

Resistive MHD Simulations of X-Line Retreat

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NIMROD Team Meeting

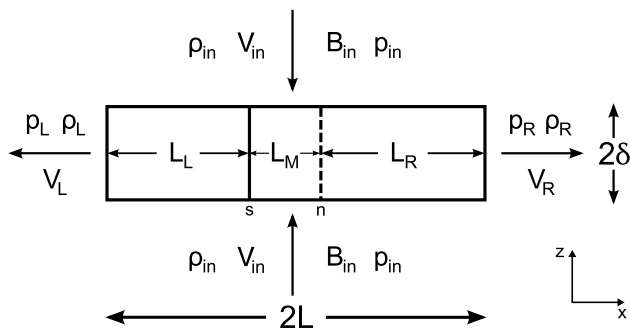
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Mitsuo Oka, Aleida Young, Jun Lin, & Chengcai Shen

Introduction

- ▶ X-line motion is frequently observed during magnetic reconnection in nature and the laboratory. Examples include:
 - ▶ X-line retreat in Earth's magnetotail (e.g., Runov *et al.* 2003; Eastwood *et al.* 2010)
 - ▶ Current sheets that form in the wakes of coronal mass ejections (CMEs)
 - ▶ Radial motion of the diffusion region during spheromak merging (e.g., Inomoto *et al.* 2006; Gray *et al.* 2010; Ono *et al.* 1997; see also Murphy & Sovinec 2008)
- ▶ Despite the prevalence of X-line retreat, it is standard practice to compare *in situ* measurements to particle-in-cell (PIC) simulations of roughly stationary reconnection layers
- ▶ In this talk, I present resistive MHD simulations of X-line retreat and competing X-lines and derive an expression for the rate of X-line retreat

Background: Murphy, Sovinec, & Cassak (2010) developed a scaling model for asymmetric outflow reconnection



- ▶ The above figure represents a long and thin reconnection layer with asymmetric downstream pressure. Reconnection is assumed to be steady in an inertial reference frame.
- ▶ 'n' denotes the magnetic field null and 's' denotes the flow stagnation point

Scaling relations for asymmetric outflow reconnection

- ▶ Following Cassak & Shay (2007), we integrate over this control volume and find relations approximating conservation of mass, momentum, and energy

$$\begin{aligned}2\rho_{in}V_{in}L &\sim \rho_L V_L \delta + \rho_R V_R \delta \\ \rho_L V_L^2 + p_L &\sim \rho_R V_R^2 + p_R \\ 2V_{in}L \left(\alpha p_{in} + \frac{B_{in}^2}{\mu_0} \right) &\sim V_L \delta \left(\alpha p_L + \frac{\rho_L V_L^2}{2} \right) \\ &\quad + V_R \delta \left(\alpha p_R + \frac{\rho_R V_R^2}{2} \right)\end{aligned}$$

where $\alpha \equiv \gamma/(\gamma - 1)$ and we ignore upstream kinetic energy/downstream kinetic energy and assume the contribution from tension along the boundary is small or even.

Deriving the outflow velocity and reconnection rate

- ▶ In the incompressible limit

$$V_{L,R}^2 \sim \sqrt{4 \left(c_{in}^2 - \frac{\bar{p}}{\rho} \right)^2 + \left(\frac{\Delta p}{2\rho} \right)^2} \pm \frac{\Delta p}{2\rho}$$

using $\bar{p} \equiv \frac{p_L + p_R}{2}$, $\Delta p \equiv p_R - p_L$, and $c_{in}^2 = \frac{B_{in}^2}{\mu_0 \rho_{in}} + \frac{\alpha p_{in}}{\rho_{in}}$

