Update on kinetic effects of energetic particles on nonlinear resistive MHD instability

R. Takahashi, D.P. Brennan
Department of Physics and Engineering Physics
The University of Tulsa

C.C. Kim
Plasma Science and Innovation Center
The University of Washington

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Results on Energetic Particle on resistive MHD linear stability

Energetic particles have significant damping and stabilizing effects at experimentally relevant $\beta_N$, $\beta_{\text{frac}}$, and $S$, and weaker damping and stabilizing effects in the ideal unstable regime, and excite a real frequency of the 2/1 mode.

- PRL 2009
- Nucl. Fusion 2009

We are on a path to research:

“Nonlinear + 2f1 effects + Energetic particles.”
Recent Results Show Energetic Particle/MHD Coupling Important and Computationally Viable

Historical focus has been on the simplified effects on the 1/1 mode.

Recent Computational Efforts Successful

- Choi, Turnbull, Chan (GA) Show highly accurate prediction of the sawtooth crash in DIII-D (PoP 2007). --> D.D. Schnack et al. (Sherwood 09)

Our resistive MHD analyses suggest possible energetic particle stabilization of resistive 2/1 modes at high energetic particle beta fractions.
PIC noise complicates study of Energetic Particles on resistive MHD stability

• PIC code injects noise into earlier linear stage (\( \beta_{frac}/\beta_{frac_c} \sim (V_{\phi}/V_{rh})^2 \)), (these errors can be decreased by increasing particle numbers), however later driven & saturation stages can be recovered.

• With 2fl, MHD linear stability will be even more complicated. (Localization, long timescale.)
The $\delta f$ PIC model

- PIC is a Lagrangian simulation of phase space $f(x,v)$
- PIC evolves the $f(x(t),v(t))$
- $\delta f$ PIC reduces the discrete particle noise associate with conventional PIC
- Vlasov equation $\frac{\partial f(z)}{\partial t} + \dot{z} \cdot \frac{\partial f(\dot{z})}{\partial t} = 0$

- Evolution equation for $\delta f$, $\delta \dot{f} = -\delta z \cdot \frac{\partial f_0}{\partial t}$.

- the drift kinetic equations of motion are used as the particle characteristics

$$\hat{x} = v_{||} \hat{b} + \frac{E \times B}{B^2} + \frac{m^2}{eB^4} \left( v_{||}^2 + \frac{v_{\perp}^2}{2} \right) ( B \times \nabla \frac{B^2}{2} ) - \frac{\mu_0 m v_{||}^2}{eB^2} J_{\perp},$$

$$m \dot{v}_{||} = -\hat{b} \cdot (\mu \nabla B - eE).$$
The slowing down distribution function for energetic particles

The slowing down distribution function

\[ f = \frac{P_0 \exp\left(\frac{P_\xi}{\psi_n}\right)}{\varepsilon^{3/2} + \varepsilon_c^{3/2}}, \quad P_\xi \propto \psi, \quad \psi_n = C\psi_0 \]

\( P_\xi = g\rho_\parallel - \psi_p \), is the canonical toroidal momentum. The initial equilibrium state, \( \exp\left(\frac{g\rho_\parallel}{\psi_n}\right) \) is ignored: an energetic isotropic pressure.

The linearized evolution equation for \( \delta f \) becomes

\[ \delta f = f_0 \left\{ \frac{mg}{e\psi_n B^3} [(v_\parallel^2 + \frac{v_\perp^2}{2}) \delta \mathbf{B} \cdot \nabla \mathbf{B} - \mu_0 \nu_\parallel \mathbf{J}_\perp \cdot \delta \mathbf{E} \right. \]

\[ + \left. \frac{\delta \mathbf{v} \cdot (\nabla \psi_p - \rho_\parallel \nabla g)}{\psi_n} + \frac{3}{2} \frac{e\varepsilon^{1/2}}{\varepsilon^{3/2} + \varepsilon_c^{3/2}} \mathbf{v}_D \cdot \delta \mathbf{E} \right\}, \]

\[ \mathbf{v}_D = \frac{mg}{eB^3} (v_\parallel^2 + \frac{v_\perp^2}{2}) (\mathbf{B} \times \nabla \mathbf{B}) + \frac{\mu_0 m v_\parallel^2}{eB^2} \mathbf{J}_\perp, \]

\[ \delta \mathbf{v} = \frac{\delta \mathbf{E} \times \mathbf{B}}{B^2} + v_\parallel \frac{\delta \mathbf{B}}{B} \cdot \delta \mathbf{E}. \]
Equilibrium pressure and safety factor profiles as a function of $\psi$ in the D shape


Pr (the ratio of the viscosity to electric diffusivity) = 100

$$f \sim \exp(\psi / C)$$

$q_{\text{min}} \approx 1.5, \quad q_{95} \approx 4.4$
Linear Growth rates (of the resistive 2/1 mode) as a function of S for MHD only cases, Exp(-4ψ), (single fluid)
Growth rates for series of equilibria \( (\beta_N / 4l_i) \)

(stability diagram sketch)

\( \beta_{frac} = 0.0\% \)
\( \beta_{frac} = 12.5\% \)
\( \beta_{frac} = 25.0\% \)

Ideal limit

\( \{ \) ideal 2/1 mode

\( \{ \) resistive 2/1 mode (S=10^6)

PEST3
Frobenius coefficient of expansion \( \mu > 1 \)

