NIMROD modeling of poloidal flow damping in tokamaks using kinetic closures

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Introduction

• Poloidal flow damping serves as a “simple” test case which can be used to benchmark NIMROD kinetic formulations.

• Both a “delta-F” and “Chapman-Enskog-like” formulation have been implemented in the code, but only the delta-F formulation will be benchmarked herein.

• Analytic results for the poloidal flow dynamics [2], will be compared to NIMROD “delta-F” kinetic results.
Delta-F

- The delta-F approach solves Hazelton’s first-order drift kinetic equation (DKE) for the perturbation away from a lowest order, static Maxwellian [3].
- Hazelton’s first-order DKE is given as:

\[
\frac{\partial f}{\partial t} + (v_{\parallel} + v_D) \cdot \nabla f + \left( \mu \frac{\partial B}{\partial t} + q(v_{\parallel} + v_D) \cdot E \right) \frac{\partial f}{\partial \epsilon} = C(f)
\]

- Moments of the kinetic perturbation are taken to obtain the relevant fluid quantities of interest, including the parallel viscous force.
The Chapman-Enskog-like (CEL) formulation solves the CEL-DKE for the perturbation away from a time-evolving, flow-shifted Maxwellian [5].

Information about temperature, number density, and flow is contained in the Maxwellian distribution. These quantities are then evolved using NIMROD’s set of fluid equations.

The DKE is evaluated for each species in the macroscopic flow reference frame. It is given for the electrons as [6]:

\[
\frac{\partial f_{\text{NM}e}}{\partial t} + v'_{i} b \cdot \frac{\partial f_{\text{NM}e}}{\partial x} + \left( \frac{T_{e}}{m_{e}} \right) b \cdot \nabla \ln n + \frac{v'^{2}_{f}}{2} b \cdot \nabla \ln B \right) \frac{\partial f_{\text{NM}e}}{\partial v'_{f}} + \frac{v'_{t} v'_{i}}{2} b \cdot \nabla \ln B \frac{\partial f_{\text{NM}e}}{\partial v'_{t}} \]

\[= \left[ \frac{v'_{t}}{2T_{e}} \left( 5 - \frac{m_{e}}{T_{e}} (v'^{2}_{i} + v'^{2}_{t}) \right) b \cdot \nabla T_{e} + \frac{3}{nT_{e}} b \cdot \left[ \frac{2}{3} \nabla (p_{ei} - p_{e}) - (p_{ei} - p_{e}) \nabla \ln B - F_{\text{coll}} \right] \right] \]

\[+ \frac{m_{e}}{6T_{e}} (2v'^{2}_{i} - v'^{2}_{t}) (\nabla \cdot u_{e} - 3b \cdot (b \cdot \nabla) u_{e}) + \frac{1}{3nT_{e}} \left[ \frac{m_{e}}{T_{e}} (v'^{2}_{i} + v'^{2}_{t}) - 3 \right] [\nabla \cdot (q_{e} - F_{\text{coll}})] \]

\[+ \frac{1}{2eB} \left[ \frac{m_{e}^{2}}{T_{e}} (v'^{4}_{i} + v'^{4}_{t} + v'^{4}_{t}) - 5 \frac{m_{e}}{3T_{e}} (4v'^{2}_{i} + v'^{2}_{t}) + 5 \right] (b \times \kappa) \cdot \nabla T_{e} \]

\[+ \frac{1}{2T_{e}} \left[ \frac{m_{e}^{2}}{2T_{e}} (v'^{2}_{i} + v'^{2}_{t}) + 5 \frac{m_{e}}{3T_{e}} (2v'^{2}_{i} + 5v'^{2}_{t}) + 5 \right] (b \times \nabla \ln B) \cdot \nabla T_{e} \]

\[+ \frac{m_{e}}{6eB T_{e}} (2v'^{2}_{i} - v'^{2}_{t}) (b \times \nabla \ln n) \cdot \nabla T_{e} \left[ f_{\text{Me}} + (C_{ei} f_{e} + f_{i}) + C_{ei} f_{e} f_{i} \right] \]

The lowest order equation for the ions is similar, with the omission of the drive terms proportional to the collisional friction and heat friction terms.
Comparison of Formulations

**Delta-F**

*Pros:*  
- Simple to implement.
- Requires that the perturbed quantities such as number density, temperature, and flow-velocity remain small relative to the equilibrium quantities.