- ▶ By assuming resistive dissipation, the electric field is then given by

$$E_y \sim B_{in} \sqrt{\frac{\eta (V_L + V_R)}{2\mu_0 L}}$$

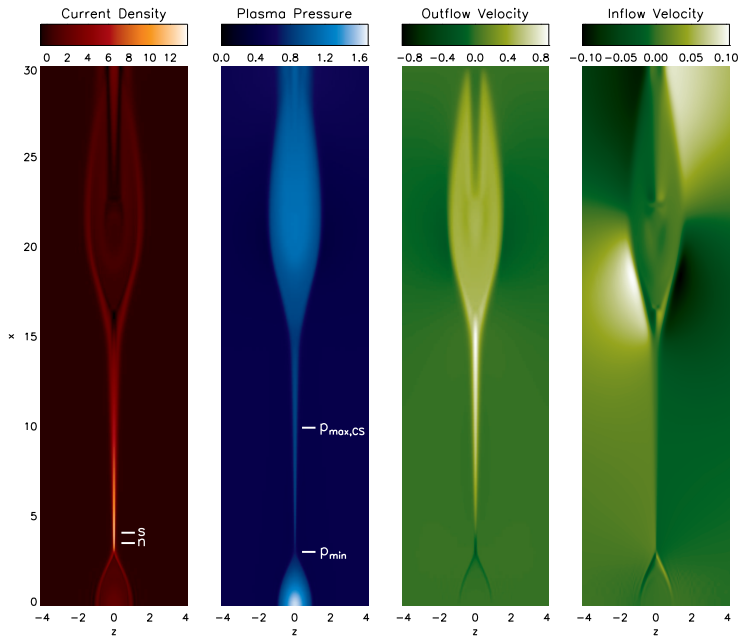
Implications of scaling model

- ▶ The scaling relations show that the reconnection rate is weakly sensitive to asymmetric downstream pressure
 - ▶ If one outflow jet is blocked, reconnection will be almost as quick
 - ▶ Reconnection will slow down greatly only if both outflow jets are blocked
 - ▶ The current sheet responds to asymmetric downstream pressure by changing its thickness or length
- ▶ However, this analysis makes three major assumptions:
 - ▶ The current sheet is stationary
 - ▶ The current sheet thickness is uniform
 - ▶ Magnetic tension contributes symmetrically along the boundaries
- ▶ To make further progress, we must do numerical simulations

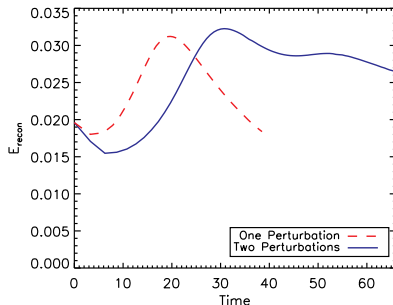
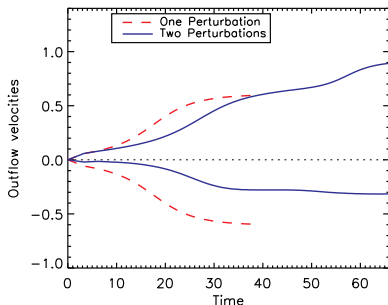
We perform resistive MHD simulations of two initial X-lines which retreat from each other as reconnection develops (see Murphy 2010)

- ▶ The 2-D simulations start from a periodic Harris sheet which is perturbed at two nearby locations ($x = \pm 1$)
- ▶ Domain: $-30 \leq x \leq 30$, $-12 \leq z \leq 12$
- ▶ Simulation parameters: $\eta = 10^{-3}$, $\beta_\infty = 1$, $S = 10^3-10^4$, $\text{Pm} = 1$, $\gamma = 5/3$, $\delta_0 = 0.1$
- ▶ Define:
 - ▶ x_n is the position of the X-line
 - ▶ x_s is the position of the flow stagnation point
 - ▶ $V_x(x_n)$ is the velocity *at* the X-line
 - ▶ $\frac{dx_n}{dt}$ is the velocity *of* the X-line
- ▶ \hat{x} is the outflow direction, \hat{y} is the out-of-plane direction, and \hat{z} is the inflow direction
- ▶ We show only $x \geq 0$ since the simulation is symmetric

