

Continuum drift hot particles and bulk ions in NIMROD

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- Added F_{hot} data structure and advance for hot particle distribution function.
- Consolidated integrand routines to solve

$$\begin{aligned}
 & \partial_t f + (\mathbf{v}_{\parallel} + \mathbf{v}_D) \cdot \nabla f \\
 & + \frac{s}{2} \left[-(\mathbf{v}_{\parallel} + \mathbf{v}_D) \cdot \nabla \ln T_0 - (1 - \xi^2) \frac{\mathbf{b}}{B} \cdot \nabla \times \mathbf{E} + \right. \\
 & \left. \frac{e}{s^2 T_0} (\mathbf{v}_{\parallel} + \mathbf{v}_d) \cdot \mathbf{E} + (1 + \xi^2) \frac{\mathbf{E} \times \mathbf{B}}{B^2} \cdot \nabla \ln B \right] \frac{\partial f}{\partial s} \\
 & + \frac{1 - \xi^2}{2\xi} \left[-(\mathbf{v}_{\parallel} + \mathbf{v}_d) \cdot \nabla \ln B + \xi^2 \frac{\mathbf{b}}{B} \cdot \nabla \times \mathbf{E} + \right. \\
 & \left. \frac{e}{s^2 T_0} (\mathbf{v}_{\parallel} + \mathbf{v}_d) \cdot \mathbf{E} + \xi^2 \frac{\mathbf{E} \times \mathbf{B}}{B^2} \cdot \nabla \ln B \right] \frac{\partial f}{\partial \xi} = C(f)
 \end{aligned}$$

for all species.

- Keeping $\partial B/\partial t$ in acceleration \mathbf{v}_D and \mathbf{v}_d terms where

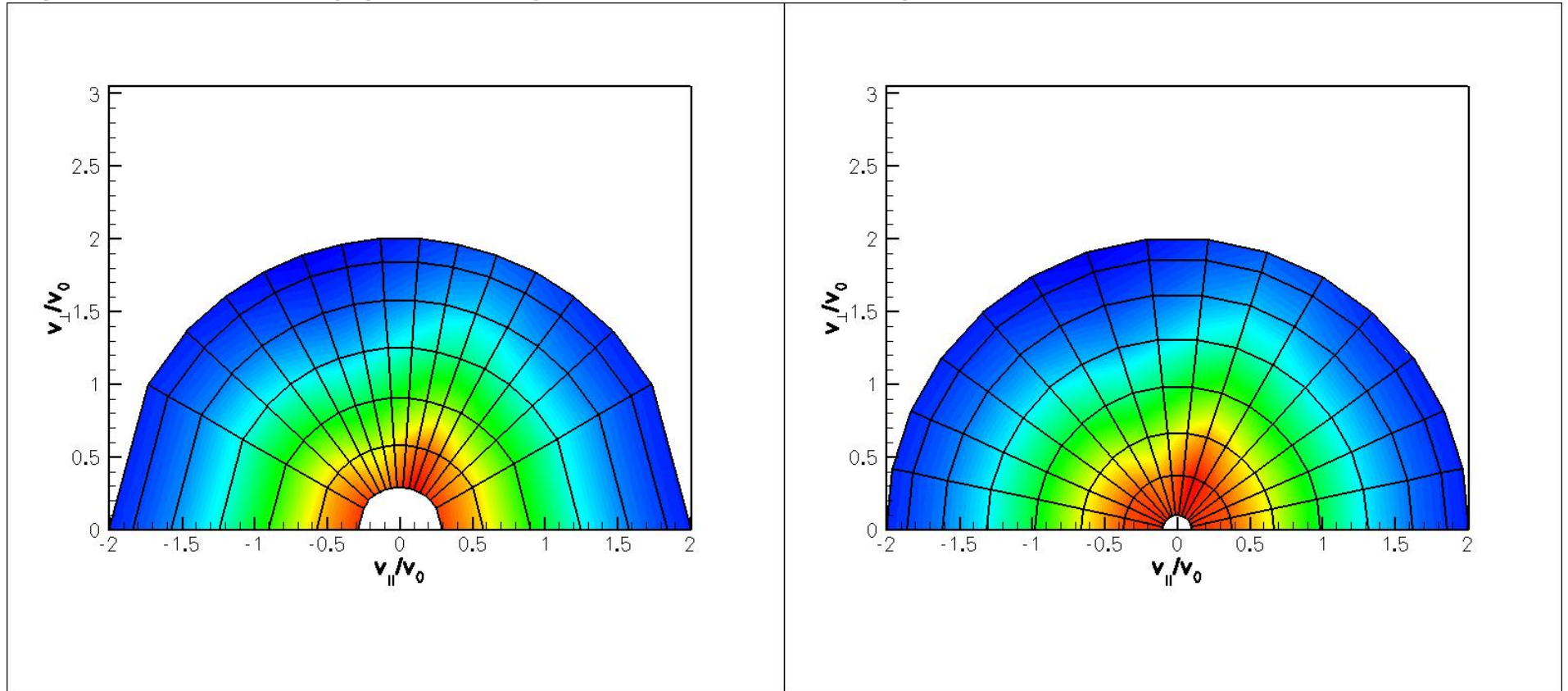
$$\mathbf{v}_D = \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \frac{\rho_0 v_0 s^2}{2B} (1 + \xi^2) \mathbf{b} \times \nabla \ln B + \frac{\rho_0 v_0 s^2}{B} \xi^2 \mu_0 \mathbf{J}_\perp +$$

