

Stabilization of interchange g-modes with gyroviscosity

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discrepancies between NIMROD and existing theory

- g mode stabilization wasn't observed with gyroviscosity (Dalton's case with nimrod3.2)
 - $\omega^*/\gamma_{mhd} = 10$ was unstable
- FLR stabilization was seen with nimuw (Ping)
 - However 2-fluid stabilization was not seen

some ideas

- problem might be related to the two versions of the code
- analytical local approximation not valid in Dalton's case
- should revisit local dispersion relation (Roberts & Taylor)

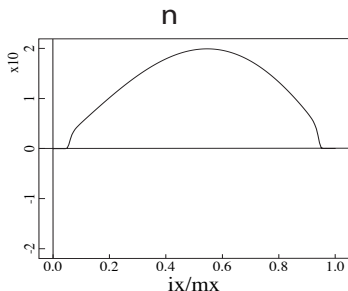
The third idea turned out to be fruitful.

Different physics cases

exponential case

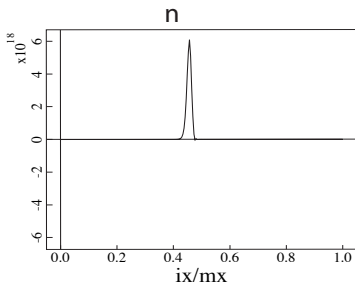
- non-isothermal
- uniform magnetic field
- global mode structure close to the walls

density perturbations:

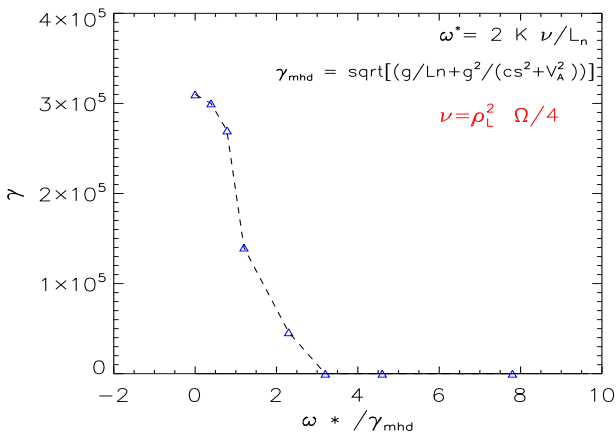


tanh case

- isothermal
- non-uniform magnetic field
- local mode structure (localized density gradient)

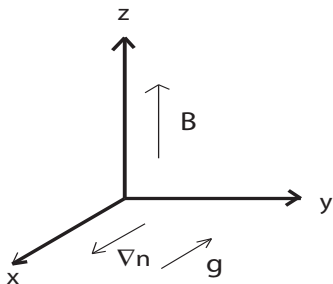


FLR stabilization for tanh case, (based on Roberts & Taylor theory - incompressible)



FLR stabilization effect arises because large-orbit ions drift slower than the electrons in an in homogeneous electric field.

geometry and assumptions



- $(p + B^2/2\mu_0)' = -mng$
- $n = n \exp(x/L_n)$,
- local approximation (x dependence is ignored)
 $kL_n \gg (k\partial/\partial x \gg 1,$

here:

- consider compressibility
 $\nabla \cdot v \neq 0$
- uniform B, but
non-isothermal

