

# Improving Braginskii theory for finite flow velocity

Jeong-Young Ji, J. Andrew Spencer, and Eric D. Held

Utah State University

NIMROD Team Meeting

October 19 2019, Fort Lauderdale, FL

# Kinetic equation and collision operator

- Kinetic equation for  $f_a(t, \mathbf{r}, \mathbf{v})$

$$\frac{\partial f_a}{\partial t} + \mathbf{v} \cdot \frac{\partial f_a}{\partial \mathbf{r}} + \frac{q_a}{m_a} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_a}{\partial \mathbf{v}} = \sum_b C(f_a, f_b)$$

- Landau collision operator
  - ▶ Landau (1937)

$$C(f_a, f_b) = \frac{\gamma_{ab}}{2m_a} \frac{\partial}{\partial \mathbf{v}} \cdot \int d\mathbf{v}' \mathbf{U} \cdot \left[ \frac{\partial f_a(\mathbf{v})}{\partial \mathbf{v}} f_b(\mathbf{v}') - f_a(\mathbf{v}) \frac{m_a}{m_b} \frac{\partial f_b(\mathbf{v}')}{\partial \mathbf{v}'} \right]$$

where  $\mathbf{U} = \frac{|\mathbf{v} - \mathbf{v}'|^2 | - (\mathbf{v} - \mathbf{v}')(\mathbf{v} - \mathbf{v}')}{|\mathbf{v} - \mathbf{v}'|^3}$  and  $\gamma_{ab} = \frac{q_a^2 q_b^2 \ln \Lambda_{ab}}{4\pi \epsilon_0^2 m_a} = \frac{3\sqrt{\pi} m_a v_{Ta}^3}{4n_b \tau_{ab}}$

- ▶ Rosenbluth, MacDonald, and Judd (1957)

$$C(f_a, f_b) = \frac{\gamma_{ab}}{2m_a} \frac{\partial}{\partial \mathbf{v}} \cdot \left[ \frac{\partial}{\partial \mathbf{v}} \cdot \left( f_a \frac{\partial}{\partial \mathbf{v}} \frac{\partial G_b^+}{\partial \mathbf{v}} \right) - 2 \left( 1 + \frac{m_a}{m_b} \right) f_a \frac{\partial G_b^-}{\partial \mathbf{v}} \right]$$

where

$$G_b^\pm(\mathbf{v}) = \int d\mathbf{v}' |\mathbf{v} - \mathbf{v}'|^{\pm 1} f_b(\mathbf{v}')$$

# General moment expansion [Ji and Held 2006 2008 2009]

## Irreducible Hermite polynomials [Chapman 1916 Enskog 1917 Grad 1963]

$$\mathbf{c}_a = \frac{\mathbf{w}_a}{v_{Ta}} = \frac{\mathbf{v} - \mathbf{V}_a}{v_{Ta}}, \quad v_{Ta} = \sqrt{\frac{2T_a}{m_a}}, \quad \text{vs.} \quad \mathbf{s}_a = \frac{\mathbf{v}}{v_{Ta}}$$

- Moment expansion:  $m_a^{lk}$  (vs.  $M_a^{lk}$ ) are symmetric traceless fluid moments

$$f_a(t, \mathbf{r}, \mathbf{v}) = f_a^M \sum_{lk} m_a^{lk}(t, \mathbf{r}) \cdot \hat{\mathbf{p}}_a^{lk} \quad \text{vs.} \quad f_a(t, \mathbf{r}, \mathbf{v}) = f_a^m \sum_{lk} M_a^{lk}(t, \mathbf{r}) \cdot \hat{\mathbf{P}}_a^{lk}$$

$$n_a^{lk}(t, \mathbf{r}) \equiv n_a m_a^{lk} = \int d\mathbf{v} \hat{\mathbf{p}}_a^{lk} f_a \quad N_a^{lk}(t, \mathbf{x}) \equiv n_a M_a^{lk} = \int d\mathbf{v} \hat{\mathbf{P}}_a^{lk} f_a$$

- $\hat{\mathbf{p}}^{lk}$ 's are orthonormal, irreducible, tensorial polynomials and form a complete set

$$\int d\mathbf{v} \hat{\mathbf{p}}^{jp} \hat{\mathbf{p}}^{lk} \cdot m^{lk} f^M = \delta_{jl} \delta_{pk} n^{jp}, \quad \hat{\mathbf{p}}^{lk} = \frac{1}{\sqrt{\sigma_{lk}}} \mathbf{p}^{lk}$$

$$\begin{aligned} \mathbf{p}_a^{lk} &= \mathbf{P}^l(\mathbf{c}_a) L_k^{(l+1/2)}(c_a^2) \quad \text{vs.} \quad \mathbf{P}_a^{lk} = \mathbf{P}^l(\mathbf{s}_a) L_k^{(l+1/2)}(s_a^2) \\ &= (\text{harmonic tensor})(\text{associated Laguerre polynomial}) \end{aligned}$$

$$f_a^M = \frac{n_a}{\pi^{3/2} v_{Ta}^3} e^{-c_a^2} \quad (\text{Maxwellian distribution}) \quad \text{vs.} \quad f_a^m = \frac{n_a}{\pi^{3/2} v_{Ta}^3} e^{-s_a^2}$$

