Progress in theoretical and numerical modeling of RF/MHD coupling using NIMROD

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What are we trying to accomplish?

- Major physics issue being addressed by SWIM project (Center for the Simulation of RF Wave Interactions with Magnetohydrodynamics)— How can RF sources optimally be used to suppress or reduce the negative effects of MHD instabilities in fusion plasmas?

- At present, no theoretical framework exists for self–consistently including the effects of arbitrary RF sources in the MHD model

- Relevant issues in developing such a formalism:
  
  —Can a small expansion parameter be found, such that the lowest order distribution function is a local Maxwellian, \( f_M(x, v, t) \equiv n(x, t) \left( \frac{m}{2\pi T(x, t)} \right)^{3/2} \exp \left[ -\frac{m|v - V(x, t)|^2}{2T(x, t)} \right] \) ?

  —If so, how do RF effects enter the fluid equations? What is the proper closure?

  —If not, what is the lowest order distribution function? What are the appropriate fluid–like quantities describing the plasma?

- This work will focus on the effects of electron cyclotron current drive
Specific case — ECCD stabilization of neoclassical tearing modes

- Experimental efforts to stabilize neoclassical tearing modes via electron cyclotron current drive (ECCD) have been very successful; R. J. La Haye [Phys. Plasmas 13, 055501 (2006)] gives a detailed overview and many references.

- For ECCD, the RF–induced current is relatively small [of the same order as the current driven by the electric field] ⇒ small expansion parameter.

- General kinetic equation:

\[
\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla f_\alpha + \frac{q_\alpha}{m_\alpha} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_\alpha}{\partial \mathbf{v}} = C(f_\alpha) + Q(f_\alpha)
\]

- \( Q(f_\alpha) \) is a gyrophase–averaged quasilinear diffusion operator:

\[
Q(f_\alpha) = \frac{\partial}{\partial \mathbf{v}} \cdot \mathbf{D} \cdot \frac{\partial}{\partial \mathbf{v}}
\]

where the diffusion tensor \( \mathbf{D} \) arises from the RF source.

- \( C(f_\alpha) \) is the gyrophase–averaged Fokker–Planck Coulomb collision operator.
RF terms appear in the fluid equations

- Taking fluid moments in the conventional manner yields

\[
\frac{\partial n_\alpha}{\partial t} + \nabla \cdot (n_\alpha v_\alpha) = 0 \quad \text{(RF produces no particles)}
\]

\[
m_\alpha n_\alpha \left( \frac{\partial v_\alpha}{\partial t} + v_\alpha \cdot \nabla v_\alpha \right) = n_\alpha q_\alpha (E + v_\alpha \times B) - \nabla p_\alpha - \nabla \cdot \pi_\alpha + R_\alpha + F_{\alpha 0}^{rf}
\]

\[
F_{\alpha 0}^{rf} = \int m_\alpha v Q(f_\alpha) dv \quad \text{(additional momentum imparted by RF waves)}
\]

\[
3 n_\alpha \left( \frac{\partial T_\alpha}{\partial t} + v_\alpha \cdot \nabla T_\alpha \right) + n_\alpha T_\alpha \nabla \cdot v_\alpha = -\nabla \cdot q_\alpha - \pi : \nabla v_\alpha + Q_\alpha + S_{\alpha 0}^{rf}
\]

\[
S_{\alpha 0}^{rf} = \int \frac{1}{2} m_\alpha v^2 Q(f_\alpha) dv \quad \text{(additional energy imparted by RF waves)}
\]
Calculation of RF terms in fluid equations