Dcon: ideal unstable
Wall at a\(^*\)0.25
MHD only nonlinear (resistive) results

\( \left( \frac{\beta_N}{4l_i} = 0.83, S=10^6 \right) \)

\[ t = 4.8 \text{(ms)} \]

Saturation stage can be resolved at higher modes --> too expensive for 2-fl & energetic particles.
MHD only nonlinear (ideal) results

$\left( \beta_N / 4/_{i} = 0.90, S = 10^6 \right)$

t = 4.5(ms)

Higher toroidal modes need to be resolved …

Also, need to evolve nonlinear stage longer …

$m/n=4/2$ islands $\rightarrow n1 \sim n2$ (magnetic)
Nonlinear results \((\beta_N / 4/ = 0.83, S=10^6)\)

With energetic particles

Magnetic

Kinetic

Damping and stabilizing effects

\[
\log\left(\frac{E}{E_0}\right) \propto \log\left(\frac{V^2_{\phi h}}{V^2_{\text{th}}}ight) \approx 8
\]
Nonlinear results \((\beta_N / 4i = 0.83, S=10^6)\)

\(n=1\) Growth rates

**Magnetic**

**Kinetic**
Nonlinear results \((\beta_N / 4_i = 0.90, S = 10^6)\)

With energetic particles

Magnetic  

Kinetic

\[ V_\phi \]

\[ \log E \] vs. \( t(s) \times 10^3 \)

Graphs showing the behavior of magnetic and kinetic energies over time for different values of \( n \). The graphs compare multiple cases with varying \( n \) values, including \( n=0, n=1, n=2 \), and specific cases like \( n=0(\beta_{frac}=12.5\%) \) and \( n=2(\beta_{frac}=12.5\%) \).
Nonlinear results ($\beta_N / 4l_i = 0.90, S=10^6$)

$n=1$ Growth rate
A real frequency of the 2/1 mode \( (\beta_N / 4/\iota = 0.90) \)

nonlinear (ideal) results

Energetic particle: 12.5%
Two-fluid effect ($\beta_N / 4i = 0.98$), close to ideal limits, linear results (MHD & Hall, $dtm = 5 \times 10^{-9}$)

No Energetic particle
Two-fluid effect ($\beta_n / 4/ = 0.83$), resistive linear results (MHD & Hall, $dtm = 5 \times 10^{-9}$)

No Energetic particle
Precession rates (analytic calculations)

Ballpark estimation

The ion banana orbits drift toroidally with a frequency $\omega_B$

$$\omega_B \approx \frac{qV_{th}^2}{\Omega_c Rr} \leq 8.0 \times 10^2, \omega_B \tau_A \sim 3.0 \times 10^{-4}$$

(Hu et al, PoP 2005)

Diamagnetic Rotation

$$\omega_{*e,-i} = \frac{c}{ne Br} \frac{dp_{e,i}}{dr} = \frac{1}{m_{e,i} n r \omega_{ce,i}} \frac{dp_{e,i}}{dr}$$

$$\omega_{*e} \sim 2.2 \times 10^3, \omega_{*i} \sim 1.1 \times 10^3$$

$$\left(\omega(\omega - \omega_{*i})(\omega - \omega_{*e})^3 = i \gamma_{MHD}^5\right)$$

(Coppi, PFs 1965)

$$|\omega| \leq 1.5 \times 10^3, \omega \tau_A \sim 7.0 \times 10^{-4}$$
Conclusion and Discussion

• Nonlinear 2fl with energetic particles will be important!

• Nonlinear (single fluid with energetic particles)
  • Real frequencies will increase or decrease at nonlinear stage?
  • nlayers needs for nonlinear higher modes, NIM(RE)SET.

• 2fl linear results
  • Close to the Ideal limit, small damping effects $\gamma$, and small $\omega$
  • Resistive cases, small damping effects, however $\omega$ is larger.
  • Need to resolve separatrix region, add $n_{hypd}$, … etc.
Can a Kinetic - MHD model Explain the Stabilization of the 2/1 in JET

Experimental data from the DIII-D, Asdex, JT-60U and JET experiments show only JET breaks the model of onset of the 2/1 near ideal MHD limit.

- Model: parametric $\Delta'$ near ideal limit (Brennan 2002/3) in modified Rutherford equation for a $\rho^*_i$ dependence of onset (La Haye 2008).

$$\frac{\tau_R}{r} \frac{dw}{dt} = \Delta' r + a_2 \varepsilon^{1/2} \left( \frac{L_q}{L_p} \right) \beta_\theta (r/w) \left( 1 - \frac{w_{mag}^2}{3w^2} \right)$$

Fit with pole at 1.2 to $\rho^*_i \Delta' r$

(La Haye et al. N. Fusion 2008)

Classic theory:
The linear tearing stability index

$$\Delta' r = -(m - k) - k\alpha x [\cot(\alpha x)], \quad x \equiv \frac{\beta_N}{4l_i}$$
Can a Kinetic - MHD model Explain the Stabilization of the 2/1 in JET, the 2/1 is stable in JET

Buttery et al (IAEA, 2008) 
Can a Kinetic - MHD model Explain the Stabilization of the 2/1 in JET

Puzzle: Why does the JET experiment not show instability like the others?

Likely reason: energetic particles stabilize the 2/1 mode.
- JET ($\beta_{\text{frac}} > 30\%$),
- DIII-D, JT-60U ($\beta_{\text{frac}} < 20\%$


OTHER Possible Causes?

- Accurate $\Delta'$ calculation (Brennan 2002/3/6).
- Accurate equilibrium.
- Other physics, two-fluid effects … ?