# Several low order moments (21 moments)

$\mathbf{p}^{lk} = \mathbf{P}^l(\mathbf{c})L_k^l(c^2)$	$L_k^l = L_k^{(l+\frac{1}{2})}$	$n^{lk}$	fluid moment equation	indep.
$\mathbf{P}^0 = 1$	$L_0^0 = 1$ $L_1^0 = \frac{3}{2} - c^2$	$n$ $0$	density ( $n$ ) temperature ( $T$ )	1 1
$\mathbf{P}^1 = \mathbf{c}$	$L_0^1 = 1$ $L_1^1 = \frac{5}{2} - c^2$ $L_2^1 = \frac{35}{8} - \frac{7}{2}c^2 + \frac{1}{2}c^4$	$0$ $n^{11}$ $n^{12}$	flow velocity ( $\mathbf{V}$ ) heat flow ( $\mathbf{h}$ ) heat w. heat flow ( $\mathbf{r}$ )	3 3 3
$\mathbf{P}^2 = \mathbf{c}\mathbf{c} - \frac{c^2}{3}\mathbf{I}$	$L_0^2 = 1$ $L_1^2 = \frac{7}{2} - c^2$	$n^{20}$ $n^{21}$	viscosity ( $\boldsymbol{\pi}$ ) heat viscosity ( $\boldsymbol{\theta}$ )	5 5

$$\begin{aligned}
 f = f^{\mathbf{M}\{n, \mathbf{V}, T\}} & \left( 1 + \dots \right. && \leftarrow \text{scalar moments} \\
 & + \sigma_{\mathbf{h}} \mathbf{p}^{11} \cdot \mathbf{h} + \sigma_{\mathbf{r}} \mathbf{p}^{12} \cdot \mathbf{r} + \dots && \leftarrow \text{vector moments} \\
 & + \sigma_{\boldsymbol{\pi}} \mathbf{p}^{20} \cdot \boldsymbol{\pi} + \sigma_{\boldsymbol{\theta}} \mathbf{p}^{21} \cdot \boldsymbol{\theta} + \dots && \leftarrow \text{rank-2 tensor moments} \\
 & + \dots \left. \right) && \leftarrow \text{higher rank tensor moments}
 \end{aligned}$$

# Fluid equations and closures

Maxwellian moment  $(n_a, \mathbf{V}_a, T_a)$  equations

$$(0,0) \quad d_t n_a + n_a \nabla \cdot \mathbf{V}_a = 0 \quad (d_t \equiv \partial_t + \mathbf{V}_a \cdot \nabla)$$

$$(0,1) \quad \frac{3}{2} n_a d_t T_a + n_a T_a \nabla \cdot \mathbf{V}_a + \nabla \cdot \mathbf{h}_a + \nabla \mathbf{V}_a : \boldsymbol{\pi}_a = Q_a$$

$$(1,0) \quad m_a n_a d_t \mathbf{V}_a - n_a q_a (\mathbf{E} + \mathbf{V}_a \times \mathbf{B}) + \nabla p_a + \nabla \cdot \boldsymbol{\pi}_a = \mathbf{R}_a$$

General moment equations  $Dn + \Omega \mathbf{b} \check{\times} n = Cn$  ( $n^{lk} \rightarrow v^{l+2k}$  moment)

$$(1,1) \quad d_t \mathbf{h} + \Omega \mathbf{b} \times \mathbf{h} + \frac{7}{5} (\nabla \cdot \mathbf{V}) \mathbf{h} + \frac{7}{5} \mathbf{h} \cdot (\nabla \mathbf{V}) + \frac{2}{5} (\nabla \mathbf{V}) \cdot \mathbf{h} + \frac{5p}{2m} \nabla T \\ + \frac{T}{m} \nabla \cdot \boldsymbol{\pi} + \frac{7}{2} \frac{\nabla T}{m} \cdot \boldsymbol{\pi} - \mathbf{a} \cdot \boldsymbol{\pi} + \nabla \cdot \boldsymbol{\theta} + \frac{1}{3} \nabla u^{02} + \nabla \mathbf{V} : u^{30} \\ = C_{10}^1 \mathbf{V}_{ei} + C_{11}^1 \mathbf{h} + C_{12}^1 \mathbf{r} + \dots \quad (\mathbf{h} \text{ heat flow})$$

$$(1,2) \quad d_t \mathbf{r} + \Omega \mathbf{b} \times \mathbf{r} + \dots = C_{10}^1 \mathbf{V}_{ei} + C_{21}^1 \mathbf{h} + C_{22}^1 \mathbf{r} + \dots \quad (\mathbf{r} \text{ heat heat flow})$$

$$(2,0) \quad d_t \boldsymbol{\pi} + \Omega \mathbf{b} \check{\times} \boldsymbol{\pi} + (\nabla \cdot \mathbf{V}) \boldsymbol{\pi} + 2 \boldsymbol{\pi} \cdot (\nabla \mathbf{V}) + pW + \frac{4}{5} \overline{\nabla \mathbf{h}} + \nabla \cdot u^{30} \\ = C_{00}^2 \boldsymbol{\pi} + C_{01}^2 \boldsymbol{\theta} + \dots \quad (\boldsymbol{\pi} \text{ viscosity})$$

