Ballooning Instability of a Prototype Tokamak

Part 1: Equilibrium

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- Motivation
  - To simulate nonlinear ballooning instability and compare with analytic theory for prototype tokamak configurations
  - Edge localized mode (ELM) in a prototype tokamak
  - Other ballooning dominant/associated processes in prototype tokamaks
- Outline
  - Grad-Shafranov solvers: ESC, etc.
  - Coupling to NIMSET

Choose a Grad-Shafranov solver

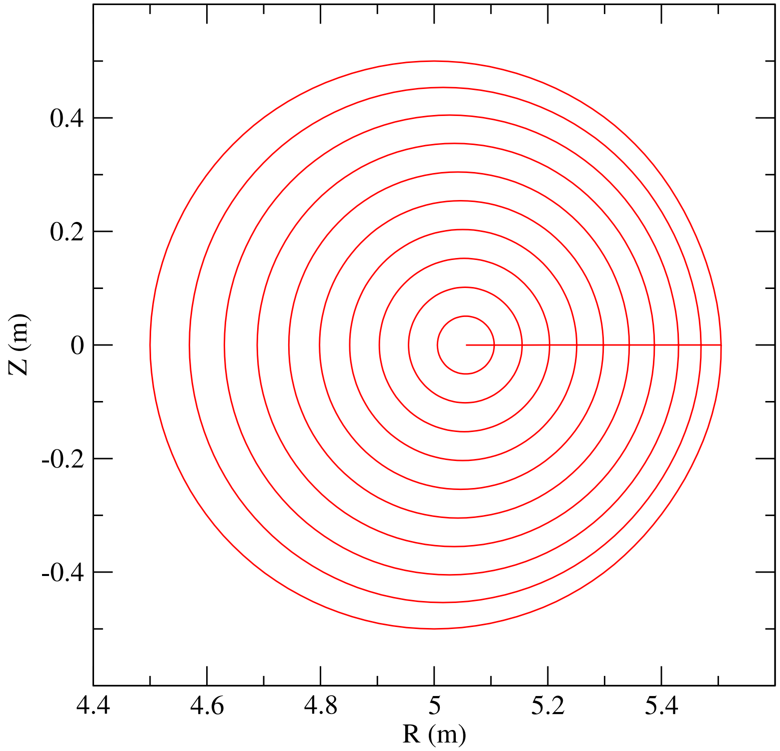
- Existing solvers
    - Supported by NIMROD: TOQ, GALKIN, CHEASE, CHUM, EFIT, RSTEQ, SOLOVIEV, ...
    - Public from NTCC: TEQ, PEST, JSOLVER, ESC, ...
    - Other often used ones: VMEC, ...
- Eulerian: prescribed mesh
    - good for complicated geometry/configuration; experimental interpretation and control
    - limiting range of formulations of equilibrium
- Larangian: flux coordinate
    - allows large range of prescribed current/pressure profiles; good for comparison with analytical model
    - difficulty with free boundary with a separatrix.

ESC: Equilibrium and Stability Code

- Developed in PPPL since 1994, by Zakharov and Pletzer.
- Downloadable from NTCC website.
- Lagrangian approach, Newton iteration scheme: solving perturbed Grad-Shafranov equations (Zakharov and Pletzer [1999]).
- Fast (most other solvers use slower Piccard iteration scheme)
- Good to be embedded in other such as transport code; less robust for high β equilibrium.
- A light-weight solver, may be suitable for prototype tokamak study.
- Disadvantage: fixed boundary; kernel not (well) documented, difficult to understand.

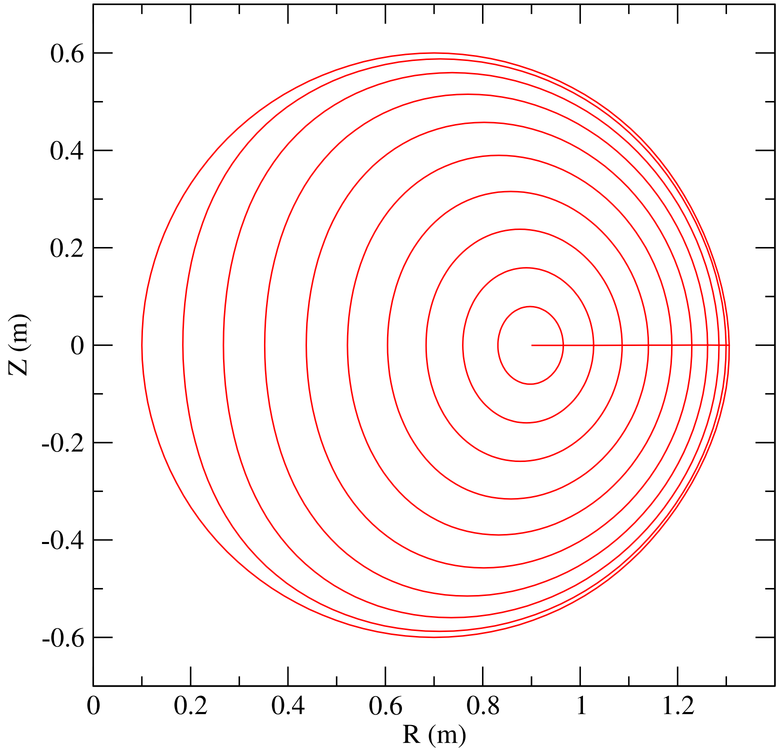
Circular Cross Section: large R_0/a, low β

