Implementation of parallel moment equations in NIMROD

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Motivation

- Braginskii closures in Nimrod
  - In large Knudsen number, trying to capture some kinetic effects about parallel dynamics for magnetized plasmas
- Integral closures in Nimrod
  - Adiabatic and linear approxiamtions to obtain the integral closures
  - Trying to incorporate non-adiabatic and nonlinear effects into parallel closures
  - Calculating exact parallel moment equations without integrals along the field lines

Total velocity moment equations

- Simpler than random velocity moments
  - Landau (Fokker-Planck) kinetic equation

$$\frac{\partial f_a}{\partial t} + \mathbf{v}\cdot\nabla f_a + \frac{q_a}{m_a}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f_a}{\partial \mathbf{v}} = \sum_b C(f_a, f_b)$$

  - Moment expansion

$$f_a(t,\mathbf{x},\mathbf{v}) = f_a^{\mathrm{M}} \sum_{lk} \mathsf{M}_a^{lk}(t,\mathbf{x})\cdot\hat{\mathsf{P}}_a^{lk} \text{ with } f_a^{\mathrm{M}} = \frac{n_a}{\pi^{3/2}v_{Ta}^3}\exp(-s_a^2)$$

$\mathbf{s_a} = \frac{\mathbf{v_a}}{v_{T_a}}$,$\mathbf{c_a} = \frac{\mathbf{v_a}-\mathbf{V_a}}{v_{T_a}}$ (in random velocity moments)

$\mathsf{N}_a^{lk} \equiv n_a \mathsf{M}_a^{lk}(t,\mathbf{x}) = \int d\mathbf{v}\hat{\mathsf{P}}_a^{lk} f_a(t,\mathbf{x},\mathbf{v})$

where $\hat{\mathsf{P}}^{lk}$'s are orthonormal polynomials of $\mathbf{s}_a$

Several low order moment equations

$\mathsf{P}^{lk}(\mathbf{s}) = \mathsf{P}^l(\mathbf{s})L_k^l(s^2)$	$L_k^l = L_k^{(l+\frac{1}{2})}$	M^{lk}	fluid moments	indep.
$\mathsf{P}^0 = 1$	$L_0^0 = 1$	1	1	1
	$L_1^0 = \frac{3}{2} - s^2$	M^{01}	$-\sqrt{\frac{2}{3}}\frac{V^2}{v_T^2}$	1
$\mathsf{P}^1 = \mathbf{s}$	$L_0^1 = 1$	M^{10}	$\sqrt{2}\frac{\mathbf{V}}{v_T}$	3
	$L_1^1 = \frac{5}{2} - s^2$	M^{11}	$-\sqrt{\frac{4}{5}}\frac{1}{nv_TT}(\mathbf{H} - \frac{5}{2}p\mathbf{V})$	3

When $\mathbf{V} \to 0$, $(\mathbf{H} - \frac{5}{2}p\mathbf{V}) \to \mathbf{h}$ in the random velocity moment expression

- Variables are $\mathsf{N}_a^{lk} \equiv n_a \mathsf{M}_\mathsf{a}^\mathsf{lk}(\mathsf{t}, \mathbf{x})$

Parallel moment equations in NIMROD

- Total velocity moment equations (no $\nabla \mathbf{V}$ coupled terms)[Ji, Jeong-Young, and Eric D. Held. "A framework for moment equations for magnetized plasmas." Physics of Plasmas 21.4 (2014): 042102.]

$$
\partial_t \bar{N}_{\|}^{lp} + \sum_k \Bigg[\bar{\Xi}_{pk}^{l} (\partial_t \ln T) \bar{N}_{\|}^{lk} + v_T (\bar{\Psi}_{pk}^{l+} \partial_{\|}^{l+} + \bar{\Phi}_{pk}^{l+} \partial_{\|} \ln T + \frac{q}{2T} \bar{\Theta}_{pk}^{l+} E_{\|}) \bar{N}_{\|}^{l+1,k}
$$

$$
+ v_T (\bar{\Psi}_{pk}^{l-} \partial_{\|}^{l-} + \bar{\Phi}_{pk}^{l-} \partial_{\|} \ln T + \frac{q}{2T} \bar{\Theta}_{pk}^{l-} E_{\|}) \bar{N}_{\|}^{l-1,k} \Bigg] = \sum C^{lpk} \bar{N}_{\|}^{lk}
$$

$$
+ C^{(2)lp}(\bar{N}_{\|}, \bar{N}_{\|}) + G^{lp}
$$

$$
\begin{aligned}
\partial_{\|}^{l+} &= \partial_{\|} - \frac{l+2}{2} (\partial_{\|} \ln B) \\
\partial_{\|}^{l-} &= \partial_{\|} + \frac{l-1}{2} (\partial_{\|} \ln B)
\end{aligned}
$$