$$(2,1) \quad d_t \boldsymbol{\theta} + \Omega \mathbf{b} \check{\times} \boldsymbol{\pi} + \dots = C_{10}^2 \boldsymbol{\pi} + C_{11}^2 \boldsymbol{\theta} + \dots \quad (\boldsymbol{\theta} \text{ heat viscosity})$$

$$\text{where } \mathbf{a} = \frac{q}{m} (\mathbf{E} + \mathbf{V} \times \mathbf{B}) - d_t \mathbf{V} \text{ and } W = \nabla \mathbf{V} + (\nabla \mathbf{V})^T - \frac{2}{3} \nabla \cdot \mathbf{V} \mathbf{I}$$

Closures: express  $\mathbf{h}_a(n_a^{11})$ ,  $\boldsymbol{\pi}_a(n_a^{20})$ ,  $Q_a$ ,  $\mathbf{R}_a$  in terms of  $n_a, \mathbf{V}_a, T_a$

$$\mathbf{h}_e = -\kappa_{\parallel}^e \nabla_{\parallel} T_e - \kappa_{\perp}^e \nabla_{\perp} T_e - \kappa_{\times}^e \nabla_{\times} T_e + \beta_{\parallel} T_e \mathbf{V}_{ei\parallel} + \beta_{\perp} T_e \mathbf{V}_{ei\perp} + \beta_{\times} T_e \mathbf{V}_{ei\times}$$

$$\mathbf{R}_e = -\alpha_{\parallel} \mathbf{V}_{ei\parallel} - \alpha_{\perp} \mathbf{V}_{ei\perp} + \alpha_{\times} \mathbf{V}_{ei\times} - \beta_{\parallel} \nabla_{\parallel} T_e - \beta_{\perp} \nabla_{\perp} T_e - \beta_{\times} \nabla_{\times} T_e$$

# Moments of the Landau collision operator

$$\frac{\gamma_{ab}}{2m_a} \int d\mathbf{v} \mathbf{P}^{jp}(\mathbf{c}_a) \frac{\partial}{\partial \mathbf{v}} \cdot \left[ \frac{\partial}{\partial \mathbf{v}} \cdot \left( f_a \frac{\partial^2 G_{+,b}}{\partial \mathbf{v} \partial \mathbf{v}} \right) - 2 \left( 1 + \frac{m_a}{m_b} \right) \frac{\partial}{\partial \mathbf{v}} \cdot \left( f_a \frac{\partial G_{-,b}}{\partial \mathbf{v}} \right) \right]$$

Rosenbluth potential  $G_{\pm,b}(\mathbf{v}) = \int d\mathbf{v}' f_b(\mathbf{v}') |\mathbf{v} - \mathbf{v}'|^{\pm 1}$

- Exact collisional moments in the total velocity expansion [Ji and Held 2006]

$$\int d\mathbf{v} \mathbf{P}^{jp}(\mathbf{s}_a) C(f_a^M \mathbf{M}_a^{lk} \cdot \mathbf{P}^{lk}(\mathbf{s}_a), f_b^M) = \delta_{jl} \sigma_j A_{ab}^{jpk} \mathbf{M}_a^{lk}$$

$$\int d\mathbf{v} \mathbf{P}^{jp}(\mathbf{s}_a) C(f_a^M, f_b^M \mathbf{M}_b^{lk} \cdot \mathbf{P}^{lk}(\mathbf{s}_b)) = \delta_{jl} \sigma_j B_{ab}^{jpk} \mathbf{M}_b^{lk}$$

- Small mass ratio approximation in the random velocity expansion [Ji and Held 2009]

$$\int d\mathbf{v} \mathbf{P}^{jp}(\mathbf{c}_e) C(f_e^M \mathbf{m}_e^{lk} \cdot \mathbf{P}^{lk}(\mathbf{c}_e), f_i^M) = \sigma_j A_{ei}^{jpk, lk} \mathbf{m}_e^{lk}$$

$$\int d\mathbf{v} \mathbf{P}^{jp}(\mathbf{c}_i) C(f_e^M, f_i^M \mathbf{m}_i^{nq} \cdot \mathbf{P}^{nq}(\mathbf{c}_i)) = \sigma_j B_{ei}^{jpk, lk} \mathbf{m}_i^{lk}$$

- Assumed  $v_{Ti} \ll v_{Te}$  and  $|\mathbf{V}_e - \mathbf{V}_i| \ll v_{Te}$ : not valid for runaway electrons
- Need to calculate the collisional moments for arbitrary relative flow velocity

