0}$$

$$\bar{G}^{21(1)}_{a\parallel A} = -v_{Ta0}^{-1}\bar{\Theta}^{2-}_{11}\mathbf{bb} : \overline{\mathbf{a}_0 \mathsf{n}^{11(1)}_a} = -\frac{1}{3}\sqrt{\frac{14}{3}}\frac{n_{a0}}{T_{a0}}\mathbf{V}_{a1\perp}\cdot\nabla T_{a0}$$

Parallel δ^1-order equations $\Rightarrow n_{\|}^{(1)}$

$$[\Psi]\partial_z \bar{n}_{\|}^{(1)} = [c]\bar{n}_{\|}^{(1)} + g_{\|}^{(1)} - \{\partial_z \ln B[\Psi_B] + \partial_z \ln T_0[\Phi] + a_{0\|}[\Theta]\}\bar{n}_{\|}^{(1)}$$

- Solution of $[\psi]\partial_z \bar{n}_{\|}^{(1)} = [c]\bar{n}_{\|}^{(1)} + g_{\|}^{(1)}$ $\Rightarrow$ integral closures

$$\bar{n}_{\|}^{jp(1)}(y) = \int K^{jp,lk}(y-z) g_{\|}^{lk(1)}(z) dz$$

where $dz = d\ell/v_T\tau = d\ell/L_C$ [Ji Held Sovinec 2009 PoP]

$$g_{\|}^{lk(1)} \Leftarrow \frac{\partial T_1}{\partial z}, \frac{V_{ei1\|}}{v_{Te}}, \left(\overline{\nabla \mathbf{V}_1}\right)_{\|}, \boldsymbol{\kappa} \cdot \frac{\mathbf{B} \times \nabla T_0}{B^2}, \nabla \cdot \frac{\mathbf{B} \times \nabla T_0}{q_a B^2}, \mathbf{V}_{1\perp} \cdot \nabla T_0$$

- Axi-symmetric geometry $\mathbf{B} = I(\psi)\nabla\varphi + \nabla\varphi \times \nabla\psi$, $\dfrac{\mathbf{B} \times \nabla\psi}{B^2} = \dfrac{I\mathbf{B}}{B^2} - R\hat{\varphi}$

$$\nabla \cdot \frac{\mathbf{B} \times \nabla T_0}{qB^2} = \mathbf{B} \cdot \nabla \left(\frac{nI}{qB^2} \frac{dT_0}{d\psi} \right)$$

$$\int K(y-z) \nabla \cdot \frac{\mathbf{B} \times \nabla T_0}{qB^2} dz = \frac{nI}{q} \int K(y-z) \mathbf{B} \cdot \nabla B^{-2} dz \frac{dT_0}{d\psi}$$

Perpendicular δ^1-order $(\hat{D}\mathsf{n})^{l(1)} + \mathbf{b}\breve{\times}\mathsf{n}^{l(2)} = C\mathsf{n}^{l(1)} + \mathsf{G}^{l(1)} \Rightarrow \mathsf{n}_\perp^{(2)}$

- Perpendicular vector moments

$$\mathsf{n}_\perp^{1p(2)} = -\Omega^{-1}\mathbf{b} \times [C^{1p1}\mathsf{n}^{11(1)} + \mathsf{g}^{11(1)} - \mathbf{D}_0^{11,0k}\mathsf{n}^{0k(1)} - \mathbf{D}_0^{11,2k} \cdot \mathsf{n}^{2k(1)}]$$

Heat flow across the flux surface $\hat{\psi} = \nabla\psi/|\nabla\psi|$, $\mathbf{b} \times \hat{\psi} = \mathbf{t}$

$$\begin{aligned} \hat{\psi} \cdot \mathbf{h}^{(2)} \quad = \quad & \frac{5}{2}\frac{c^{111}}{\tau}\frac{n_0 T_0}{m\Omega^2}\hat{\psi} \cdot \nabla T_0 - \frac{5}{2}\frac{n_0 T_0}{m\Omega}\mathbf{t} \cdot \nabla T_1 \\ & -\frac{T_0^2}{m\Omega}\mathbf{t} \cdot \nabla(\sqrt{\frac{10}{3}}n^{02} - \sqrt{\frac{1}{2}}n_{zz}^{20} + \sqrt{\frac{7}{2}}n_{zz}^{21}) + \sqrt{\frac{1}{2}}T_0\hat{\psi} \cdot \mathbf{V}_1 n_{zz}^{20} \end{aligned}$$

