Hyper-resistivity and Other "Stabilization" Methods

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Thesis

Like other implicit differential operators of 4th order, hyper-dissipation can serve to “stabilize” the Hall B-advance.

Outline

- Introduction
- Hyper-dissipation implementation
- Initial RFP-related results
- Discussion points

Introduction

- The numerical stability analysis of the implicit leapfrog in (Sovinec and King, JCP **229**) focuses on the "semi-discrete" temporal advance, i.e. spatial approximation is not considered.
- Considering the function-space for the differential operators is also required with finite-elements.
- When the Hall term dominates the **B**-advance at all represented scales, that equation does not have the property of being mathematically coercive.

Wolfram MathWorld, "A bilinear functional ϕ on a normed space *E* is called coercive (or elliptic) if there exists a positive constant such that

$$\phi(x,x) \geq K\|x\|^2$$

for all $x \in E$."

The anisotropic, indefinite nature of the Hall term implies that it is not coercive.

- The cross product is anti-symmetric, and the dot product indicates a nontrivial null space (kernel).

$$\int \left[\mathbf{A}^* \cdot \Delta\mathbf{B} - \frac{\Delta t}{2\mu_0 ne} \mathbf{B}^n \cdot \left(\nabla \times \mathbf{A}^* \times \nabla \times \Delta\mathbf{B} \right) \right] dVol = \text{stuff}$$

- A related point noted by Chacon, Lukin, and others is that resistive Hall-MHD without hyper-resistivity allows the 2fl reconnection scale to vanish.

$$E_{rec} \sim \frac{\eta}{\mu_0 \delta} B \sim \frac{k}{\mu_0 ne} B^2$$

  There is no scale where dissipation dominates.

- Hyper-dissipation is one way to improve the properties of the Hall B-advance.

Formulation: The NIMROD representation requires an auxiliary field for hyper-dissipation.

- For simplicity, consider just the Hall + hyper-dissipation terms, for all **A** and **F** in the vector-function space:

$$\int\Bigg[\mathbf{A}^*\cdot\Delta\mathbf{B}-\frac{\Delta t}{2\mu_0 ne}\mathbf{B}^n\cdot\left(\nabla\times\mathbf{A}^*\times\nabla\times\Delta\mathbf{B}\right)$$

$$-\sqrt{f\eta_h\Delta t}\left(\nabla\times\mathbf{A}^*\cdot\nabla\times\mathbf{G}\right)-\sqrt{f\chi_h\Delta t}\left(\nabla\cdot\mathbf{A}^*\nabla\cdot\mathbf{G}\right)$$

$$\mathbf{F}^*\cdot\mathbf{G}+\sqrt{f\eta_h\Delta t}\left(\nabla\times\mathbf{F}^*\cdot\nabla\times\Delta\mathbf{B}\right)+\sqrt{f\chi_h\Delta t}\left(\nabla\cdot\mathbf{F}^*\nabla\cdot\Delta\mathbf{B}\right)\Bigg]dVol$$

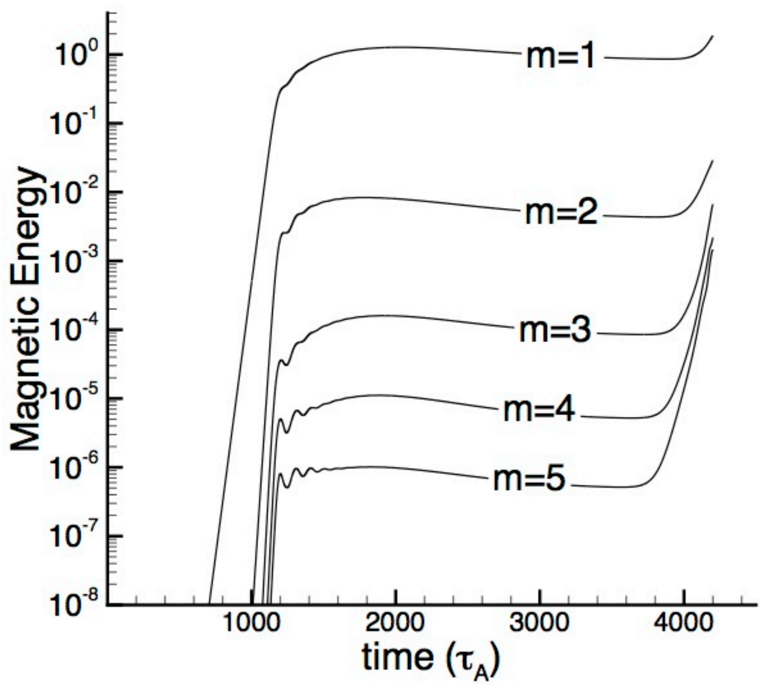
$$=\int\Bigg[\frac{\Delta t}{\mu_0 ne}\mathbf{B}^n\cdot\left(\nabla\times\mathbf{A}^*\times\nabla\times\mathbf{B}^n\right)$$