# Exact collisional moments for arbitrary relative flow velocity: Calculation steps

- Change derivatives,  $\frac{\partial G_{\pm}(\mathbf{v} - \mathbf{V}_b)}{\partial \mathbf{v}} = -\frac{\partial G_{\pm}}{\partial \mathbf{V}_b}$
- Integrate by parts to differentiate each term of polynomial  $P^{jp}(\mathbf{c}_a)$
- Change velocity variables [Schunk and Nagy 2000]

$$\begin{aligned} \mathbf{w}_{\star} &= X_{ba} \mathbf{w}_a + X_{ab} \mathbf{w}'_b & \leftrightarrow & \quad \mathbf{w}_a = v_{\star} (\mathbf{c}_{\star} + \chi_a \mathbf{c}_{\dagger}) & \quad v_{\star}^{-2} &= v_{Ta}^{-2} + v_{Tb}^{-2} \\ \mathbf{w}_{\dagger} &= \mathbf{w}_a - \mathbf{w}'_b & & \quad \mathbf{w}'_b = v_{\star} (\mathbf{c}_{\star} + \chi'_b \mathbf{c}_{\dagger}) & \quad v_{\dagger}^2 &= v_{Ta}^2 + v_{Tb}^2 \end{aligned}$$

where  $X_{ab} = (1 + v_{Ta}^2/v_{Tb}^2)^{-1}$ ,  $X_{ba} = 1 - X_{ab}$ ,  $\chi_a = v_{Ta}/v_{Tb}$ ,  $\chi'_b = -v_{Tb}/v_{Ta}$

$$\int d\mathbf{v} \int d\mathbf{v}' f_a^M f_b^M |\mathbf{v} - \mathbf{v}'|^{\pm} = n_a n_b v_{\dagger}^{\pm 1} \int d\mathbf{c}_{\star} \int d\mathbf{c}_{\dagger} \frac{e^{-c_{\star}^2}}{\pi^{3/2}} \frac{e^{-c_{\dagger}^2}}{\pi^{3/2}} |\mathbf{c}_{\dagger} - \mathbf{x}|^{\pm}$$

where  $\mathbf{x} = (\mathbf{V}_b - \mathbf{V}_a)/v_{\dagger}$

- Expand all  $\mathbf{c}_a$  and  $\mathbf{c}_b$  variables: involve multiple summations
- Perform  $\mathbf{c}_{\star}$  integration using  $\int d\mathbf{c}_{\star} \mathbf{c}_{\star}^n c_{\star}^{2u} \frac{e^{-c_{\star}^2}}{\pi^{3/2}} = \frac{2[u + (n+1)/2]!}{\pi^{1/2}(n+1)} \{I^{n/2}\}$
- Simplify all inner-products and symmetrizations
- Perform  $\mathbf{c}_{\dagger}$  integration using the Rosenbluth potentials [Ji and Held 2006]
- Differentiate with respect to  $\mathbf{V}_b(\mathbf{x})$
- Final results involve only algebraic summations of  $G_{\pm}^{np}(x) \{\mathbf{x}^j \cdot r m^{lk} |^i\}$  terms

## Examples: vector moments due to relative flow

$$\begin{aligned}
 A_{ab}^{10,00} &= \frac{3\sqrt{\pi}(1+\mu)X}{4\tau_{ab}\mu} \frac{E - xE'}{x^2} \hat{\mathbf{x}}_{ba} n_a \\
 A_{ab}^{11,00} &= \frac{9\sqrt{\pi}X}{4\tau_{ab}\mu} \frac{[\mu x + (1+\mu)x^3 X] E' - \mu E}{x^2} \hat{\mathbf{x}}_{ba} n_a \\
 A_{ab}^{20,00} &= \frac{3\sqrt{\pi}\sqrt{X}}{8\tau_{ab}\mu} \left[ \left( \frac{3\mu + 6X(1+\mu)}{x^3} - \frac{2\mu}{x} \right) E \right. \\
 &\quad \left. - \left( \frac{3\mu + 6X(1+\mu)}{x^2} + 4X(1+\mu) \right) E' \right] \overline{\hat{\mathbf{x}}_{ba} \hat{\mathbf{x}}_{ba}}
 \end{aligned}$$

Small mass ratio approximation for  $ab = ei$ :

$$\mu^{-1} = m_e/m_i \ll 1, \quad X = (1 + v_{Ti}^2/v_{Te}^2)^{-1} \rightarrow 1, \quad x \rightarrow |\mathbf{V}_i - \mathbf{V}_e|/v_{Te}$$

$$\begin{aligned}
 A_{ei}^{10,00} &\approx \frac{3\sqrt{\pi}}{4\tau_{ei}} \frac{E - xE'}{x^2} \hat{\mathbf{x}}_{ie} n_e \xrightarrow{x \ll 1} \frac{1}{\tau_{ei}} x \hat{\mathbf{x}}_{ie} n_e \\
 A_{ei}^{11,00} &\approx \frac{9\sqrt{\pi}}{4\tau_{ei}} \frac{x(x^2 + 1)E' - E}{x^2} \hat{\mathbf{x}}_{ie} n_e \xrightarrow{x \ll 1} \frac{3}{2\tau_{ei}} x \hat{\mathbf{x}}_{ie} n_e \\
 A_{ei}^{20,00} &\approx \frac{3\sqrt{\pi}}{8\tau_{ei}} \left[ \left( \frac{9}{x^3} - \frac{2}{x} \right) E - \left( \frac{9}{x^2} + 4 \right) E' \right] \overline{\hat{\mathbf{x}}_{ie} \hat{\mathbf{x}}_{ie}} n_e \xrightarrow{x \ll 1} \mathcal{O}(x^2)
 \end{aligned}$$



