Implementations and applications of continuum closures

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Outline

1. Solve full plasma kinetic equation using continuum method.
   - Apply 2-D finite-elements (FE) in Chapman-Enskog-like kinetic equation.
   - Slowing down of diffuse ion beam.
2. Solve drift kinetic equation using continuum method.
   - Recap of recursive drift kinetic theory.
   - Application to sound wave damping problem.
   - One recursion introduces drift effects.

Solve Chapman-Enskog-like (CEL) kinetic equation.

- Couple solutions of 6-D, time-dependent Chapman-Enskog-like kinetic equation to evolving fluid equations through closures for **q**and **Π**:

$$\frac{\partial F}{\partial t} + \mathbf{v} \cdot \nabla F + \mathbf{a} \cdot \nabla F = C(F + f_M) - \frac{m}{T}\left(v'\mathbf{v}' - v'^2 \frac{I}{3}\right):\nabla \mathbf{u} - \frac{2f_m}{3p} L_1^{(3/2)} [\nabla \cdot \mathbf{q} + \mathbf{\Pi}:\nabla \mathbf{V} - Q] + \mathbf{v}' \cdot \left[\frac{f_m}{p}(\nabla \cdot \mathbf{\Pi} - \mathbf{R}) + \frac{f_m}{T} L_1^{(5/2)} \nabla T\right]$$

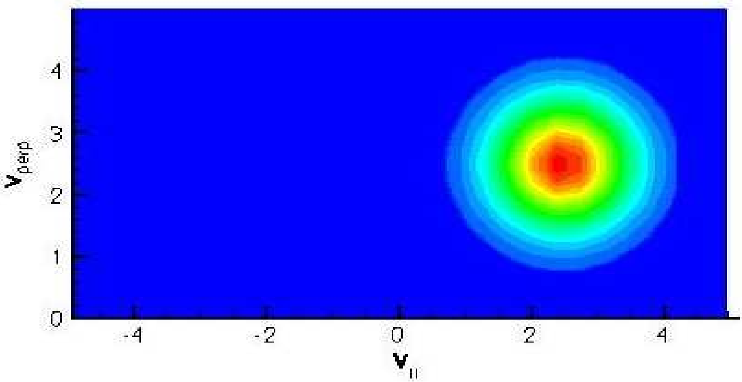
where $f_M = n(\mathbf{x}, t) \left(\frac{m}{2\pi T(\mathbf{x}, T)}\right)^{3/2} e^{-\frac{m(\mathbf{v}-\mathbf{u})^2}{2T}}$

and $\mathbf{v} = v_{||}\mathbf{b} + v_{\perp}(\mathbf{e}_2 sin\gamma - \mathbf{e}_3 cos\gamma)$.

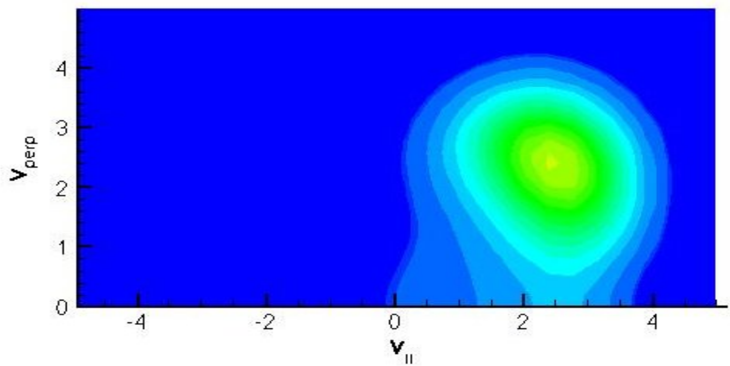
Use 2-D FE, 1-D Fourier basis for F.

- Expand distribution function as
  $F = F_0 + \sum_{n>0} F_n e^{in\gamma} + F_n^\dagger e^{-in\gamma}$,
  with Fourier coefficients expanded as
  $F_n = \sum_j F_{nj}(\mathbf{x}, t)\, \alpha_j(v_\perp, v_\|)$.
- At present, test particle portion of linearized Coulomb collision operator is implemented.
  $\int d\mathbf{v} \alpha C(f_\alpha, f_{M\beta}) = -\frac{2\pi q_\alpha^2 q_\beta^2 ln\Lambda_{ab}}{m_\alpha^2} \frac{n_\beta}{n_\alpha} \frac{1}{z_\beta} \int d\mathbf{v}(4\frac{m_\alpha}{m_\beta} \frac{1}{v_{T\beta}} G(\mathbf{z}_\beta \cdot \frac{\partial\alpha}{\partial\mathbf{v}})\, f_\alpha + (E-G)(\frac{\partial\alpha}{\partial\mathbf{v}} \cdot \frac{\partial f_\alpha}{\partial\mathbf{v}}) + (\frac{3G-E}{z_\beta^2})(\mathbf{z}_\beta \cdot \frac{\partial\alpha}{\partial\mathbf{v}})(\mathbf{z}_\beta \cdot \frac{\partial f_\alpha}{\partial\mathbf{v}}))$.