The dispersion relation with compressibility

$$\frac{\partial n}{\partial t} + \nabla \cdot (nV) = 0$$

$$\frac{\partial p}{\partial t} + \Gamma p \nabla \cdot V + V \cdot \nabla p = 0$$

$$\frac{\partial B}{\partial t} = (B \cdot \nabla)V - (V \cdot \nabla)B - B(\nabla \cdot V)$$

$$mn \frac{\partial V}{\partial t} = -\nabla \left(p + \frac{B^2}{2\mu_0} \right) + mn\bar{g} + \nu \bar{\lambda}$$

$$\lambda_x = \frac{\partial}{\partial x}(\Pi_{yy}) - \frac{\partial}{\partial y}(\Pi_{xy}), \quad \lambda_y = -\frac{\partial}{\partial y}(\Pi_{yy}) - \frac{\partial}{\partial x}(\Pi_{xy})$$

$$\Pi_{yy} = -\Pi_{xx} = \frac{nT}{2\Omega} \left(\frac{\partial V_y}{\partial x} + \frac{\partial V_x}{\partial y} \right), \quad \Pi_{xy} = \frac{nT}{2\Omega} \left(\frac{\partial V_x}{\partial x} - \frac{\partial V_y}{\partial y} \right)$$

λ = anisotropic ion pressure tensor, $\nu = \rho_L^2 \Omega / 4 = \frac{T}{2m\Omega}$

The dispersion relation with compressibility

local dispersion relation with gyroviscosity and compressibility:

$$\begin{aligned} & \omega^2 \left(\omega^2 - \frac{2\nu k}{L_n} \omega - \frac{2\nu k}{L_T} \omega + \frac{2\nu k}{L_B} \omega + \frac{g}{L_n} \right) \\ &= (g - \nu k \omega) \left(\frac{\omega^2}{L_n} + k^2 g - \nu k^3 \omega + \frac{2\nu k \omega}{L_B L_n} - \frac{\nu k \omega}{L_n^2} - \frac{\nu k \omega}{L_B^2} \right) \\ &+ (k^2 c_s^2 + k^2 v_A^2 + \frac{\nu k}{L_n} \omega + \frac{\nu k}{L_T} \omega - \frac{\nu k}{L_B} \omega) \left(\omega^2 - \frac{2\nu k}{L_n} \omega - \right. \\ &\left. \frac{2\nu k}{L_T} \omega + \frac{2\nu k}{L_B} \omega + \frac{g}{L_n} \right) \end{aligned}$$

without gyroviscosity $\nu = 0$: MHD growth rate is

$$\omega^2 = - \left[\frac{g}{L_n} + \frac{g^2}{(c_s^2 + v_A^2)} \right] = -\gamma_{MHD}^2$$

For uniform magnetic field and non-isothermal:

$$\omega^2 + \omega^* \omega + \gamma_{mhd}^{*2} = 0$$

where,

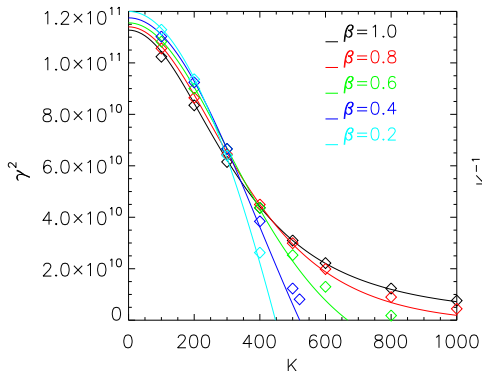
$$\omega^* = -2\nu k \frac{\left(\frac{1}{L_n} + \frac{1}{L_T} + \frac{g}{(c_s^2 + V_A^2)}\right)}{\left(1 + \frac{\nu^2 k^2}{(c_s^2 + V_A^2)}\right)} = -k \frac{\left(-2\frac{g}{\Omega} + \frac{\beta g}{2\Omega(1+\Gamma\beta/2)}\right)}{\left(1 + k^2 \rho_L^2 \frac{\beta}{16(1+\Gamma\beta/2)}\right)}$$