$$\frac{\rho_0 v_0 s^2}{2B} (1 - \xi^2) \mu_0 \mathbf{J}_\parallel + \rho_0 s \xi \mathbf{b} \times \partial_t \ln B$$

$$\mathbf{v}_d = \mathbf{v}_D - \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \frac{\rho_0 v_0 s^2}{2B} (1 + \xi^2) \mathbf{b} \times \nabla \ln B$$

Improved continuum algorithm III.

- Generalized mapping between logical and pitch-angle space for 1D finite elements: $\frac{v_{\parallel}}{v} = \sum_i (\frac{v_{\parallel}}{v})_i \phi_i(x)$.
- Option for adding pre-assigned nodes into s grid.



- Evolve δf for hot, drifting, minority ion species as in C. C. Kim, *Phys. Plasmas* **15**, 072507 (2008):

$$\frac{\delta f}{f_0(P_\zeta)} = \left\{ \frac{mRB_\phi}{e\psi_n B^3} \left[\left(v_{\parallel}^2 + v_{\perp}^2/2 \right) \delta \mathbf{B} \cdot \nabla B - \mu_0 v_{\parallel} \mathbf{J} \cdot \mathbf{E} \right] + \frac{1}{\psi_n} \left(\mathbf{E} \times \mathbf{B} / B^2 + v_{\parallel} \delta \mathbf{B} / B \right) \cdot \left(\nabla \psi_p - m v_{\parallel} \nabla (RB_\phi) / (qB) \right) + \frac{3}{2} \frac{e\epsilon^{1/2}}{\epsilon^{3/2} + \epsilon_c^{3/2}} \mathbf{v}_D \cdot \mathbf{E} \right\}$$

- Slowing down distribution, $f_0(P_\zeta, \epsilon) = P_0 \exp(P_\zeta/\psi_n) / (\epsilon^{3/2} + \epsilon_c^{3/2})$ used in weight equation but

$f_0(\psi, \epsilon) = P_0 \exp(\psi/\psi_n) / (\epsilon^{3/2} + \epsilon_c^{3/2})$ used when taking moments:

$$\delta p_{\perp} = \int d\mathbf{v} \mu B \frac{\delta f}{f_0(P_\zeta)} f_0(\psi) \quad \text{and} \quad \delta p_{\parallel} = \int d\mathbf{v} v_{\parallel}^2 \frac{\delta f}{f_0(P_\zeta)} f_0(\psi).$$

- Continuum approach predicts different growth rates and real frequencies when using $f_0(\psi, \epsilon)$ instead of $f_0(P_\zeta, \epsilon)$.