The current sheets have characteristic single wedge shapes

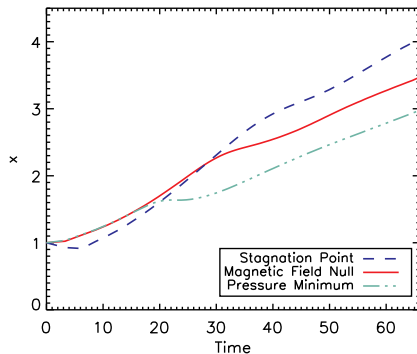


The outflow away from the obstruction is much faster



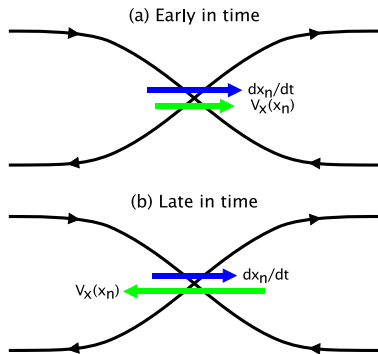
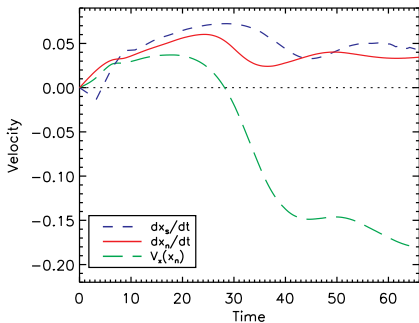
- ▶ In agreement with Seaton (2008) and Reeves *et al.* (2010), most of the energy goes away from the obstructed exit
- ▶ Eventually, reconnection proceeds more quickly in retreat simulations than in otherwise equivalent symmetric, non-retreating simulations
- ▶ The comparison with the single perturbation case is halted around $t \approx 40$ because of the formation of an island at $x = 0$

The flow stagnation point and X-line are not colocated



- ▶ Surprisingly, the relative positions of the X-line and flow stagnation point switch!
- ▶ This occurs so that the stagnation point will be located near where the tension and pressure forces cancel
- ▶ Reconnection develops slowly because the X-line is located near a pressure minimum early in time

Late in time, the X-line diffuses against strong plasma flow



- ▶ The stagnation point retreats more quickly than the X-line
- ▶ Any difference between $\frac{dx_n}{dt}$ and $V_x(x_n)$ must be due to diffusion (e.g., Seaton 2008)
- ▶ The velocity *at* the X-line is not the velocity *of* the X-line

What sets the rate of X-line retreat?

- ▶ The inflow (z) component of Faraday's law for the 2D symmetric inflow case is

$$\frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x} \quad (1)$$

- ▶ The convective derivative of B_z at the X-line taken at the velocity of X-line retreat, dx_n/dt , is

$$\left. \frac{\partial B_z}{\partial t} \right|_{x_n} + \frac{dx_n}{dt} \left. \frac{\partial B_z}{\partial x} \right|_{x_n} = 0 \quad (2)$$

The RHS of Eq. (2) is zero because $B_z(x_n, z=0) = 0$ by definition for this geometry.