- $(l, k) = (0,0), (1,0), (0,1)$ driving terms are in the rhs.
- For example, $\frac{1}{n} \partial_{\|}^{2-} \bar{N}_{\|}^{1,0} = \partial_{\|}^{2-} \bar{M}_{\|}^{1,0} = (\partial_{\|} + \frac{1}{2}(\partial_{\|} \ln B))(\sqrt{2}\frac{\mathbf{b}\cdot\mathbf{V}}{v_T}) =$ $\mathbf{b} \cdot \nabla \sqrt{2}\frac{\mathbf{b}\cdot\mathbf{V}}{v_T} + \frac{1}{2}(\partial_{\|} \ln B)(\sqrt{2}\frac{\mathbf{b}\cdot\mathbf{V}}{v_T})$ (by using parallel strain tensor)

Time advance scheme

- In matrix form

$$\partial_t [n]^{\rm even} = [c]\,[n]^{\rm even} + [A]\,[n]^{\rm odd} + \left[C^{(2)}(n,n)\right] + [g(n,T,\nabla\mathbf{V})]$$

$$\partial_t [n]^{\rm odd} = [c]\,[n]^{\rm odd} + [A]\,[n]^{\rm even} + \left[C^{(2)}(n,n)\right] + [g(n,T,\mathbf{V}_{\rm ei})]$$

$$[A] = -[\psi]\,\partial_\parallel^\pm - [\phi]\,\partial_\parallel \ln T - [\theta]\,E_\parallel$$

- Predictor-corrector time advance $t^k \rightarrow t^{k+1}$

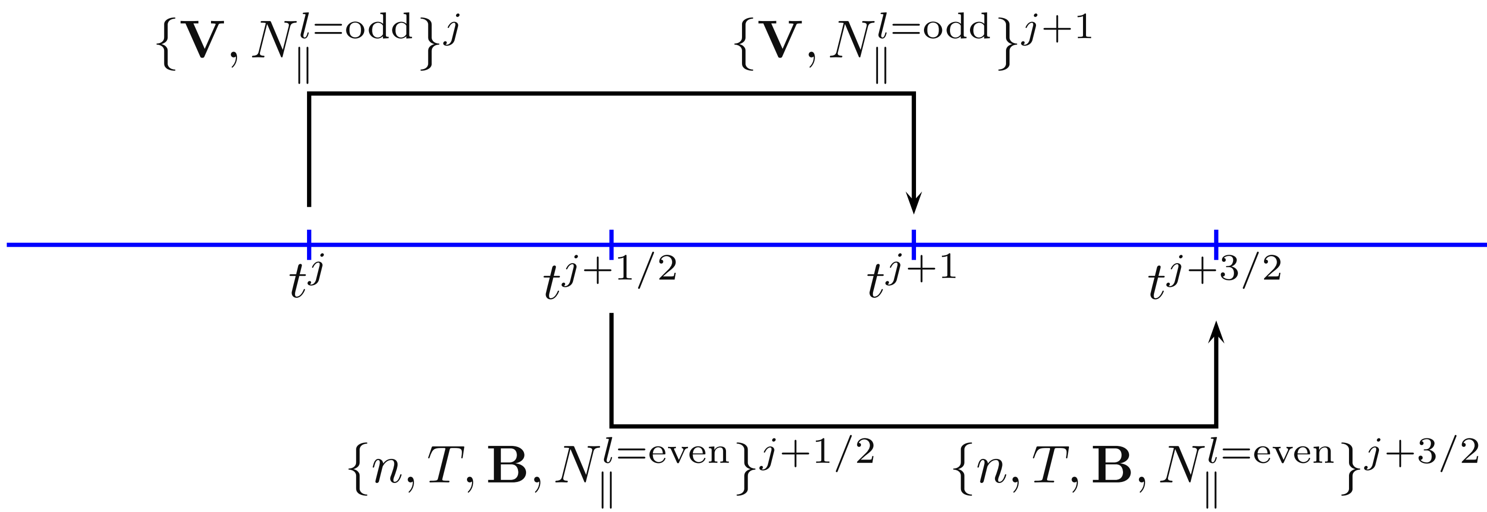
$$(1+\Delta t f_{[n]}\,[c])\,[\Delta n]_{\rm pass} = \Delta t([c]\,[n]^k + [A]\,[n]^{k+1/2} + \left[C^{(2)}([n]^*,[n]^*)\right] + [g]^*)$$