- Fu's approach (Phys. Plasmas 13, 052517 (2006)) with M3D used:

$$f_0(\psi, \epsilon) = P_0 \exp(\langle \psi \rangle / \psi_n) / (\epsilon^{3/2} + \epsilon_c^{3/2}),$$

where

$$\langle \psi \rangle = P_\zeta / e - \frac{m}{e} \langle v_{\parallel} R \frac{B_\phi}{B} \rangle \approx P_\zeta / e$$

for trapped particles and

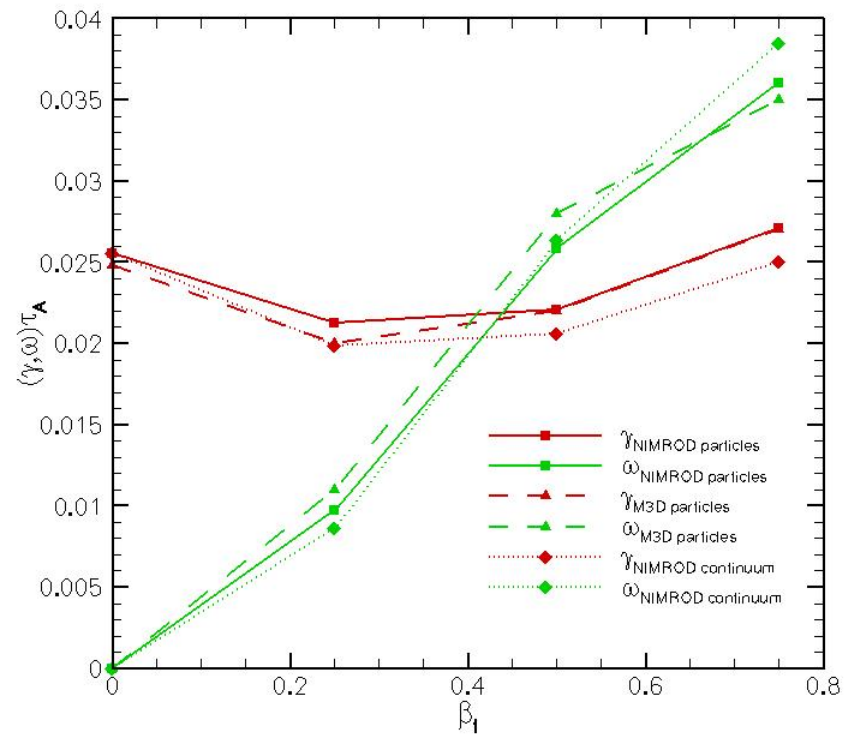
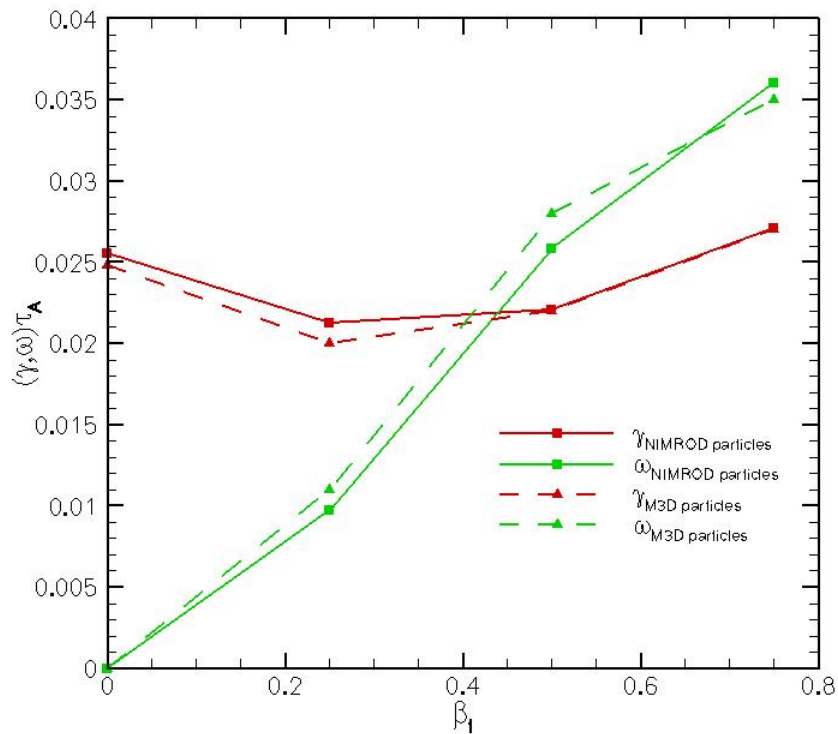
$$\langle \psi \rangle = P_\zeta / e - \frac{m}{e} \langle v_{\parallel} R \frac{B_\phi}{B} \rangle \approx P_\zeta / e - v R_0 \text{sign}\left(\frac{v_{\parallel}}{v}\right) \sqrt{1 - \mu B_0 / \epsilon}$$