and

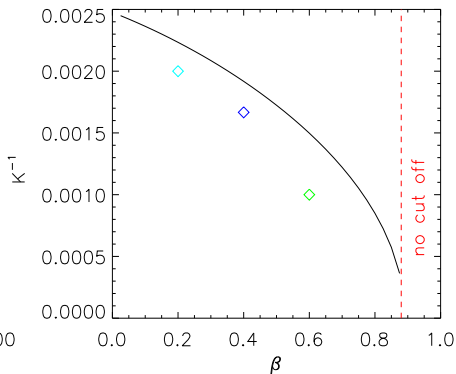
$$\gamma_{MHD}^{*2} = \frac{\frac{g}{L_n} + \frac{g^2}{(c_s^2 + V_A^2)}}{1 + \frac{\nu^2 k^2}{(c_s^2 + V_A^2)}} = \frac{\frac{g}{L_n} + \frac{g^2}{(c_s^2 + V_A^2)}}{\left(1 + k^2 \rho_L^2 \frac{\beta}{16(1+\Gamma\beta/2)}\right)}$$

In the limit of incompressible plasma, $\Gamma \rightarrow \infty$, and $L_T \rightarrow \infty$, Roberts & Taylor is recovered, $\omega^* = -k\rho_L^2\Omega/2L_n$.

growth rate



K^{-1}



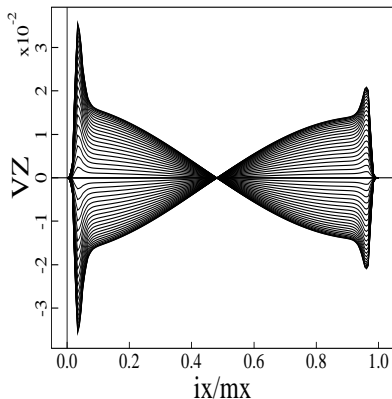
- solid lines analytic ; symbols nimrod
- $\beta = 0.6 - 0.8$ at high k reveal oscillatory behavior (convergence study underway)

Some examples of mode structure with gyroviscosity at high k ($k=800$)

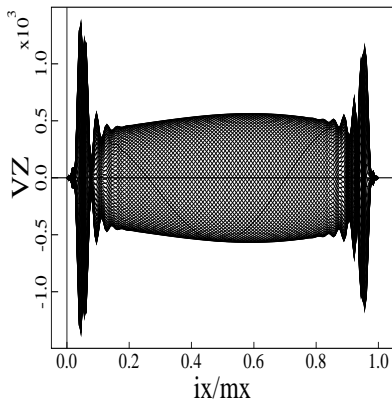
MHD ($\omega^*/\gamma_{mhd}^* = 0$)

$\beta = 1$ ($\omega^*/\gamma_{mhd}^* = 0.8$)

Re V_z vs. i



Re V_z vs. i

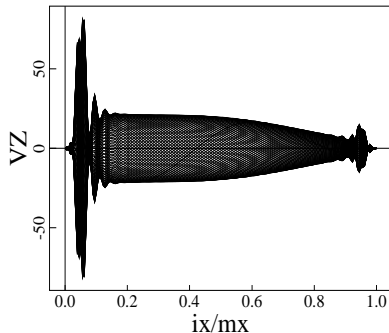


Some examples of mode structure with gyroviscosity at high k ($k=800$)

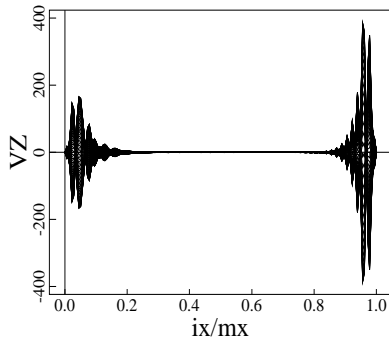
$$\beta = 0.8 (\omega^* / \gamma_{mhd}^* = 1.)$$

$$\beta = 0.6 (\omega^* / \gamma_{mhd}^* = 1.5)$$

Re V_Z vs. i



Re V_Z vs. i



might need more spatial resolution.

Possible next steps

- gyroviscosity effect at high k
- FLR effect on $k_{||}$ modes, quasi-interchange (Suzuki)