# Collision matrix element: Heat flux - heat flux

$$\begin{aligned}
 A_{ab}^{11,11} &= \frac{3\sqrt{\pi}X^{3/2}}{32\mu\tau_{ab}} \left[ n_a^{11} \left( A_{0E}^{11,11} \frac{E}{x^3} + A_{0e}^{11,11} \frac{E'}{x^2} \right) \right. \\
 &\quad \left. + \hat{\mathbf{x}}\hat{\mathbf{x}} \cdot \mathbf{n}_a^{11} \left( A_{1E}^{11,11} \frac{E}{x^3} + A_{1e}^{11,11} \frac{E'}{x^2} \right) \right] \\
 B_{ab}^{11,11} &= \frac{3\sqrt{\pi}X^{3/2}\chi}{32\mu\tau_{ab}} \left( n_a^{11} B_{0e}^{11,11} + \hat{\mathbf{x}}\hat{\mathbf{x}} \cdot \mathbf{n}_a^{11} B_{1e}^{11,11} \right) E'
 \end{aligned}$$

where  $\hat{\mathbf{x}} = \hat{\mathbf{x}}_{ba}$  and  $\chi = \chi_{ab}$  and

$$A_{0E}^{11,11} = -2(9 + 17\mu)$$

$$\begin{aligned}
 A_{0e}^{11,11} &= (18 + 34\mu) - 2x^2 (4 + 6\mu + 15X^2(1 + \mu) - 2X(11 + 14\mu)) \\
 &\quad + 4x^4 X(-2 + 3X)(1 + \mu)
 \end{aligned}$$

$$A_{1E}^{11,11} = 6(9 + 17\mu)$$

$$\begin{aligned}
 A_{1e}^{11,11} &= -6(9 + 17\mu) - 4x^2(9 + 17\mu) + 4x^4 X(-16 - 22\mu + 21X(1 + \mu)) \\
 &\quad - 24x^6 X^2(1 + \mu)
 \end{aligned}$$

$$B_{0e}^{11,11} = 6(-1 + X)(2\mu - 5X(1 + \mu)) + 12x^2(-1 + X)X(1 + \mu)$$

$$B_{1e}^{11,11} = -12x^2(-1 + X)(2\mu - 7X(1 + \mu)) - 24x^4(-1 + X)X(1 + \mu)$$

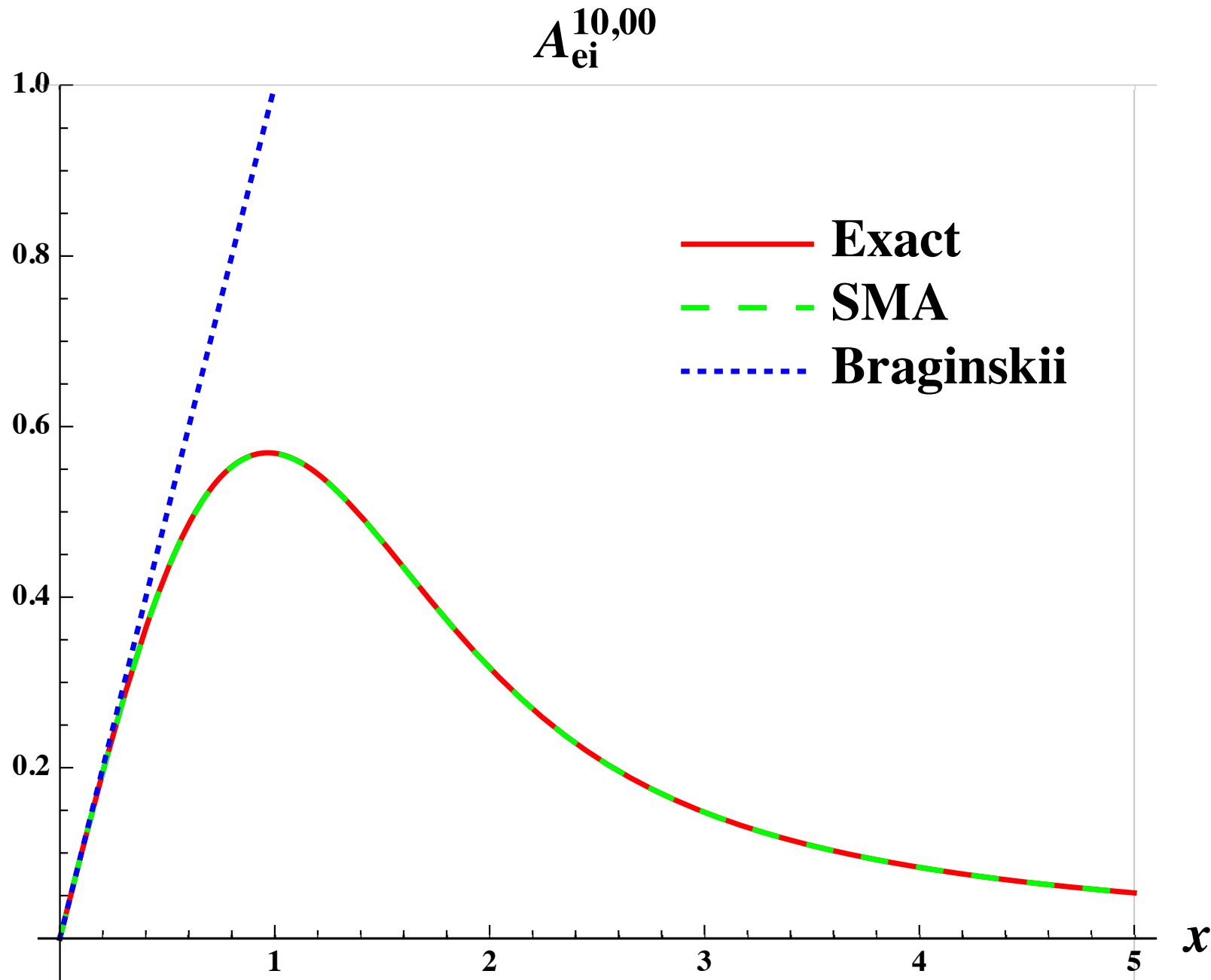
# Collision matrix element: Viscosity - viscosity