Deriving an expression for X-line retreat

- ▶ From Eqs. 1 and 2:

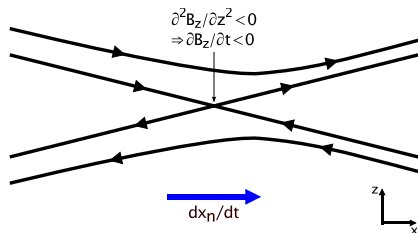
$$\frac{dx_n}{dt} = \left. \frac{\partial E_y / \partial x}{\partial B_z / \partial x} \right|_{x_n} \quad (3)$$

- ▶ Using $\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J}$, we arrive at

$$\frac{dx_n}{dt} = V_x(x_n) - \eta \left[\frac{\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial z^2}}{\frac{\partial B_z}{\partial x}} \right]_{x_n} \quad (4)$$

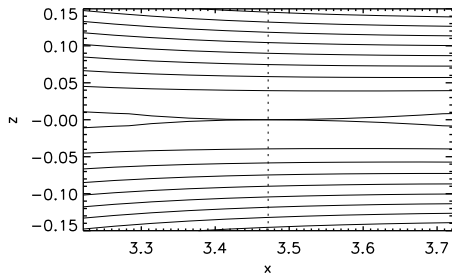
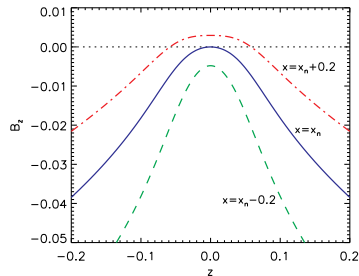
- ▶ In the simulations $\frac{\partial^2 B_z}{\partial z^2} \gg \frac{\partial^2 B_z}{\partial x^2}$, so X-line retreat is caused by diffusion of the normal component of the magnetic field along the inflow direction
- ▶ Equation (3) can also be evaluated using additional terms in the generalized Ohm's law

Mechanism and implications



- ▶ The X-line moves in the direction of increasing total reconnection electric field strength
- ▶ X-line retreat occurs through a combination of advection by bulk plasma flow and diffusion of the normal component of the magnetic field
- ▶ X-line motion depends intrinsically on local parameters evaluated at the X-line

The magnetic field structure near the X-line is characteristic of this mechanism for X-line retreat



- ▶ Negative B_z diffuses from above and below into the region of the X-line
- ▶ *Left:* $B_z(z)$ along three locations near the X-line.
- ▶ *Right:* Magnetic flux contours near the X-line.

Conclusions

- ▶ X-line motion occurs frequently during reconnection in space, astrophysics, and the laboratory
- ▶ Resistive MHD simulations of X-line retreat/asymmetric outflow reconnection show that most of the energy is directed away from the obstructed exit
- ▶ Late in time there is significant flow across the X-line in the opposite direction of X-line retreat
- ▶ An expression for the rate of X-line retreat shows that X-line motion is due to either advection by the bulk plasma flow or by diffusion of the normal component of the magnetic field

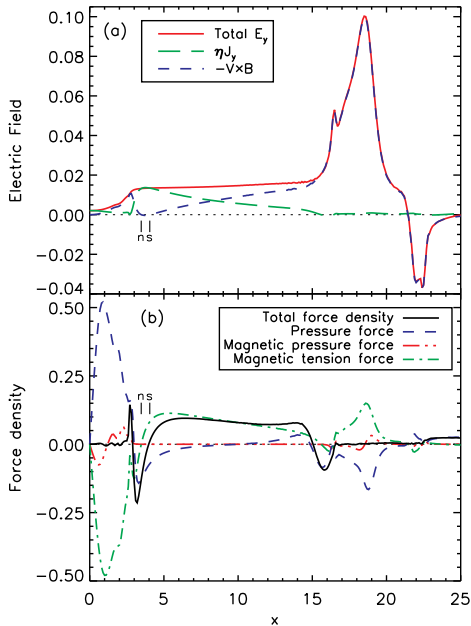
Probable directions for NIMROD simulations