for passing particles.

- Here ψ_n used to match shape of MHD pressure profile and P_0 used to replace fraction of MHD pressure at magnetic axis.

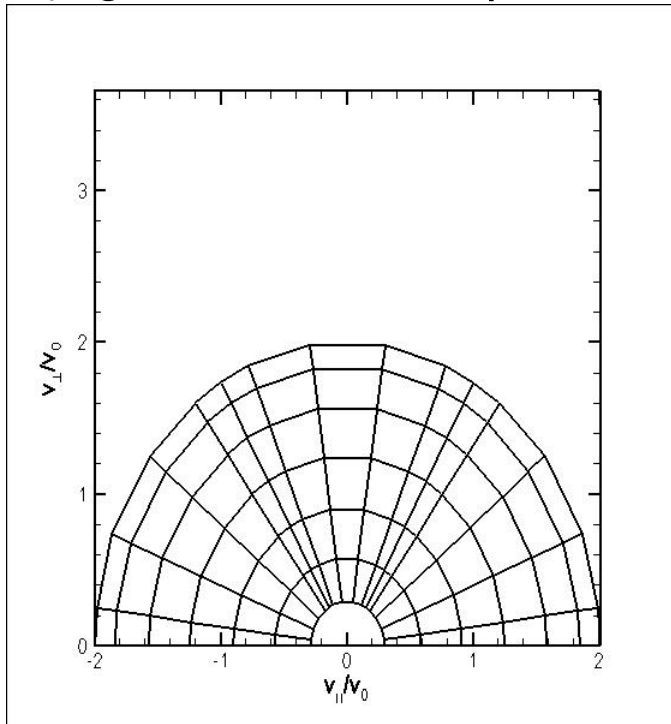
Reasonable agreement with δf PIC obtained.

- Increasing hot particle pressure first stabilizes and then destabilizes mode.
- Growth rates and real frequencies similar.

