Recent progress and results from the NIMROD extended MHD simulations of HIT-SI

C. Akcay, C. C. Kim, T. R. Jarboe, and B. A. Nelson

NIMROD APS 2012 Meeting

Oct 29th-Nov 2nd, 2012
Highlights

- Calculations show amplification of the axisymmetric mode \((n = 0)\) at Lundquist numbers \(S\) comparable to that of the experiment.
- 2-fluid MHD (2fl MHD) shows
  - greater current amplification \(\frac{I_{\text{tor}}}{I_{\text{inj}}}\) and faster current rise times than rMHD.
  - different toroidal mode spectrum and dynamo activity from rMHD.
  - good agreement with the experimental data, i.e. validation.
- Resistive MHD (rMHD):
  - Amplification of \(n = 0\) at ideal rates and \(I_{\text{tor}}\) production.
  - 3D visualization shows a competition between \(n = 0\) amplification via the relaxation of the injector columns from \(n = 1\) to largely \(n = 0\) and kink instability responsible for destroying the injector columns.
  - Extensive parameter space scans.
Helicity Injected Torus with Steady Inductive Helicity Injection (HIT-SI) is a current drive experiment

The spheromak plasma is formed and sustained by Steady Inductive Helicity Injection (SIHI) provided by two semi-toroidal injectors, X and Y. Each injector oscillates magnetic flux and voltage in phase resulting in injection of \( n = 1 \) helical magnetic fields.

- Driving the injectors 90° out of phase results in a constant helicity injection rate
- Insulating layer guarantees inductive drive by preventing arcing to the flux conserver
- No electrodes: Closed flux device
The present model assumes a fully-ionized plasma with uniform density, finite $m_e$ and $\beta = 0$

- NIMROD advances the remaining equation of motion and induction equation:

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = \frac{\mathbf{J} \times \mathbf{B}}{\rho} - \nabla \cdot \Pi$$  

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left( \eta \mathbf{J} - \mathbf{v} \times \mathbf{B} + \frac{1}{ne} \mathbf{J} \times \mathbf{B} + \frac{m_e}{ne^2} \frac{\partial \mathbf{J}}{\partial t} \right)$$

- $\mathbf{v}$ is the center-of-mass velocity of the fluid, $\rho$ is the mass density, $\eta$ is the electrical resistivity, $m_e$ is the electron mass, $\Pi$ is the viscosity tensor, $\mathbf{B}$ is the magnetic field, and $\mathbf{J} = \nabla \times \mathbf{B}/\mu_0$ is the current density in the low-frequency limit of Ampère’s law.

- $\frac{m_i}{m_e} = 36 - 100$ to make the induction equation more diagonally dominant.
SIHI is implemented as a set of normal magnetic field $B_n$ and tangential electric field $E_t$ boundary conditions imposed at the annular boundaries.

A MATLAB GS solver is used to obtain the functional form for $B_n$ and $E_t$, which are hard-coded in NIMROD in $B_{\text{norm}}$ and $E_{\text{tang}}$. 
The applied $B_n$ and $E_t$ required for SIHI result in a helical total magnetic field on the annulus.
All rMHD simulations that produce a significant $I_{tor}$ exhibit a three-stage evolution.

1. Initial strong coupling to $n = 1$ and other odd modes followed by the linear growth of the unstable ($n =$even) modes at ideal growth rates ($\gamma\tau_A = 2$).

2. Nonlinear saturation culminating in a drastic ME exchange between $n = 1$ and $n = 0$.

3. And steady-state sustainment where $n = 0$ dominates ME spectrum with significant $I_{tor}$.

(a) Log ME vs time(ms)  
(b) ME per mode vs time(ms)
Current columns symmetric with respect to the centerline are observed during $n = 1$ dominance.

(c) $J \equiv |\mathbf{J}|$ at $t=0.6$ ms

(d) $\lambda$ at $t=0.6$ ms
Full distortion of the current columns result in $n = 0$ amplification and saturation.

(e) $J$ after saturation $t=1.07$ ms

(f) $\lambda$ at $t=1.0$ ms
Visualization shows a competition between column formation and kink instability.