- Need a small expansion parameter to proceed
- For ECCD, electric field imparted by RF is small ($E \ll E_D$, the Dreicer field strength) $\Rightarrow$ RF terms are small
- Assume the lowest order distribution function is a local Maxwellian — $f_\alpha = f_{M\alpha} + \delta f_\alpha$
- Source terms in fluid equations come from velocity moments of $Q(f_{M\alpha})$; become functions of low-order fluid moments in this approximation

\[
\mathbf{F}_{\alpha 0}^{rf} \rightarrow \int m_\alpha \mathbf{v} Q(f_{M\alpha}) d\mathbf{v} \quad S_{\alpha 0}^{rf} \rightarrow \int \frac{1}{2} m_\alpha v^2 Q(f_{M\alpha}) d\mathbf{v}
\]

\[
f_{M\alpha} = n(\mathbf{x}, t) \left( \frac{m}{2\pi T(\mathbf{x}, t)} \right)^{3/2} \exp \left[ \frac{-m|\mathbf{v} - \mathbf{V}(\mathbf{x}, t)|^2}{2T(\mathbf{x}, t)} \right]
\]
RF effects also modify closure scheme

- Effects of RF must also be included in closure calculations for heat fluxes and stress tensors
- Use a Chapman–Enskog–like approach; assume kinetic distortion $\delta f_\alpha$ has no density, momentum, or temperature moments

\[
\int \delta f_\alpha dv = \int \delta f_\alpha m_\alpha v dv = \int \delta f_\alpha \frac{m_\alpha v^2}{2} dv = 0
\]

- Equation for kinetic distortion:

\[
\frac{d\delta f_\alpha}{dt} - C(\delta f_\alpha) - Q(\delta f_\alpha) = -\frac{df_{M\alpha}}{dt} + C(f_{M\alpha}) + Q(f_{M\alpha})
\]

reduces to

\[
\frac{d\delta f_\alpha}{dt} - C(\delta f_\alpha) = -\frac{df_{M\alpha}}{dt} + Q(f_{M\alpha})
\]

- Can touch base with Spitzer problem (modified by RF terms)
- Use moment formalism (Eric, Jeong–Young)
Resistive and neoclassical tearing modes — island widths

For resistive tearing modes, Rutherford equation predicts algebraic growth of island width in nonlinear regime:

\[
\frac{dw}{dt} = 1.22 \frac{\eta}{\mu_0} \Delta'
\]

Nonlinear saturation of island width — \( \Delta' \rightarrow \Delta'(w) \):

\[
\frac{dw}{dt} = 1.22 \frac{\eta}{\mu_0} \Delta'(w)
\]

Neoclassical modifications to tearing mode (perturbed bootstrap currents, curvature stabilization, ion polarization currents, resistive interchange, etc.) enter Rutherford equation additively

\[
\frac{dw}{dt} = 1.22 \frac{\eta}{\mu_0} [\Delta'(w) + \Delta_{NTM}(w)]
\]

Heuristically, with RF included,

\[
\frac{dw}{dt} = 1.22 \frac{\eta}{\mu_0} [\Delta'(w) + \Delta_{NTM}(w) + \Delta_{rf}(w)]
\]
Physical effects captured by Rutherford equation are independent

- RF modifications to Rutherford equation enter additively, on the same footing as other neoclassical modifications
- Physical effects can be treated independently
- Consequently, can study effects of RF on (ordinary) resistive tearing mode simulations as a prototype problem — no need to start with (more complicated) neoclassical simulations
- Mock up RF effects by heuristically modifying Ohm’s law; move progressively to more complicated models for RF interaction

Eventual goal — self-consistent coupling of MHD (NIMROD), Fokker–Planck (CQL3D), and RF codes (GENRAY – short term, AORSA or TORIC – long term)
Insert an ad hoc term (general RF effects) in Ohm’s law

- MHD: Ohm’s law:
  \[ \mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J} \]

- Determine effect of current drive (not self-consistent): ad hoc force on electrons
  \[ \mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J} - \frac{\mathbf{F}_{\text{rf}}}{e} \]

- Should modify plasma equilibrium. Assume forcing term has form
  \[ \mathbf{F}_{\text{rf}}(R, Z, t) = e\eta \lambda_{\text{rf}} \exp \left[ - \frac{[(R - R_{\text{rf}})^2 + (Z - Z_{\text{rf}})^2]}{w_{\text{rf}}^2} \right] \frac{\mathbf{B}}{\mu_0} f(t) \]

- Can also consider this as \( \mathbf{E} \rightarrow \mathbf{E} - \eta \mathbf{J}_{\text{rf}} \), where \( \eta \mathbf{J}_{\text{rf}} \) is an emf-per-unit-length inducing magnetic field in the plasma (mocking up effects of current density \( \mathbf{J}_{\text{rf}} \)).