$$\begin{aligned}
 A_{ab}^{20,20} = & \frac{3\sqrt{\pi}X^{3/2}}{32\mu\tau_{ab}} \left[ n_a^{20} \left( A_{0E}^{20,20} \frac{E}{x^5} + A_{0e}^{20,20} \frac{E'}{x^4} \right) \right. \\
 & + \overline{\hat{\mathbf{x}}\hat{\mathbf{x}} \cdot \mathbf{n}_a^{20}} 4 \left( A_{1E}^{20,20} \frac{E}{x^5} + A_{1e}^{20,20} \frac{E'}{x^4} \right) \\
 & \left. + \overline{\hat{\mathbf{x}}\hat{\mathbf{x}}\hat{\mathbf{x}}\hat{\mathbf{x}}} : \mathbf{n}_a^{20} \left( A_{2E}^{20,20} \frac{E}{x^5} + A_{2e}^{20,20} \frac{E'}{x^4} \right) \right]
 \end{aligned}$$

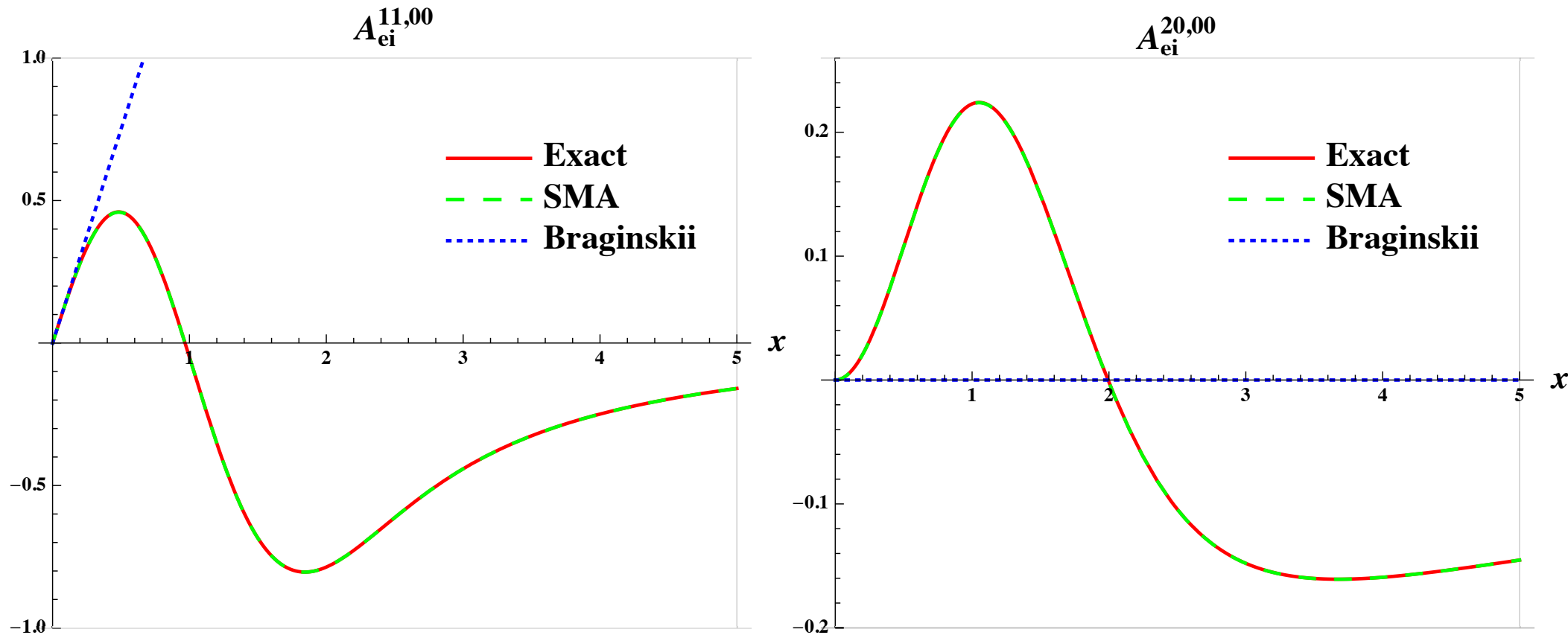
where

$$\begin{aligned}
 A_{0E}^{20,20} &= 6(\mu + 2X(1 + \mu)) - 4x^2(2 + 3\mu) \\
 A_{0e}^{20,20} &= (-6\mu - 12X(1 + \mu)) + 8x^2X(1 + \mu)\chi^2 \\
 A_{1E}^{20,20} &= -15(\mu + 2X(1 + \mu)) + 6x^2(1 + 2\mu) \\
 A_{1e}^{20,20} &= (15\mu + 30X(1 + \mu)) + x^2(4\mu + 2(-3 + 10X)(1 + \mu)) \\
 &\quad + 2x^4(-2 + 4X)(1 + \mu) \\
 A_{2E}^{20,20} &= 15(7\mu + 14X(1 + \mu)) - 30x^2\mu \\