$$-\sqrt{\frac{\eta_h\Delta t}{f}}\left(\nabla\times\mathbf{F}^*\cdot\nabla\times\mathbf{B}^n\right)-\sqrt{\frac{\chi_h\Delta t}{f}}\left(\nabla\cdot\mathbf{F}^*\nabla\cdot\mathbf{B}^n\right)\Bigg]dVol$$

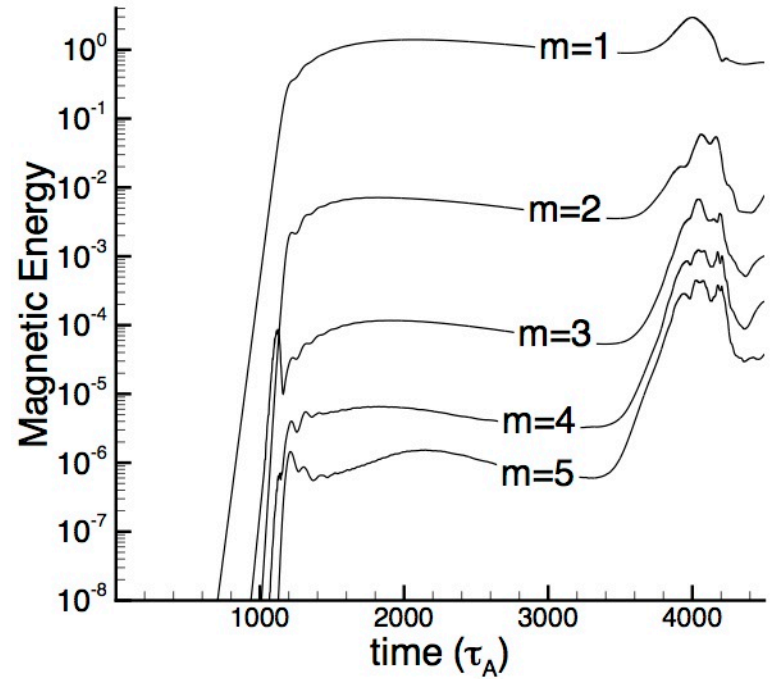
- Here, η_h is the hyper-magnetic diffusivity and χ_h is a hyper-divergence-cleaning diffusivity.

Results: The hyper-resistivity and hyper-cleaning operations have been applied in cylindrical two-fluid RFP computations.

- Josh has run S=20k computations with 120×64 meshes of degree 3.
- Results from a 60×32 mesh of degree 3 are compared below.



Fluctuation energy evolution w 120×64.



Fluctuation energy evolution w 60×32.

- Both computations have $\eta / \mu_0 = 5 \times 10^{-5}$, $\eta_h = 4 \times 10^{-8}$, and Pm=1.
- The low-resolution case also has $\chi_b = 1 \times 10^{-3}$.

Computations with Pm=0.1, low resolution, and varied η_h indicate how the hyper-dissipation terms improve the numerical properties of the implicit operator.

- With $\eta_h = 4 \times 10^{-9}$, the 3D solver takes ~40 iterations to converge the B-advance at small perturbation amplitude then fails after about 1000 steps.
- With $\eta_h = 4 \times 10^{-8}$, the 3D solver takes 7 iterations at small perturbation amplitude.
- With $\eta_h = 4 \times 10^{-7}$, the 3D solver takes 6 iterations at small perturbation amplitude.

Discussion Points

- The hyper-dissipation term for the B-advance provide another tool to help 2-fluid computations.
- Their use will require care to check that these terms (like other numerical dissipation terms) do not affect converged results.
- The 6-vector solve is more costly than 3-vector solves, but the added robustness is expected to be worth the cost in some cases.
- Other 6-vector solves can be developed by modifying the new terms.