*Cons:*  
- Harder to develop self-consistent formulism.

**CEL**

*Pros:*  
- Perturbed number density, temperature, and flow-velocity are allowed to become comparable to their equilibrium values.
- Only information that is solved for is that which is needed to close the fluid equations.
- Easier to develop self-consistent formulism.

*Cons:*  
- More difficult to implement
Flow Equations

Assuming that to lowest order the perpendicular flow for the ions is given by the diamagnetic and \( E \times B \) flows, and assuming number density, temperature, and electric potential are flux functions, then it can be shown that the following relations hold for the ion flow [2,7]:

\[
\begin{align*}
    u_\parallel &= BU_\theta - \frac{RB_t}{B} \left( \frac{p'}{qn} + \phi' \right) \\
    u_\zeta &= B_tU_\theta - R \left( \frac{p'}{qn} + \phi' \right)
\end{align*}
\]

Here \( u_\zeta \) = toroidal flow, \( u_\parallel \) = flow parallel to magnetic field, and \( U_\theta \) is a flux function, defined as:

\[
U_\theta = \frac{u \cdot \nabla \theta}{B \cdot \nabla \theta}
\]

where \( \theta \) = poloidal angle.
Testing of Delta-F Formulism

- The first order drift kinetic equation for the ions was evolved assuming a stationary magnetic field.
- An equilibrium from NSTX was used (see figure).
- For each case both a gaussian and flow-shifted perturbation were imposed upon the equilibrium Maxwellian (see figures).
- Results for the poloidal flow damping obtained by another code were also used for comparison (for an example see below).
Initial Try of Closing for the Electric Potential
(and $U_\theta$)

- The electric potential was computed here by subtracting the flux-surface average of $\frac{u_\parallel}{B}$ from itself [see 9,10 for other examples of using flux-surface averages to close for the electric potential].

- This gives (using the aforementioned flow equations):

$$\phi' = \frac{1}{RB_i} \left[ \left( \frac{u_\parallel}{B} - \left\langle \frac{u_\parallel}{B} \right\rangle \right) \left( \left\langle \frac{1}{B^2} \right\rangle - \frac{1}{B^2} \right)^{-1} \right] - \frac{p'}{qn}$$

- A similar procedure leads to (for $U_\theta$):

$$U_\theta = \left( \left\langle B u_\parallel \right\rangle - B u_\parallel \right) \left( \left\langle B^2 \right\rangle - B^2 \right)^{-1}$$
Potential Problem with Aforementioned Approach

- Only works when $\frac{1}{B^2}$ and $B^2$ are not equal to their own flux-surface averages (otherwise the denominators go to 0).
- Closures instead from references 9 or 10 may have to be utilized.
Poloidal Flow Condition - Gaussian Initial Condition

\[ \nu_{ii} \sim 10^{-3} \]

\[ \nu_{ii} \sim 10^{-2} \]
Poloidal Flow - Flow-Shifted Initial Condition

$\nu_{ii} \sim 10^{-3}$

$U_{\theta}$ vs $t$

$\nu_{ii} \sim 10^{-2}$

$U_{\theta}$ vs $t$
Watching evolving F-ion Contours

Fion, extrema=(1.005e-15, 3.061e+00)

Fion, extrema=(-2.507e-02, 1.085e+00)

Fion, extrema=(-3.558e-03, 8.517e-01)

Fion, extrema=(-1.857e-03, 8.019e-01)

Fion, extrema=(-1.550e-03, 7.893e-01)

Fion, extrema=(-1.047e-03, 7.813e-01)
Comparison with Analytic Results

We have shown, in agreement with previous work, that for the majority of the poloidal flow damping the decay is exponential, with a time constant on the order of the ion-ion collision time [2].
Future Work

• Compare poloidal flow evolution obtained using the CEL formulation to analytics [11], and to delta-F formulation.
• Compare efficiency and effectiveness of CEL formulation to that of delta-F for other test cases.