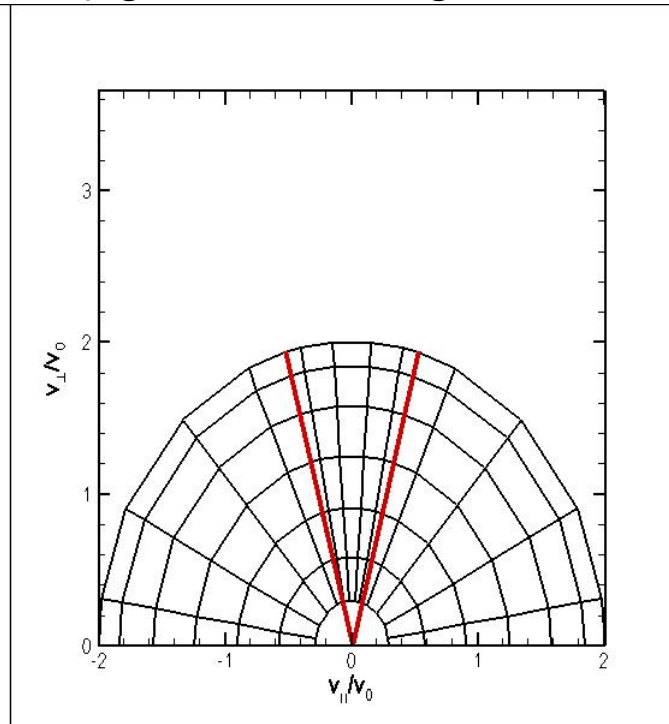


Vertex nodes in ξ grid at $\pm\xi_t(\psi)$ help convergence.