  - For pass=predict, $[n]^* = [n]^k \longrightarrow [\Delta n]_{\rm predict}$
  - For pass=correct, $[n]^{k+1} = [n]^k + f_{[n]}\,[\Delta n]_{\rm predict}$

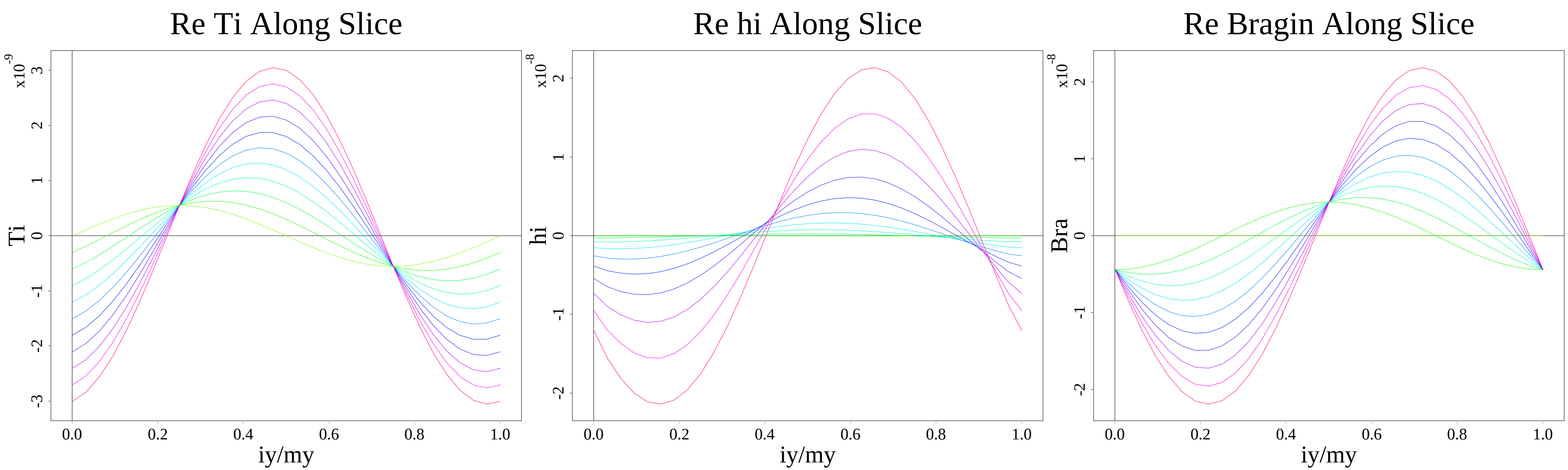
- Currently implemented for $f_{[n]} = 0$ and no $C^{(2)}$ term

Leap-frog scheme between even and odd moments

- NIMROD's flow velocity $(\mathbf{V})$ is staggered 1/2 step from number density (n), temperature (T), and magnetic field $(\mathbf{B})$ [Sovinec *et al*, International Sherwood Fusion Theory Conference 2008]