- $Z_0 = 0$, $R_0 = 5$, $a = 0.5$, $\kappa = 1$, $\delta = 0$
- $q_0 = 1.0$, $q_a = 10.0$, $B_0 = 1.0$, $p_0 = 0.01$
- $r = \sqrt{\Psi/\Psi_a}$, Ψ – toroidal flux
- $p = p_0(1 - r^2)$
- $q = q_0[1 + (q_a/q_0 - 1)r^4]$
- $J_{\|} = 2B_0(1 - r^2)/(R_0 q_0)$



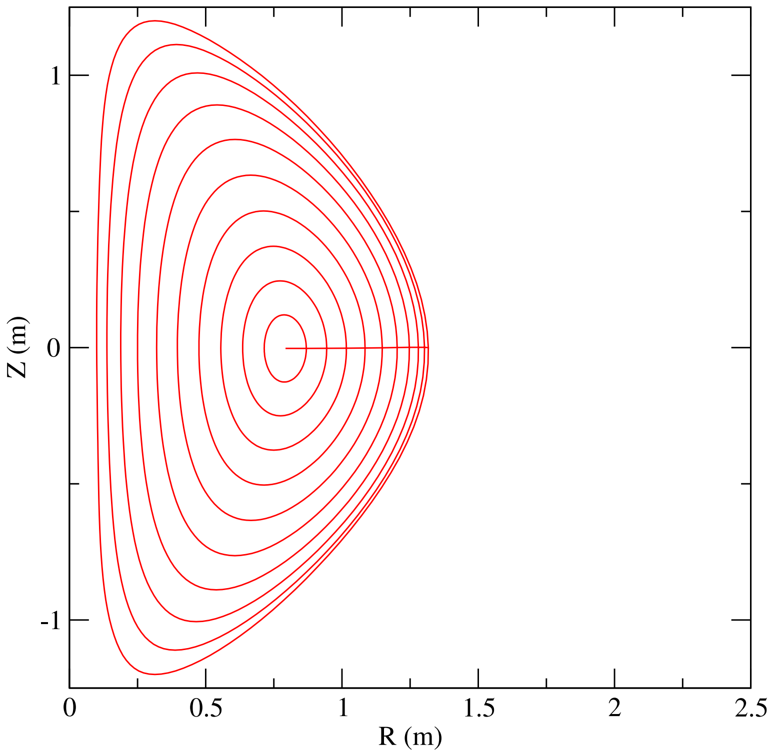
Circular Cross Section: small R_0/a, high β

- $Z_0 = 0$, $R_0 = 0.7$, $a = 0.6$, $\kappa = 1$, $\delta = 0$
- $q_0 = 1.0$, $q_a = 10.0$, $B_0 = 1.0$, $p_0 = 0.5$
- $r = \sqrt{\Psi/\Psi_a}$, Ψ – toroidal flux
- $p = p_0(1 - r^2)$
- $q = q_0[1 + (q_a/q_0 - 1)r^4]$
- $J_{||} = 2B_0(1 - r^2)/(R_0 q_0)$



