Wall Force Diagnostic Testing

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Outline

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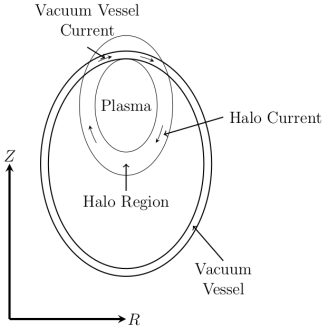
Why do we care about the force on the wall during VDEs?

- VDEs can cause large wall forces through the interaction of induced currents and magnetic fields that damage the experiment.
- Several codes have calculated wall forces during VDE events, and there is still discussion about how the wall forces are mediated by the plasma-wall system.
- We want to compare results using a force diagnostic that would be simple to implement with our resistive wall calculations in `NIMROD`.



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Let's review some general theory for calculating the force on a conducting wall.

- To find the electromagnetic force, we could find the electromagnetic force density $\mathbf{J} \times \mathbf{B}$ and integrate over the volume of interest.
- For a thin resistive wall, this method could be done but would require explicit calculation of $\mathbf{J}$ along the wall.
- Instead of using $\mathbf{J} \times \mathbf{B}$, we could instead look at the Maxwell stress $(\frac{\mathbf{BB}}{\mu_0} - \frac{|\mathbf{B}|^2}{2\mu_0}\mathbb{1})$ to find the forces in a surface integral.
- The Maxwell stress method may be easier to compare to experiments, because experiments do not need to guess what the halo current looks like.

We can separate the system into parts for calculating the wall force.

- Consider a system of coils (c), plasma (p), and the wall (w). Looking only at electromagnetic forces, we must have for the force on α due to β given by [1]

$$\mathbf{F}_{\alpha\beta} = \iiint_\alpha \mathrm{d}V \; \mathbf{J}_\alpha \times \mathbf{B}_\beta$$

$$\mathbf{F}_\alpha = \sum_\beta \iiint_\alpha \mathrm{d}V \; \mathbf{J}_\alpha \times \mathbf{B}_\beta = \iiint_\alpha \mathbf{J}_\alpha \times \mathbf{B}$$

$$\sum_\alpha \mathbf{F}_\alpha = \mathbf{0}$$

- Explicitly for the force on the wall (note $\mathbf{F}_{ww}$ is not necessarily zero because the wall is not isolated from the plasma)

$$\mathbf{F}_w = \mathbf{F}_{ww} + \mathbf{F}_{wp} + \mathbf{F}_{wc}$$

[1] This follows Pustovitov, Nucl. Fus. **55** (2015) 113032

We can then use that the plasma/wall system is an electrically isolated system.

- Call the plasma/wall system internal, then we can write

$$\mathbf{F}_p + \mathbf{F}_w \equiv \mathbf{F}_i = \mathbf{F}_{ii} + \mathbf{F}_{ic}$$

- Now we must have $\mathbf{F}_{ii} = \mathbf{0}$ because the plasma/wall is electrically isolated. So with $w+$ being a surface just outside the wall

$$\mathbf{F}_w = \mathbf{F}_{ie} - \mathbf{F}_p = \mathbf{F}_{ii} + \mathbf{F}_{ie} - \mathbf{F}_p = \mathbf{F}_i - \mathbf{F}_p \approx \mathbf{F}_i$$

$$\mathbf{F}_w \approx \mathbf{F}_i = \iiint_i \mathrm{d}V \; \mathbf{J} \times \mathbf{B} = \iiint_i \mathrm{d}V \; \left(\frac{\mathbf{BB}}{\mu_0} - \frac{|\mathbf{B}|^2}{2\mu_0} \mathbb{1} \right)$$

$$= \oint_{w+} \mathrm{d}S \; \left(\frac{\hat{\mathbf{n}} \cdot \mathbf{BB}}{\mu_0} - \frac{|\mathbf{B}|^2}{2\mu_0} \hat{\mathbf{n}} \right)$$

The calculations in NIMROD need to be transformed into Cartesian components.