 A_{2e}^{20,20} &= (-105\mu - 210X(1 + \mu)) + x^2(-40\mu - 140X(1 + \mu)) \\
 &\quad + x^4(-8\mu - 56X(1 + \mu)) - 16x^6X(1 + \mu)
 \end{aligned}$$

# Momentum (1,0) moment due to relative flow: Dreicer's friction



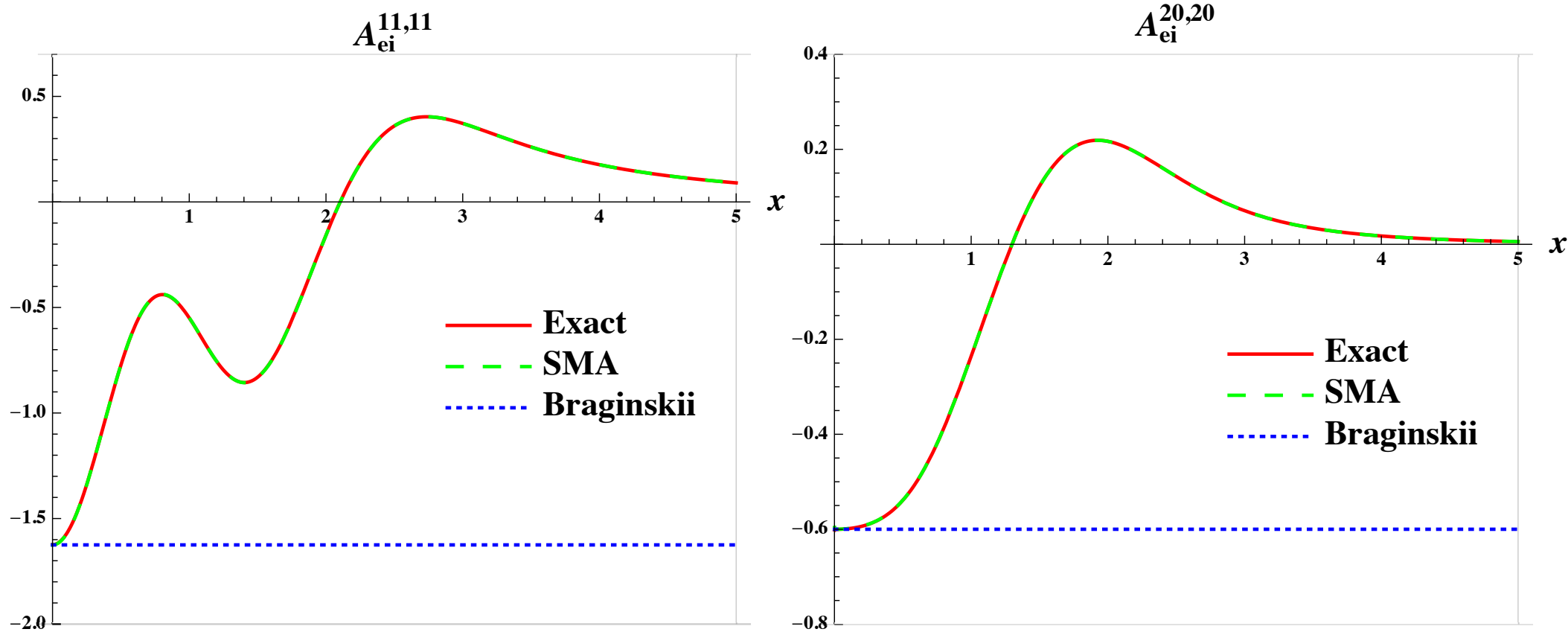
# Heat flux (1, 1) and viscosity (2, 0) moments due to relative flow



- Vector drives are proportional to  $x$  for  $x \ll 1$  in Braginskii theory  
 $\rightarrow \mathbf{h}_e = -\kappa_e(x)\nabla T_e + \beta(x)T\mathbf{V}_{ei}$  (where  $\mathbf{V}_{ab} = \mathbf{V}_a - \mathbf{V}_b$ )
- Viscosity drive not considered in Braginskii ( $\pi = 2\eta\overline{\nabla\mathbf{V}}$ )

$$\overline{\mathbf{x}\mathbf{x}} = \mathbf{x}\mathbf{x} - \frac{1}{3}x^2\mathbf{I} \text{ where } \mathbf{x} = \frac{\mathbf{V}_{ie}}{v_{Te}} \rightarrow \pi = 2\eta(x)\overline{\nabla\mathbf{V}} + \gamma(x)\overline{\mathbf{V}_{ie}\mathbf{V}_{ie}} + \lambda(x)\overline{\mathbf{V}_{ie}\nabla T}$$