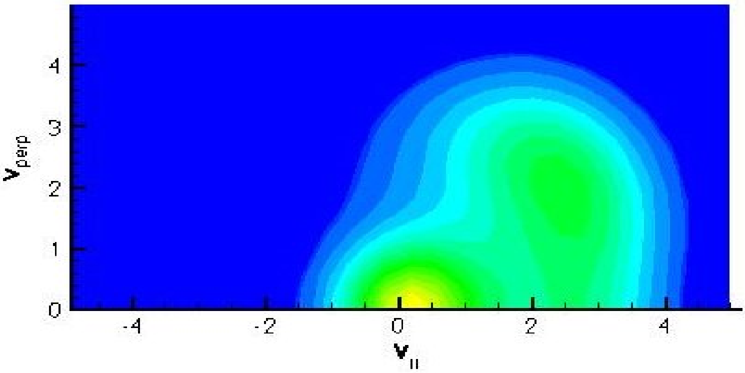
Apply test-particle operator to beam ions.



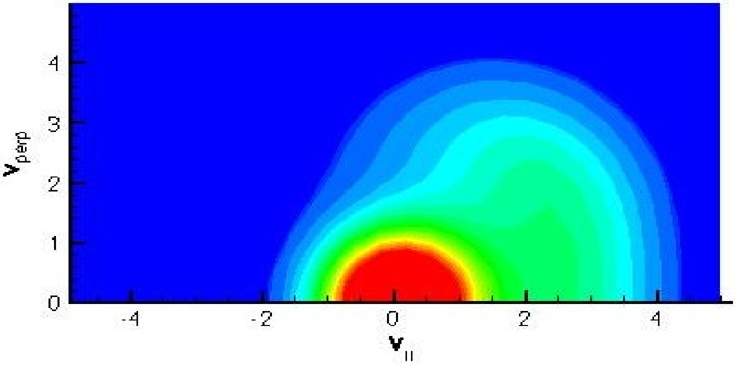
After 1 collision time.



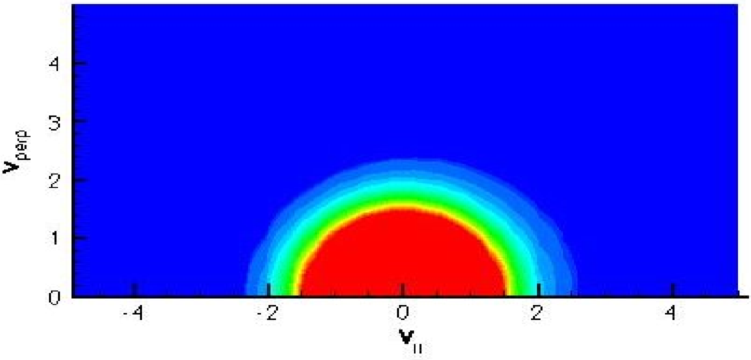
After 3 collision times.



After 5 collision times.



After 20 collision times distribution is Maxwellian.



Future work on full continuum solution.

- Test-particle operator implemented.
- Implement field terms of linearized Coulomb operator.
- Implement nonlinear terms.
- Develop coding necessary to extend solution to spatial domain.

Drift kinetic theory a la Hazeltine '73.

- Define $L = d/dt - \Omega \partial / \partial \gamma$, total time derivative without the rapid gyro-motion. Here$\Omega = qB/m$.
- Hazeltine suggests solving
  $\Omega \frac{\partial \tilde{f}}{\partial \gamma} = -(L - C)f + \langle (L - C)f \rangle$,
  by approximating f on right as $\bar{f}$ (the gyro-angle independent part).
- Can repeat this by step by using $f = \bar{f} + \tilde{f}_1$on right to get an improved f.
- Finally, insert $f = \bar{f} + \tilde{f}_j$after j recursions into $\langle (L - C)f \rangle = 0$ and solve for $\bar{f}$.

Zero recursions ($\tilde{f} = 0$) leads to lowest-order CEL-drift kinetic equation.