- ▶ I just got 4.1×10^{-10} processor-Hubble times on NASA's Pleiades cluster
- ▶ Simulate two-fluid or 3-D X-line retreat
- ▶ Examine X-line dynamics during the plasmoid instability
- ▶ Investigate X-line motion in asymmetric inflow reconnection
- ▶ Simulate line-tied asymmetric inflow reconnection to see if this can lead to current sheet drifting as observed in some events
- ▶ Simulate anemone jets to determine scaling behavior and whether or not coronal jets contribute substantially to the solar wind (recently submitted NASA TR&T proposal)
- ▶ Our group is developing postprocessing subroutines to perform time-dependent ionization analyses of MHD simulations of CME current sheets
- ▶ I have developed several tools for analyzing NIMROD simulations using IDL which I can make available

A potential numerical methods study

- ▶ Many astrophysical simulations employ numerical resistivity as the sole means of breaking magnetic field lines, even when reconnection is an important effect
- ▶ This is bad because ideal MHD simulations which show significant reconnection are inherently unconverged
- ▶ A useful project would be to investigate the dynamics of this 'numerical reconnection' to show the drawbacks of such an approach and encourage astronomers and space scientists to use a physical resistivity
- ▶ My question: What is an appropriate numerical method?
 - ▶ Solves the equations of ideal or resistive MHD, including compressibility
 - ▶ Is numerically dissipative of second (or fourth) order, with well-understood error terms
 - ▶ Pre-existing and easy to use, or simple to program

Electric field and momentum density plots



Simulations of multiple X-line reconnection

- ▶ NIMROD resistive MHD simulations with multiple initial X-line perturbations were analyzed by A. Young
- ▶ Isolated or strong perturbations are more likely to survive
- ▶ Initial X-lines surrounded by other initial X-lines are less likely to survive, but X-lines which have room to develop in one outflow direction do have a better chance of developing (see also two-fluid simulations by Nakamura *et al.* 2010)
- ▶ Early on, winning X-lines have plasma pressure facilitating outflow rather than impeding it
- ▶ When an X-line is located near one exit of the current sheet, the flow stagnation point is located between the X-line and a central plasma pressure maximum
- ▶ Flow across the X-line in the opposite direction of X-line motion does occur, but infrequently

In resistive MHD when inflow is symmetric, the number of X-lines can only change by resistive diffusion of B_z

- ▶ The induction equation is

$$\frac{\partial \mathbf{B}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{v} - \mathbf{B} (\nabla \cdot \mathbf{v}) + \eta \nabla^2 \mathbf{B}, \quad (5)$$

where in our 2-D geometry,

$$[\eta \nabla^2 \mathbf{B}]_z = \eta \left[\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial z^2} \right]. \quad (6)$$

- ▶ The term $\eta \frac{\partial^2 B_z}{\partial x^2}$ acts to smooth out the $B_z(x)$ profile. This term can move or reduce the number of X-lines (where $B_z = 0$), but not create new X-lines.
- ▶ The term $\eta \frac{\partial^2 B_z}{\partial z^2}$ brings in B_z along the inflow direction. This term can move or increase the number of X-lines, but by symmetry cannot cause pre-existing X-lines to disappear.

Future work on X-line retreat

- ▶ Simulation:
 - ▶ Two-fluid simulations of X-line retreat
 - ▶ 3-D simulations of X-line retreat with initial perturbations offset from each other in the out-of-plane direction
 - ▶ Examine X-line behavior during the plasmoid instability
 - ▶ Investigate X-line dynamics in two and three dimensions for realistic geometries such as coronal jets, CME current sheets, planetary magnetotails, and spheromak merging experiments
 - ▶ Investigate X-line motion in asymmetric inflow reconnection
- ▶ Theory:
 - ▶ Derive expressions for neutral line and X-line motion in 3-D
 - ▶ Determine the rate of retreat of the flow stagnation point
 - ▶ Examine the consequences of a separation between 3-D magnetic field nulls and flow stagnation points
- ▶ Experiment and observation (suggestions for others):
 - ▶ Characterize X-line motion during laboratory reconnection
 - ▶ Find signatures of X-line retreat in magnetotail (M. Oka)
 - ▶ Observe effects of asymmetric outflow in CME current sheets