- Variable parameters \( \lambda_{\text{rf}}, R_{\text{rf}}, Z_{\text{rf}}, w_{\text{rf}} \) (amplitude, location, spatial width), along with time dependence \( f(t) \); begin with cylindrical equilibrium.
Rapid current equilibration occurs on the flux surfaces

- Begin with time-independent perturbation; $\lambda_{rf} = 16.0$, $w_{rf} = 0.1$, $R_{rf} = 0.5$, $Z_{rf} = 0$:
Current equilibration: Alfvén timescale

- Current equilibrates over a flux surface on the Alfvén timescale.
- Not necessary to average ad hoc forcing term over flux surface — force balance does this for us
- Add time dependence, decrease perturbation amplitude, go to realistic geometry
Ad hoc RF current is slowly ramped up in time

- Specify the time dependence $f(t)$:

$$f(t) = \left[ \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{t - t_{\text{offset}}}{t_{\text{build}}} \right) \right]$$

- Build up current on timescale $t_{\text{build}}$ to suppress transient Alfvén waves ($t_{\text{build}}$ short compared to resistive diffusion timescale $\tau_R$, but long compared to Alfvén time $\tau_A$)

- Initial perturbation has amplitude comparable to random current fluctuations; $\lambda_{\text{rf}} = 8.0 \times 10^{-4}$, $w_{\text{rf}} = 0.1$, $R_{\text{rf}} = 0.5$, $Z_{\text{rf}} = 0$
Test this effect in realistic (DIII-D-like) geometry

- Grid packing used to resolve rational surfaces \( (q = 2, q = 3) \) more accurately

\[ q \text{ vs } \sqrt{\text{pol}_\text{flux}} \]


- Use axially symmetric \( F_{\text{rf}} \) (a “ring” of current in the tokamak); examine effects on axially symmetric component of equilibrium \( (n = 0) \)
Current equilibration occurs in realistic geometries

- Equilibration is remains rapid in realistic geometries, even at amplitudes comparable to background fluctuations
- $\lambda_{rf} = 1.0, Z_{rf} = 0, R_{rf} = 2.0, w_{rf} = 0.1$
- $t_{\text{offset}} = 0.02, t_{\text{build}} = 0.005, \Delta t = 1.0 \times 10^{-5}, \tau_R = 0.998, \tau_A = 6.0 \times 10^{-7}$
- $R/a \sim 3, S = 1.66 \times 10^6, Pr = 0.1$
The RF term modifies the width of the magnetic islands

- Now, study resistive tearing modes; let the islands grow up and saturate, and then turn on the RF term
The island widths are visibly reduced due to the RF

- In the absence of RF, large \((2, 1)\) islands form; \((3, 1)\) and \((5, 2)\) also visible
- In the presence of RF, the \((2, 1)\) island is reduced in size
Closeup of reduced island width

Magnetic islands, $t = 0.12815$ s

Magnetic islands, $t = 0.12721$ s
Future plans

- Investigate dependence of island width reduction on source amplitude and spatial localization

- Further exploration of physics issues associated with island width reduction

- Introduction of toroidally localized sources ($F_{\text{rf}}$)

- Discussion with RF community:
  - Numerical issues, relevant to coupling of existing RF wave propagation and deposition codes with NIMROD
  - Theoretical issues, relevant to form of quasilinear operator $Q$
  - Feasability of simulating ICRF physics in this theoretical framework