Non-Circular Cross Section

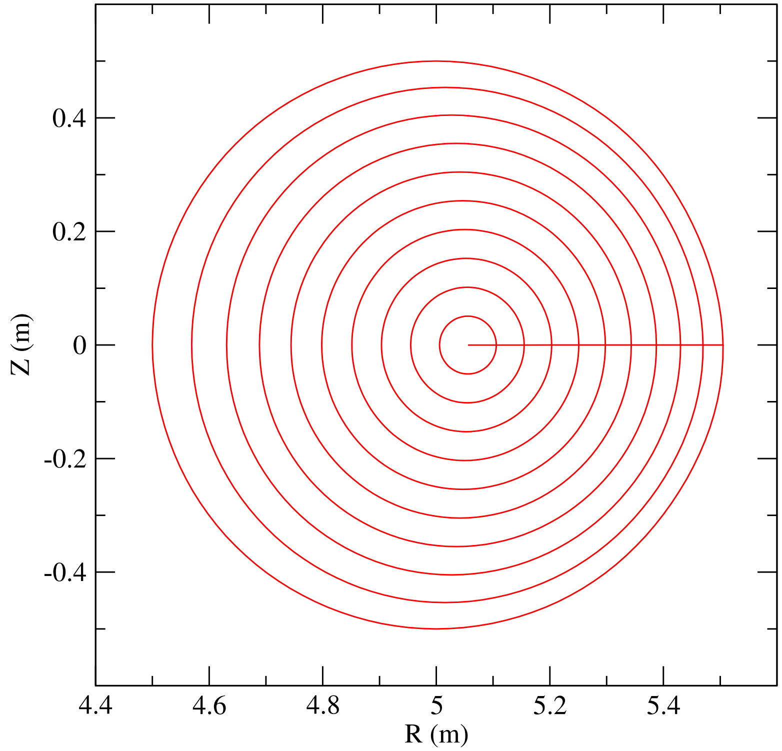
- $Z_0 = 0,\ R_0 = 0.7,\ a = 0.6,$
  $\kappa = 2,\ \delta = 0.7$
- $q_0 = 1.0,\ q_a = 10.0,\ B_0 = 1.0,$
  $p_0 = 0.04$
- $r = \sqrt{\Psi/\Psi_a},\ \Psi$ – toroidal flux
- $p = p_0(1 - r^2)$
- $q = q_0[1 + (q_a/q_0 - 1)r^4]$
- $J_{||} = 2B_0(1 - r^2)/(R_0 q_0)$