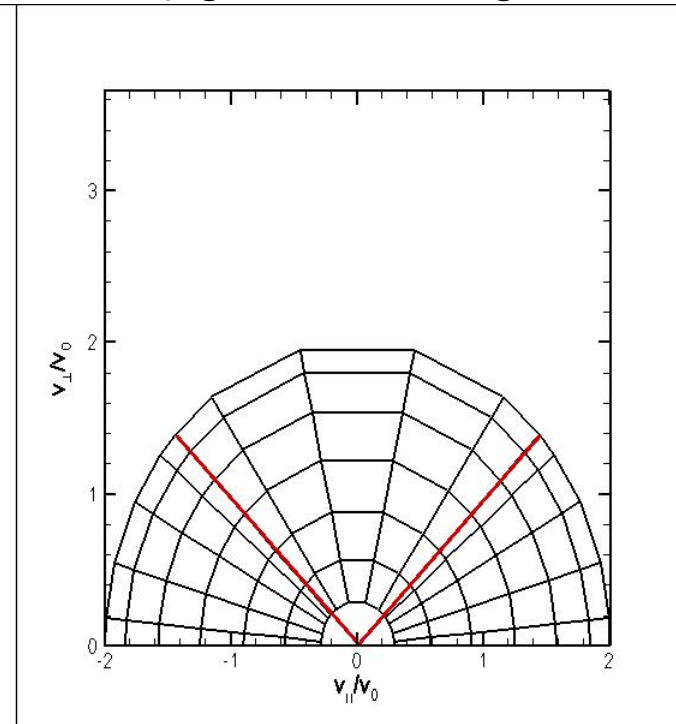
ξ grid uniform in space



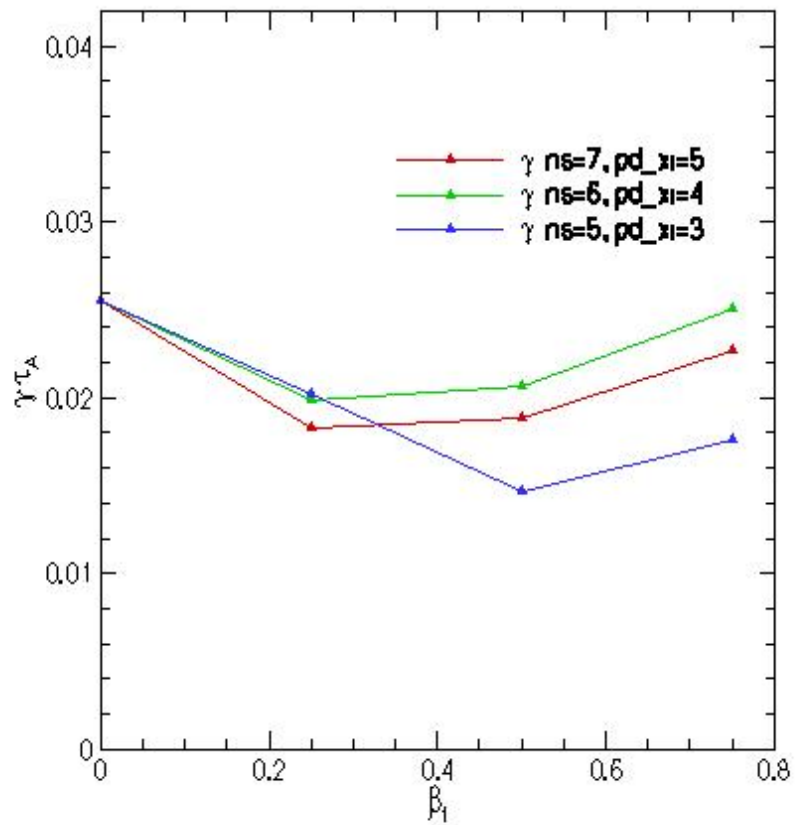
ξ grid near magnetic axis



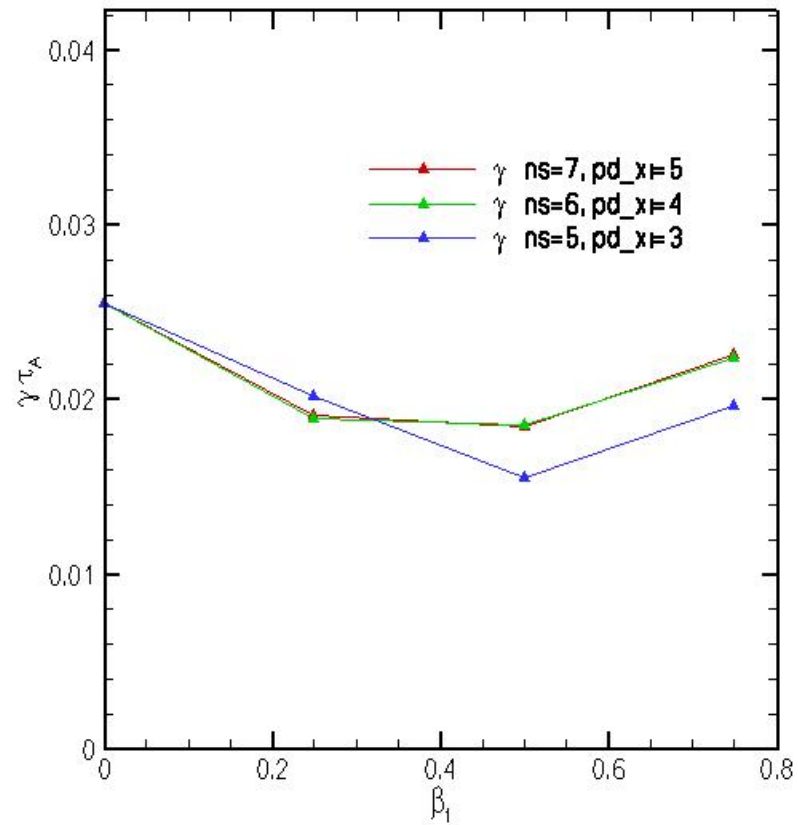
ξ grid near edge



vertex nodes uniform in $\cos^{-1}(\xi)$