where

$$\mathbf{t} \cdot \nabla n_{\|}^{lk} \propto \hat{\psi} \cdot \mathbf{b} \times \nabla \left[\left(\int K^{lk,A}(y-z) B \partial_\ell B^{-2} dz \right) \frac{nI}{q}\frac{dT_0}{d\psi} \right] + \cdots$$

$\mathbf{b} \times \nabla\theta$ and $\mathbf{b} \times \nabla\varphi$ yield terms of transport across the flux surfaces

Perpendicular δ^1-order $(\hat{D}\mathsf{n})^{l(1)} + \mathbf{b}\breve{\times}\mathsf{n}^{l(2)} = C\mathsf{n}^{l(1)} + \mathsf{G}^{l(1)} \Rightarrow \mathsf{n}_\perp^{(2)}$

- Viscous stress $\Omega\mathbf{b}\breve{\times}\mathsf{n}^{20(2)} = C^{20k}\mathsf{n}^{2k(1)} + \mathsf{G}^{20(1)} - (\hat{D}\mathsf{n})^{20(1)}$

$$\begin{aligned}\mathsf{n}^{20(2)} &= \mathsf{n}^{20(2)}_{\mathrm{CGL}} - \Omega^{-1}\mathsf{K}^{-1}[\mathsf{G}^{20(1)} - D^{2-}_{0k}\mathsf{n}^{1k(1)} - D^{2+}_{0k}\mathsf{n}^{3k(1)}] \\ &= \mathsf{n}^{20(2)}_{\mathrm{CGL}} - \Omega^{-1}\mathsf{K}^{-1}[-\sqrt{2}n\overline{\nabla\mathbf{V}} - \overline{\mathbf{D}^{2-}_{01}\mathsf{n}^{11(1)}} - \mathbf{D}^{2+}_{00}\mathsf{n}^{30(1)}]\end{aligned}$$

$$\mathsf{K}^{-1}\mathsf{W} = \frac{1}{2}\mathsf{W}_{\times\perp} + 2\mathsf{W}_{\parallel\times}$$

$$\boldsymbol{\sigma} = \int d\mathbf{v}m(\mathbf{www} - \frac{3}{5}\{\mathbf{w}\mathsf{l}\})f = \frac{\sqrt{3}}{2}v_T^3\mathsf{n}^{30}, \quad \mathbf{w} = \mathbf{v} - \mathbf{V}$$

$$\boldsymbol{\pi}^{(2)} = \boldsymbol{\pi}^{(2)}_{\mathrm{CGL}} + \Omega^{-1}\mathsf{K}^{-1}[2p\overline{\nabla\mathbf{V}} + \frac{4}{5}\overline{\nabla\mathbf{h}^{(1)}} + \nabla\cdot\boldsymbol{\sigma}^{(1)}]$$

$$\mathsf{K}^{-1}\nabla\cdot\boldsymbol{\sigma}^{(1)} = \frac{5}{2}\sigma_\parallel^{(1)}(\mathbf{b}\times\boldsymbol{\kappa}\mathbf{b} + \mathbf{bb}\times\boldsymbol{\kappa})$$

- Transport theory $\Leftarrow$ Closures and

$$mnd_t\mathbf{V} - nq(\mathbf{E} + \mathbf{V}\times\mathbf{B}) + \nabla p + \nabla\cdot\boldsymbol{\pi} = \mathbf{R}$$

Summary and future work

- Unified closures for general collisionality/magnetic geometry
  - Braginskii's theory in the high collision limit
  - More accurate neoclassical transport theory
  - Transport theory without flux surface average
- Find fitting functions for kernels
- Compare with neoclassical transport theory
  - Flux surfaces
  - Axi-symmetric geometry
- Apply to interesting fusion devices