- The surface integral implementation in NIMROD, uses Fourier modes in (R, Z, φ). We desire (x, y, z) Cartesian forces.
- Remember with $\hat{\mathbf{R}} = \cos\varphi\hat{\mathbf{x}} - \sin\varphi\hat{\mathbf{y}}$ that we'd find (linearly)

$$\mathbf{Q} = \mathbf{Q}_0 + \sum_{n=1}^{\infty}\left(\mathbf{Q}_n e^{in\varphi} + \mathbf{Q}_n^* e^{-in\varphi}\right)$$

$$\int_0^{2\pi} \mathrm{d}\varphi\, (\hat{\mathbf{R}}\cdot\mathbf{Q})\hat{\mathbf{R}} = 2\pi\left[\Re\left\{\hat{\mathbf{R}}\cdot\mathbf{Q}_1\right\}\hat{\mathbf{x}} + \Im\left\{\hat{\mathbf{R}}\cdot\mathbf{Q}_1\right\}\hat{\mathbf{y}}\right]$$

$$\hat{\mathbf{x}}\cdot\int_0^{2\pi} \mathrm{d}\varphi\, \mathbf{Q} = 2\pi\left[\Re\left\{\hat{\mathbf{R}}\cdot\mathbf{Q}_1\right\} + \Im\left\{\hat{\boldsymbol{\varphi}}\cdot\mathbf{Q}_1\right\}\right]$$

$$\hat{\mathbf{y}}\cdot\int_0^{2\pi} \mathrm{d}\varphi\, \mathbf{Q} = 2\pi\left[\Im\left\{\hat{\mathbf{R}}\cdot\mathbf{Q}_1\right\} - \Re\left\{\hat{\boldsymbol{\varphi}}\cdot\mathbf{Q}_1\right\}\right]$$

$$\hat{\mathbf{z}}\cdot\int_0^{2\pi} \mathrm{d}\varphi\, \mathbf{Q} = 2\pi\hat{\mathbf{Z}}\cdot\mathbf{Q}_0$$

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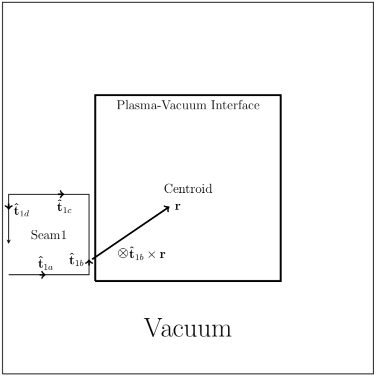
We can calculate the force with NIMROD surface integrals along the resistive wall.

- We could do these calculations in NIMROD or put into NIMPLOT.
- Originally chose NIMROD because NIMROD already can determine if boundary is the plasma-wall interface.
  - Calculating the nonlinear Maxwell stress has difficulties Fourier transform calls for single points for parallelization.
  - There is no simple way to find $\mathbf{B}_{n=0}$ for the Maxwell stress calculation in the vacuum region currently.
  - We need a way to find the $\mathbf{B}$ from the external coils used in finding an equilibrium.
- To avoid these problems, we used NIMPLOT.

We use NIMPLOT with a simple algorithm for finding the plasma-wall interface.

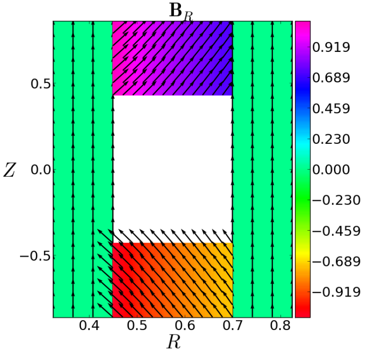
- The NIMPLOT algorithm uses that our seams all go around counterclockwise as seen to the right.
- Then, for simple geometries, we can use that the tangent vector crossed with the vector pointing to the centroid will point into the page for those points along the seam that are required.



A simple test case helps verify the force diagnostic.

- A simple rectangular vacuum was created with a $(n = 1)$ $\mathbf{B}_R$ field which would produce a horizontal force shown below at a single toroidal cross section, and a uniform $\mathbf{B}_Z$ throughout the entire vacuum.
- Vertical forces can also be tested by putting in different $\mathbf{B}_Z$ on the top and bottom of the vacuum-plasma interface.

- Given $B_R = \frac{10^{-7}}{R}$ and $\mu_0 = 1$ we expect a horizontal force of $\frac{\pi B_R R}{\mu_0} \approx 3.142 \times 10^{-7}$
- The highest resolution case yields 3.141×10^{-7} with convergence as resolution increases.



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Conclusions

- A mechanism for calculating the wall force directly from the Maxwell stress was implemented in NIMROD.
- The surface integration capabilities of NIMROD were used to find the wall force.
- A simple algorithm for finding the vacuum-plasma interface is used for current calculations.
- The force in the wall matches analytical predictions linearly and nonlinearly.

Future Work

- Implement more robust vacuum-plasma interface test.
- Perhaps calculate higher order moments on the wall in the future.
  - The surface integral of the Maxwell stress implementation would not work for higher order moments.
- Local force density calculations along the wall may also be useful.