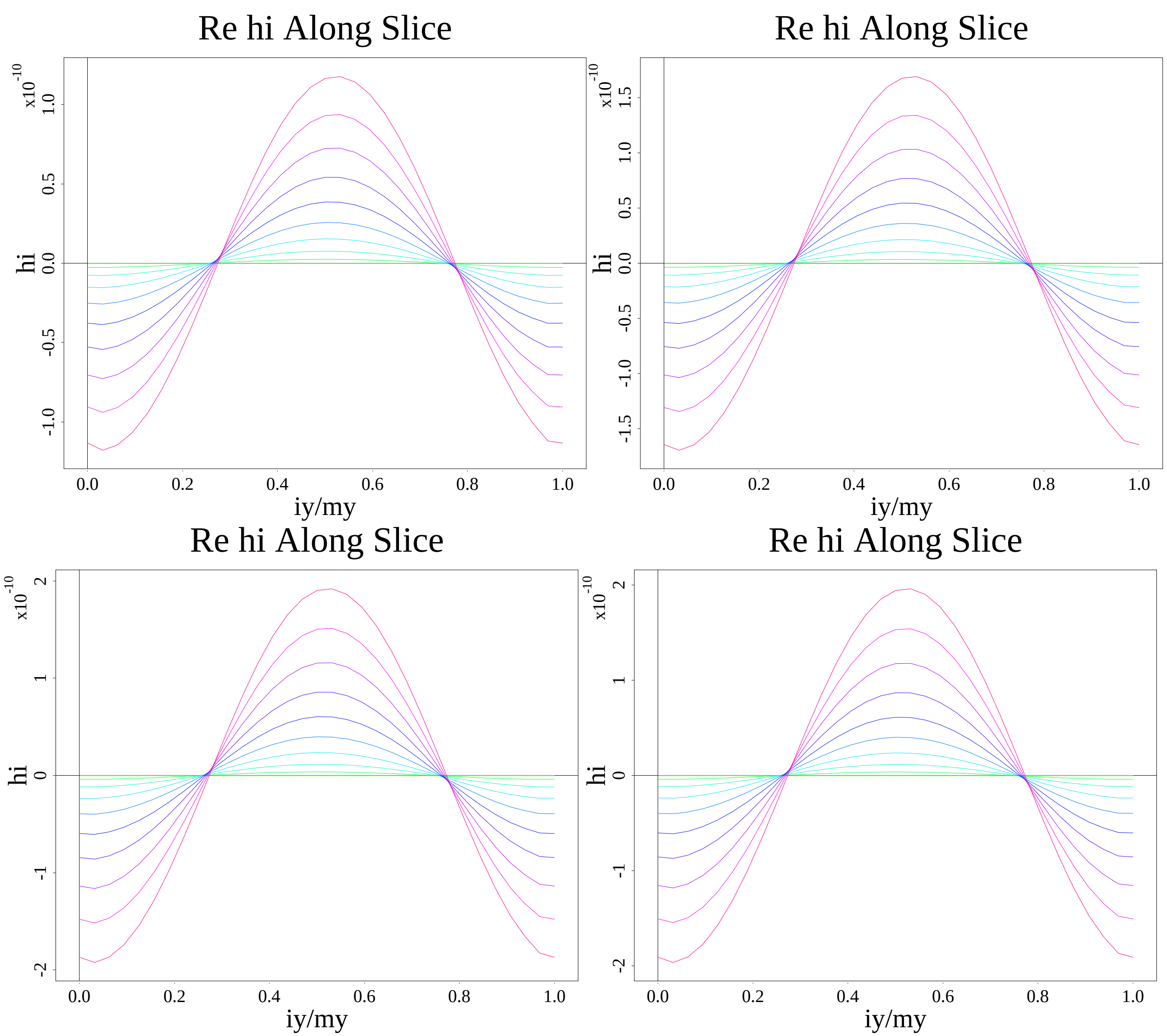


Verification by Ion acoustic wave simulation (ongoing)



- Ion temperature: 0.9 eV
- Using until $(l, k) = (2, 2)$ moments. Almost same results with Braginskii closure

Convergence along number of moments



(l max ,k max)=(2,2), (4,4), (8,8), (10,10)

Future work

- Verification in equilibrium states [Held, E. D., et al. "Verification of continuum drift kinetic equation solvers in NIMROD." Physics of Plasmas 22.3 (2015)]
  - High aspect ratio equilibrium
  - High beta DIII-D like equilibrium

- Applying to nonlinear and time dependent phenomena
  - Magnetic reconnection
  - Plasma sheath
  - Waves