(g) $t = 1.342$ ms  
(h) $t = 1.355$ ms  
(i) $t = 1.358$ ms

(j) $t = 1.370$ ms  
(k) $t = 1.374$ ms  
(l) $t = 1.380$ ms
Onset of $n = 0$ growth occurs much earlier in 2fl MHD than in rMHD

- $n = 0$ grows out of direct coupling of the injector with itself. The Hall term generates $E$ needed to drive an $n = 0 J$.
- ME still initially goes to $n = 1$.
- The onset of current growth rate and rise time are in much closer agreement with HIT-SI.

(m) ME spectrum for 2fl MHD

(n) ME spectrum for rMHD
2fl calculations yield a greater $I_{\text{tor}}$ than the rMHD calculations by $\sim 40 - 50\%$.

- 2fl calculations scan a range in $\eta$ corresponding to $T_e = 6 - 19$ eV yielding $\frac{I_{\text{tor}}}{I_{\text{inj}}} = 1.7, 3, $ and $4$ respectively for a deuterium plasma.
- $I_{\text{tor}}$ scales more favorably with $\eta$ in 2fl-MHD: $\frac{I_{\text{tor}}}{I_{\text{inj}}} \propto \eta^{-4/5}$ whereas in rMHD $\frac{I_{\text{tor}}}{I_{\text{inj}}} \propto \eta^{-1/2}$.

---

**Toroidal Current vs. Time**

- **(o)** 2-MHD and rMHD $I_{\text{tor}}$ vs time
- **(p)** $I_{\text{tor}}$ vs time for various $T_e(\eta)$
Electron fluid motion (2fl dynamo) channels more energy into $n = 0$ than ion fluid motion (MHD dynamo).

\[ \int dtdV \sum_n \langle J \rangle \cdot (v_n^* \times B_n + \text{c.c.}), \]

\[ \int dtdV \sum_n \langle J \rangle \cdot \left( \frac{1}{n_e} \mathbf{J}_n^* \times \mathbf{B}_n + \text{c.c.} \right) \]

- The Hall term enhances the MHD dynamo-mediated power transfer to $n = 0$:
  relaxation requires ion inertia.
NIMROD internal magnetic profiles show good agreement with IDCD prediction and profiles measured by the internal magnetic probe (IMP) array (shot 122385)

- All the profiles are smoothed over one injector cycle.
- Shot 122385 produced $\frac{l_{tor}}{l_{inj}} \sim 3$.

(q) 6 eV 2fl MHD with $\frac{l_{tor}}{l_{inj}} = 1.7$

(r) 13 eV (scaled) 2fl MHD with $\frac{l_{tor}}{l_{inj}} = 3$
Comparison of IMP time traces at various radial locations with NIMROD synthetic probes.

\[(s)\] 6 eV 2fl MHD with \( \frac{I_{\text{tor}}}{I_{\text{inj}}} = 1.7 \]

\[(t)\] 13eV 2fl MHD with \( \frac{I_{\text{tor}}}{I_{\text{inj}}} = 3 \)
Ongoing efforts to compare simulated plasma flow to the measurements by Ion Doppler Spectroscope†

Data from a 32-chord IDS instrument with 20µs time resolution compared to a 6eV, $f_{inj} = 36.8$ kHz 2fl-MHD run

†Figure courtesy of Aaron Hossack
The Bi-orthogonal Decomposition (BOD) applies to the analysis of multipoint measurements, in this case $B(\theta, t)$ for each toroidal location.

- The data are decomposed in a small set of linearly independent modes called a spatio-temporal ‘mode’ of the fluctuating system:
  $$B(\theta, t) \rightarrow B(\theta_i, t_j) = \sum_k \lambda_{kk} \psi_k(\theta_i) \phi_k(t_j)$$

- This decomposition is achieved using a standard singular value decomposition (SVD): $B(\theta, t) = USV^T$ where $S$ contains the eigenvalues ($\lambda_{kk}$) of the system, $U$ the temporal modes and $V$ the spatial modes.

- Insightful and accurate quantitative comparisons between the data and simulation.
Poloidal (left) and toroidal (right) eigenvalues for shot 122385, rMHD and 2fI MHD NIMROD runs with valid parameters
Correlation of NIMROD and experimental spatial eigenmodes can be used as a metric for agreement.