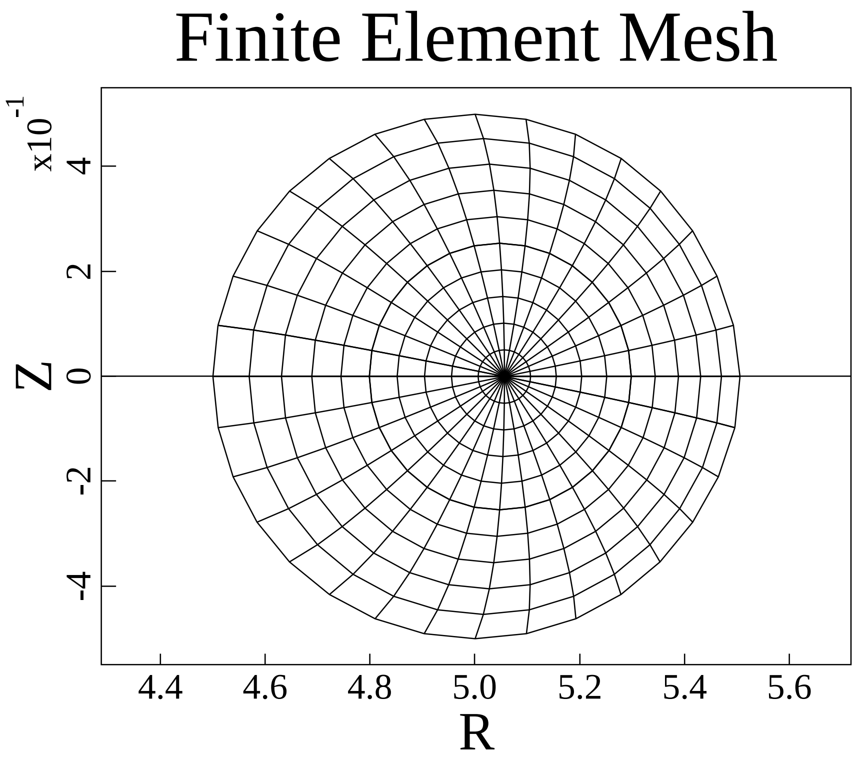


NIMSET: Calling ESC from `nimset_init.f`

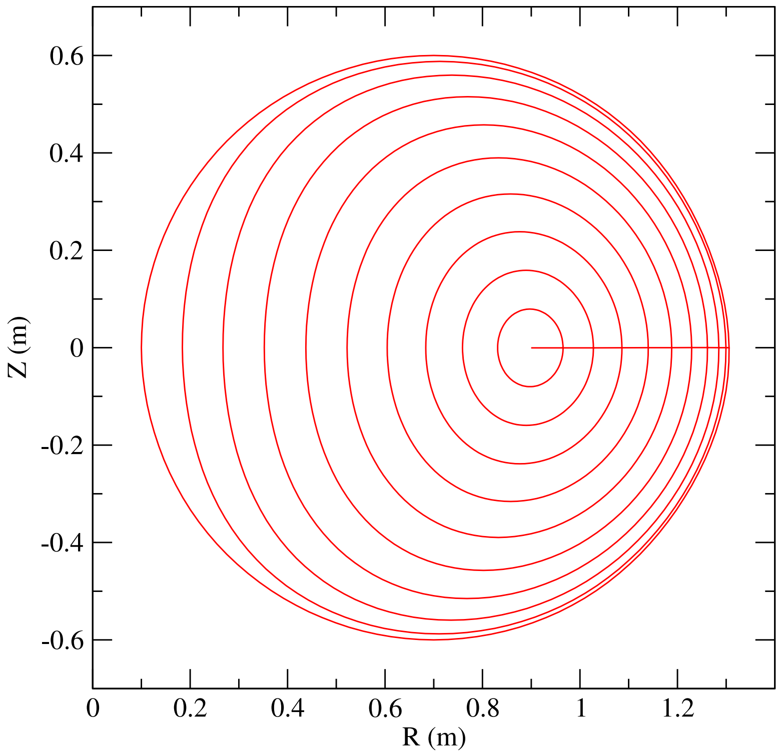
```
      MODULE polar_init
.............
      USE esc_mod
.............
      SUBROUTINE polar_escgrid_init
.............
      CALL esc_grid(mx,my)
.............
      END SUBROUTINE polar_escgrid_init
.............
      END MODULE polar_init
```

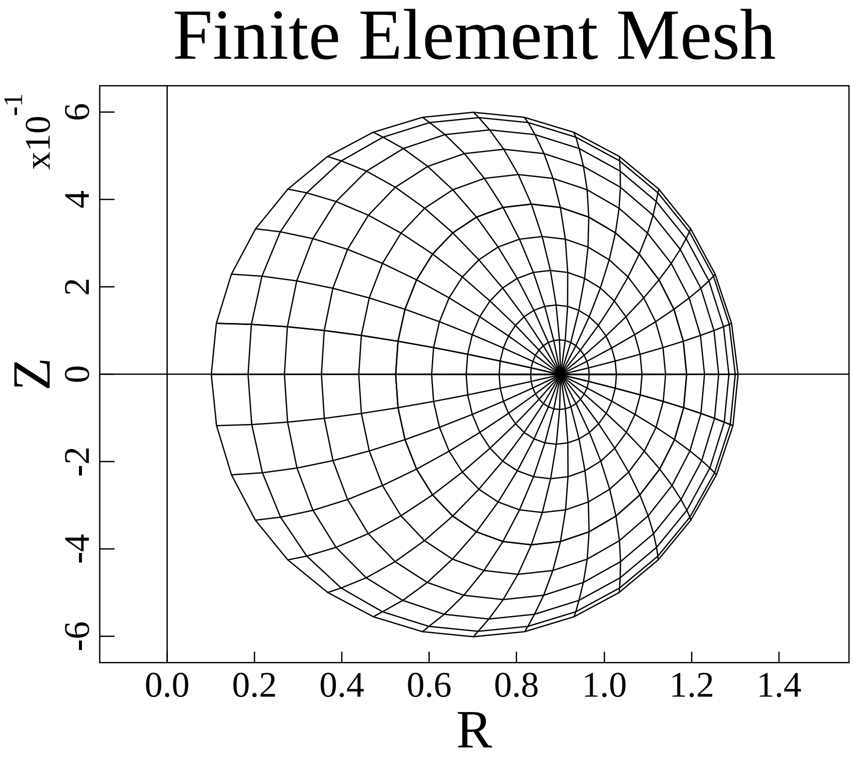
Circular Cross Section: NIMROD mesh (low β)