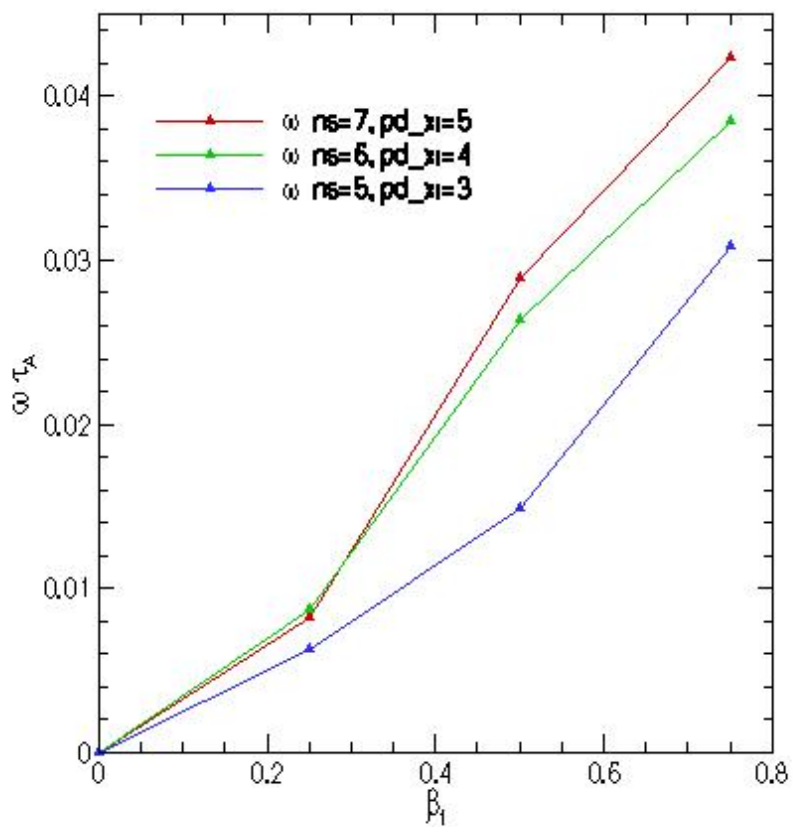


vertex nodes at $\pm\xi_t(\psi)$

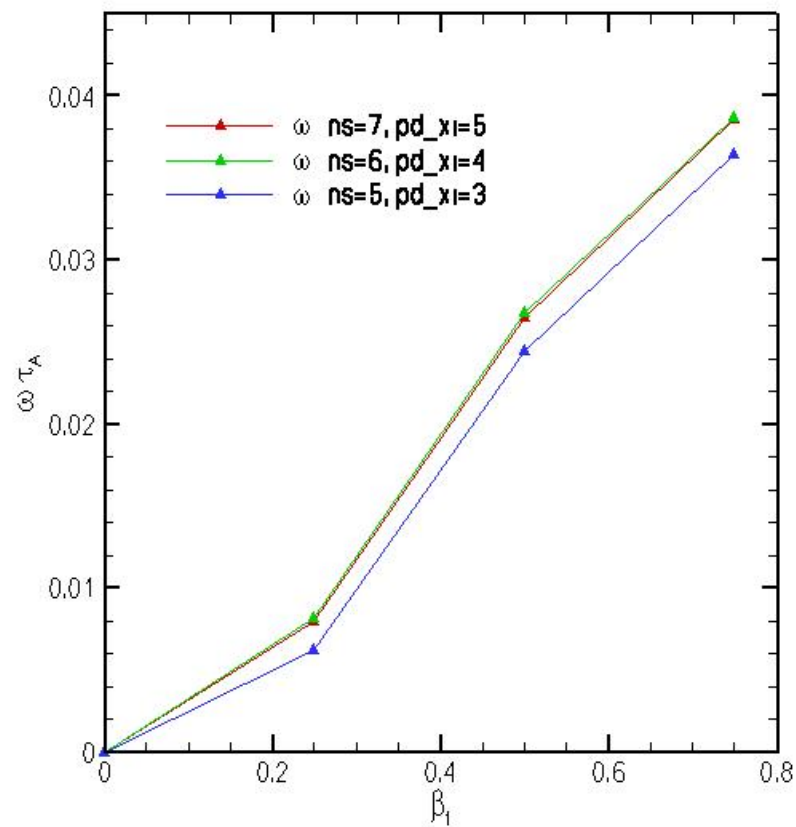


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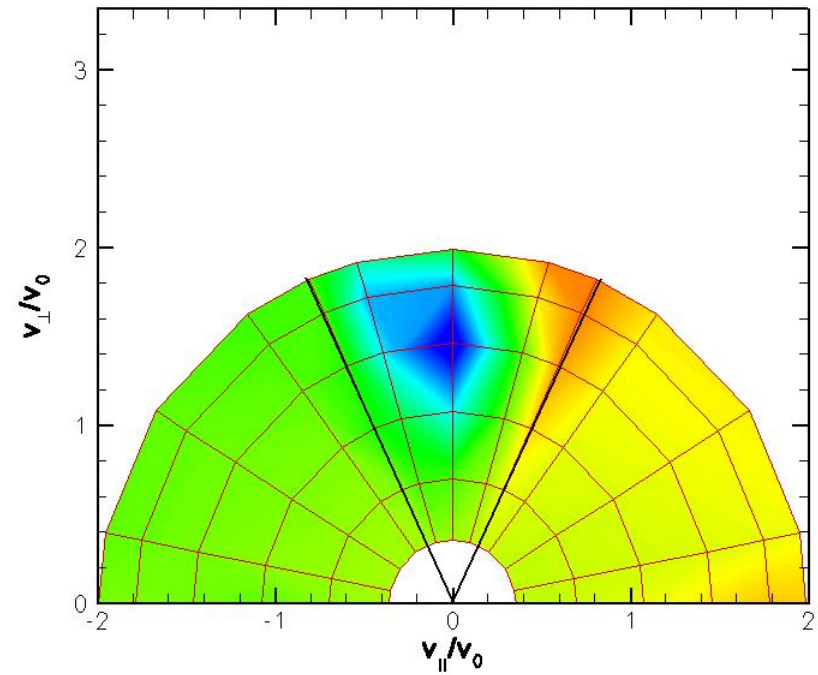
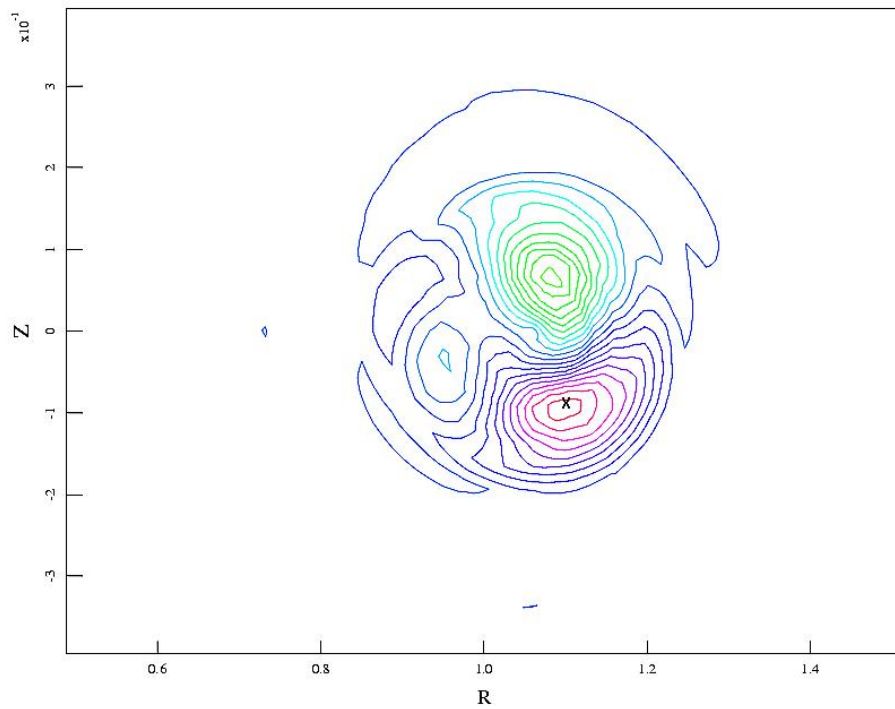


vertex nodes at $\pm\xi_t(\psi)$



Trapped particles dominate anisotropic pressure response.

- Anisotropic hot particle pressure shifted to outboard side of torus.



Future work for hot particles with continuum approach.