- First two poloidal eigenmodes show strong correlations.
- Higher data eigenmodes appear to be superposition of NIMROD eigenmodes.

(w) 10eV 2fl MHD with $\frac{l_{tor}}{l_{inj}} = 1.7$

(x) 20eV 2fl MHD with $\frac{l_{tor}}{l_{inj}} = 3$
Correlation of NIMROD and experimental spatial eigenmodes can be used as a metric for agreement.

- First two toroidal eigenmodes show strong correlations.
- Higher data eigenmodes appear to be superposition of NIMROD eigenmodes.

\( \text{(y) 10eV 2fl MHD with } \frac{l_{tor}}{l_{inj}} = 1.7 \)

\( \text{(z) 20eV 2fl MHD with } \frac{l_{tor}}{l_{inj}} = 3 \)
Comparison of the 1\textsuperscript{st} and 2\textsuperscript{nd} toroidal eigenmodes between shot 122385 (left) and NIMROD (right)
Comparison of the 1\textsuperscript{st} and 2\textsuperscript{nd} poloidal eigenmodes between shot 122385 (left) and NIMROD (right)
Two-fluid MHD (2fl MHD) calculations show greater (1.5×) current amplification and faster current rise times than rMHD.
- 2 fl dynamo channels more energy into \( n = 0 \) than MHD dynamo.
- Internal magnetic structure shows good agreement with the data.
- SVD indicates similar spatial eigenmodes for the first 2 modes.

rMHD campaign is written up and soon to be published.

We are currently continuing analysis of 2fl-MHD results. Publication to follow.
Parameters for the two scans are based on HIT-SI operations with He and D gas assuming $T_i=T_e=10$ eV

<table>
<thead>
<tr>
<th></th>
<th>Sim.I</th>
<th>HIT-SI He</th>
<th>Sim. II</th>
<th>HIT-SI D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injector flux (mWb)</td>
<td>0.47</td>
<td>0.6-1.4</td>
<td>1.8</td>
<td>0.6-1.4</td>
</tr>
<tr>
<td>Injector current (kA)</td>
<td>11</td>
<td>10-20</td>
<td>21</td>
<td>10-20</td>
</tr>
<tr>
<td>$\lambda_{inj}$ (m$^{-1}$)</td>
<td>30</td>
<td>15-25</td>
<td>20</td>
<td>15-25</td>
</tr>
<tr>
<td>$\langle n_e \rangle$ (m$^{-3}$)</td>
<td>$3 \times 10^{19}$</td>
<td>$3 \times 10^{19}$</td>
<td>$1.5 \times 10^{19}$</td>
<td>$1.5-3 \times 10^{19}$</td>
</tr>
<tr>
<td>$\eta_{</td>
<td></td>
<td>}/\mu_0$ (m$^2$/s)</td>
<td>25-5</td>
<td>5</td>
</tr>
<tr>
<td>$\nu$ (m$^2$/s)</td>
<td>100</td>
<td>100-1000?</td>
<td>260</td>
<td>100-1000?</td>
</tr>
<tr>
<td>$f_{inj}$ (kHz)</td>
<td>5</td>
<td>5.8, 14.5</td>
<td>14.5</td>
<td>14.5</td>
</tr>
<tr>
<td>$S$</td>
<td>10-60</td>
<td>5-20</td>
<td>20-200</td>
<td>20-50?</td>
</tr>
</tbody>
</table>

- The rMHD system is characterized by two dimensionless parameters: Lundquist number $S$ and magnetic Prandtl number $Pm$.
- $S$ is the ratio of the two important time scales: $S = \frac{\tau_L/R}{\tau_A} \sim \left[ \frac{B}{\eta n} \right]^{1/2}$.
- $Pm$ is the ratio of the two diffusivities: $Pm = \frac{\nu}{\eta}$.
- $S$ is increased by lowering $\eta/\mu_0 = (25, 11.7, 8, 5)$. 

Cihan Akcay (University of Washington)  Extended MHD Simulations of HIT-SI  APS 2012  25 / 24
Specify a $B_n$ and $E_t$ consistent with Faraday’s law to inject flux.
An additional $E_t$ is applied to drive normal current $J_z$ along $B_n$.

This causes a tangential $B$ on the annulus.