- Solve lowest-order equation for electrons and ions to compute parallel closures, $\mathbf{q}_{||}$and ˝$_{||}$:

$$\frac{\partial F}{\partial t} + \mathbf{v}_{||} \cdot \nabla F + \frac{q}{m} E_{||} \frac{v_{||}}{v} \frac{\partial F}{\partial v} =$$

$$\langle C(F + f_M) \rangle - \frac{mv^2}{T} f_M \left(\mathbf{bb} - \frac{\mathbf{I}}{3} \right) : \nabla \mathbf{u} -$$

$$+ \frac{2 f_m}{3p} L_1^{(3/2)} \left[\nabla \cdot \mathbf{q} + \mathbf{\Pi} : \nabla \mathbf{V} - Q - S_0^{rf} \right] +$$

$$\mathbf{v}_{||} \cdot \left[\frac{f_m}{p} \left(\nabla \cdot \mathbf{\Pi} - \mathbf{R} - F_0^{rf} \right) + \frac{f_m}{T} L_1^{(5/2)} \nabla T \right]$$

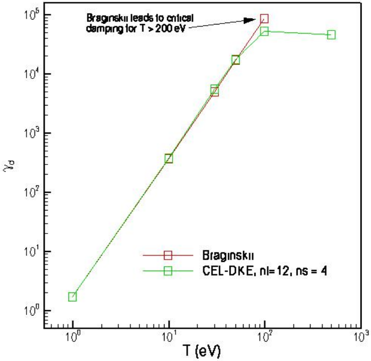
- Expand $F = \sum_{l=0}^{nl} F_l(\mathbf{x}, t, s) P_l(v_{||}/v)$, where the coefficients $F_l(\mathbf{x}, t, s)$ are determined on a grid of ns grid points in the normalized speed, $s = v/v_T$.

Time-discretization and centering issues.

- Present implementation centers $F_e$and $F_i$with n, **B**, T_e, $T_i$in time.
- n, **B**, $T_e$and $T_i$dependence in Maxwellian drives approximately centered in implicit F advance.
- Advance **V**, n, F_e,F_i, compute closures.
- Advance T_e,$T_i$and **B**, solve again for F_e,F_i with approximately centered n, **B**, T_e, T_i, and recompute closures.
- Advance T_e,$T_i$and **B**with approximately centered closures.

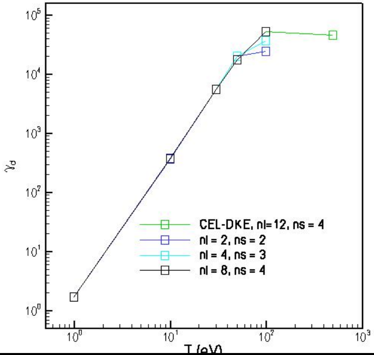
Apply to sound wave damping problem.

- Test stress damping of sound waves with adiabatic equation of state.



Test convergence in velocity space.

1. Number of Legendre polynomials = nl. Number of grid points in speed = ns.
2. Limits on workstation with 4 GB of memory: nl=12 and ns=4.



Future work on lowest-order continuum solution to CEL-DKE.

- Implement RF quasilinear source terms imported from GENRAY.
- Implement finite flow corrections in Maxwellian drives.
- Implement calculation of self-consistent, equilibrium F_e and F_i (needed when there is equilibrium current and flow).
- Test continuum solution to CEL-DKE on various problems.

Compare with Chang/Callen '92.

- One-pole approximations for linear closures were given as:
$$\mathbf{b}\cdot\nabla\cdot\tilde{\mathbf{\Pi}} \approx n_0 m_i \frac{3}{5}\left[\sqrt{\pi}k_{||}v_{Ti} - \frac{6}{5}\left(\frac{\partial}{\partial t} + 0.32\nu_{ii}\right)\right]\tilde{u}_{||i}$$
$$+\frac{2}{5}\left[k_{||}v_{Ti} - \sqrt{\frac{3\pi}{10}}\left(\frac{\partial}{\partial t} + 0.42\nu_{ii}\right)\right]\frac{n_0}{k_{||}v_{Ti}}(\nabla_{||}\tilde{T})$$
$$\tilde{q}_{||} \approx \frac{2}{5}\left[k_{||}v_{Ti} - \sqrt{\frac{3\pi}{10}}\left(\frac{\partial}{\partial t} + 0.42\nu_{ii}\right)\right]\frac{p_i}{k_{||}v_{Ti}}\tilde{u}_{||i}$$
$$-\frac{9}{5\pi}\left[\sqrt{\pi}k_{||}v_{Ti} - \frac{72}{25\pi}\left(\frac{\partial}{\partial t} + 0.42\nu_{ii}\right)\right]\frac{n_0}{k_{||}^2}(\nabla_{||}\tilde{T})$$
- Compare linear damping results between theory, kinetic MHD and continuum closure calculations.

Drift corrections needed for many problems.

- Do one level of recursive theory to incorporate drift effects.
- Integrate, $\Omega \frac{\partial \tilde{f}_1}{\partial \gamma} = -(L - C)\bar{f} + \langle (L - C)\bar{f} \rangle$.
- Insert $f = \bar{f} + \tilde{f}_1$ into $\langle (L - C) f \rangle = 0$ and solve for $\bar{f}$.
- Solve: $\frac{\partial \bar{F}}{\partial t} + (\mathbf{v}_{||} + \mathbf{v}_D) \cdot \nabla \bar{F} + a \frac{\partial \bar{F}}{\partial \epsilon}$ = Maxwellian drives including drift effects.