# Collision matrix elements: Heat flux - heat flux and viscosity - viscosity

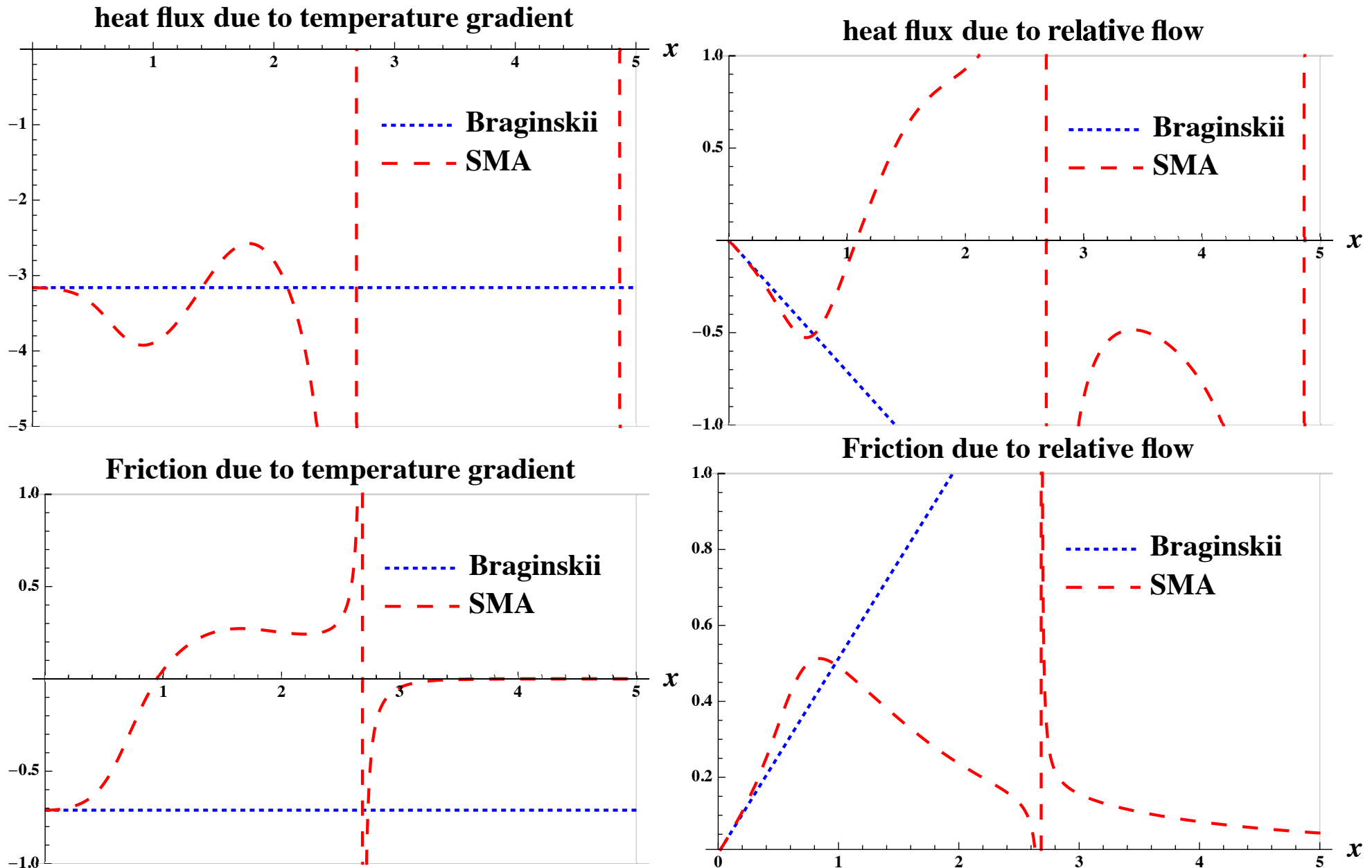


- Electron collision operator  $\hat{C}_e = \hat{C}_{ee} + Z\hat{A}_{ei}$

$$\hat{C}_{ee}^{11,11} = -\frac{2\sqrt{2}}{5} \approx -0.566$$

$$\hat{C}_{ee}^{20,20} = -\frac{3\sqrt{2}}{5} \approx -0.849$$

# Closure coefficients ( $2 \times 2$ calculation) - incomplete



# Summary and future work

---

- Formulas are implemented in Mathematica and verified against existing results
- Closures obtained in high collisionality
  - Check convergence with increasing the number of moments
- Investigate the behavior of closures low collisionality regimes
  - Parallel integral closures
- Consider magnetic field effects
  - Inhomogeneous along the field line
  - Curvature effects (centrifugal force)
  - Modify perpendicular closures