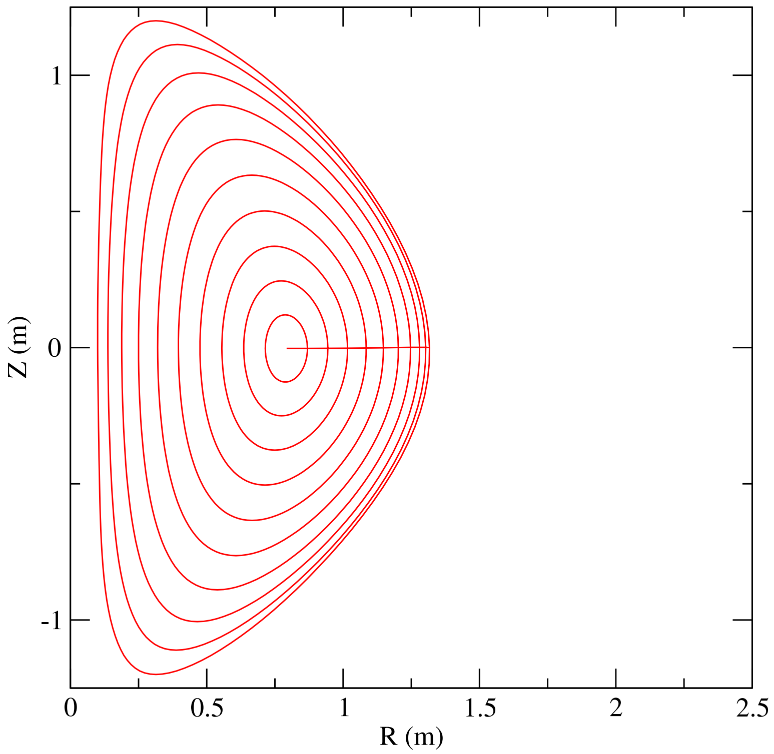

Finite Element Mesh

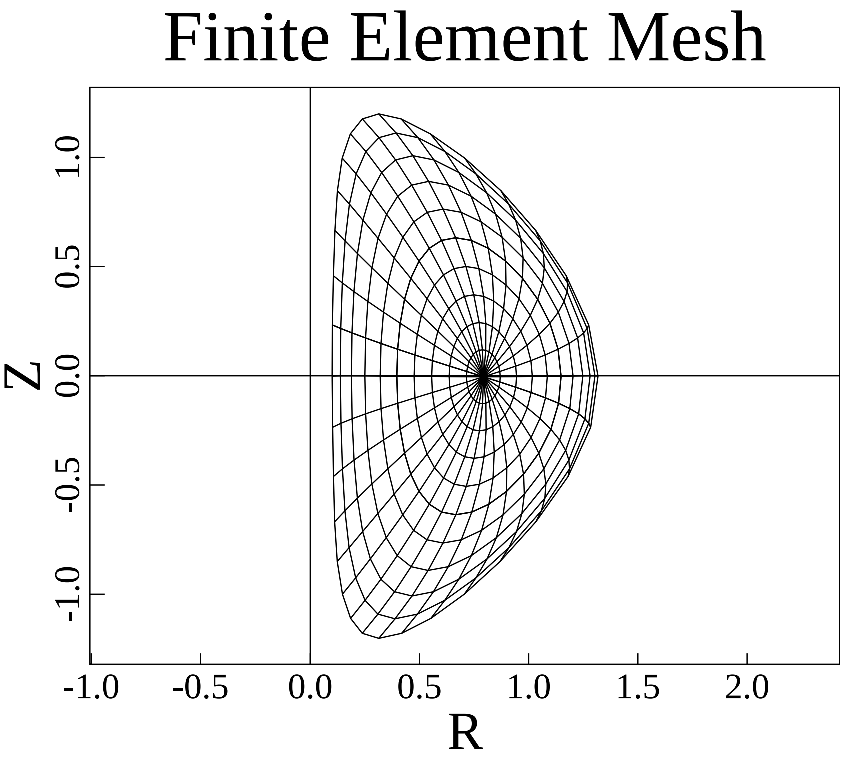
Circular Cross Section: NIMROD mesh (high β)





Non-Circular Cross Section: NIMROD mesh





To do list

- Evaluate equilibrium **B** and **J**
  - ESC only outputs Ψ, $p(\Psi)$, $q(\Psi)$, $I(\Psi)$, need to evaluate the followings

$$\mathbf{B} = \frac{1}{R}\nabla\Psi \times \hat{\phi} + \frac{I}{R}\hat{\phi} \tag{1}$$

$$\mu_0\mathbf{J} = \frac{I'}{R}\nabla\Psi \times \hat{\phi} - \frac{1}{R}\Delta^*\Psi\hat{\phi} \tag{2}$$

$$\Delta^*\Psi = -\mu_0 R^2 p' - II' \tag{3}$$

  - Calculate **B** and **J** in ESC, and/or
  - Calculate **B** and **J** in NIMSET
- Part 2: ballooning simulations