Benchhmark of NIMROD kinetic electron closures with NEO

APS-DPP 2010, Chicago, Illinois

E. Held, S. Kruger, J. Callen, E. Belli, J-Y Ji

November 11, 2010

0.1 First-order DKE in the (ξ,s) velocity variables

Hazeltine's form for the drift kinetic equation (ϵ, μ) :

$$\partial_t f + (\mathbf{v}_{||} + v_D) \cdot \nabla f + \left(\mu \frac{\partial B}{\partial t} + e(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E}\right) \partial_\epsilon f = C + Q.$$

Using $\xi = v_{||}/v$ and $s = v/v_0$ yields

$$\partial_t f + (\mathbf{v}_{||} + v_D) \cdot \nabla f - +(\mathbf{v}_{||} + v_D) \cdot \left[\frac{1-\xi^2}{2\xi} \nabla \ln B \partial_\xi + s \nabla \ln v_0 \partial_s\right] f + \left(\frac{e}{2e_0 s^2}(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E}\right)(s\partial_s f + 2g(\xi)\partial_\xi f) = C + Q$$

with general form for drift

$$v_D = \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{e_0 s^2}{eB}\left[\mathbf{b}\times\left(\left(1-\xi^2\right)\nabla \ln B + 2\xi^2\kappa - \frac{v_0 s\xi}{e_0 B}\nabla\times\mathbf{E}\right) + (1-\xi^2)\frac{\mu_0 \mathbf{J}_{||}}{B}\right].$$

0.2 1D FE basis in ξ

- Use 1D FE grid in pitch angle. In each element expand f as :

$$f(\mathbf{r}, t, \xi, s) \;=\; \sum_i f_i(\mathbf{r}, t, s)\phi_i(x)$$

.

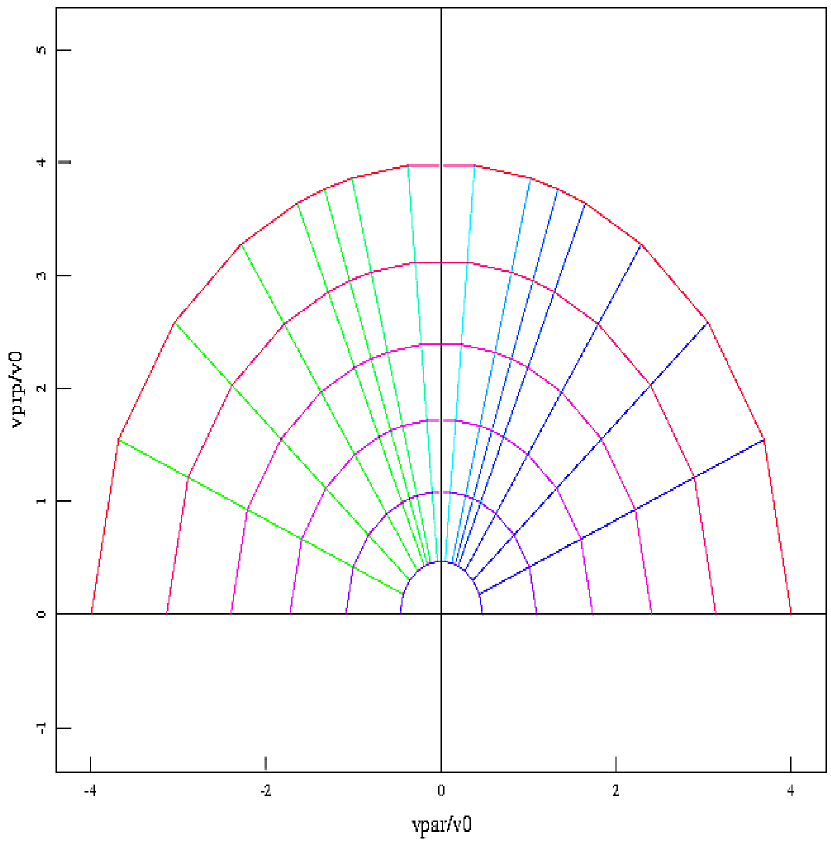
- Modal (built from Legendre polynomials) and nodal (Lagrange and Gauss-Lobatto-Legendre) bases have been implemented.

- Test packing in pitch angle.

- Pitch-angle coefficients, f_i, computed on speed grid determined by Gauss-Laguerre ($s\epsilon[0, \infty)$) or Gauss-Legendre ($s\epsilon[0, s_{max}]$) quadrature.

0.3 Sample velocity grid

- 3 cells in ξ with 5th-order polynomials and 6 speed grid points = 96 unknowns. Naturally packed near trapped/passing boundary.



0.4 Coulomb collision operator for like particle collisions

- Full, linearized Coulomb collision operator taken from Ji and Held, PoP (2006) :

$$C^{aa} = \frac{1}{n_a v_{Ta}^{l+2k}} \sum_{lk} \frac{f_a^{(0)}}{\sigma_k^l} P_l(v_{||}/v) M_{||}^{lk}(\mathbf{r},t) \nu_{aa}^{lk},$$

where $f_a^{(0)}$ is Maxwellian, ν_{aa}^{lk}'s are speed dependent collision frequency and

$$M_{||}^{lk} = \frac{l!}{(2l-1)!!} v_T^{l+2k} \int d\mathbf{v} L_k^{l+1/2}(s^2) s^l P_l(v_{||}/v) F$$

- Applying quadrature yields:

$$C^{aa} = \frac{2}{\sqrt{\pi}} e^{-s_{is}^2} \sum_{js} w_{js} \begin{pmatrix} \frac{1}{2} s_{js}^{l+1-2n} e^{s_{js}^2} \\ s_{js}^{l+2} \end{pmatrix} L_k^{l+1/2}(s_{js}^2) \sum_j \sum_{lk} \frac{l!}{(2l-1)!!} \frac{\nu_{aa}^{lk*}(s_{is})}{\sigma_k^l} P_{li} P_{lj} f_j(\mathbf{r}, t, s_{js}).$$

0.5 Coulomb collision operator for unlike particle collisions

- Full, linearized Coulomb collision operator taken from Ji and Held, PoP (2006) :

$$C^{aa} = \frac{1}{n_a v_{Ta}^{l+2k}} \sum_{lk} \frac{f_a^{(0)}}{\sigma_k^l} P_l(v_{||}/v) M_{||}^{lk}(\mathbf{r},t) \nu_{aa}^{lk},$$

where $f_a^{(0)}$ is Maxwellian, ν_{aa}^{lk}'s are speed dependent collision frequency and

$$M_{||}^{lk} = \frac{l!}{(2l-1)!!} v_T^{l+2k} \int d\mathbf{v} L_k^{l+1/2}(s^2) s^l P_l(v_{||}/v) F$$

- Applying quadrature yields:

$$C^{aa} = \frac{2}{\sqrt{\pi}} e^{-s_{is}^2} \sum_{js} w_{js} \begin{pmatrix} \frac{1}{2} s_{js}^{l+1-2n} e^{s_{js}^2} \\ s_{js}^{l+2} \end{pmatrix} L_k^{l+1/2}(s_{js}^2) \sum_j \sum_{lk} \frac{l!}{(2l-1)!!} \frac{\nu_{aa}^{lk*}(s_{is})}{\sigma_k^l} P_{li} P_{lj} f_j(\mathbf{r}, t, s_{js}).$$

0.6 Ordering for neoclassical transport calculations

Order $v_D << v_{||}$ and assume weak (relative to Dreicer) electric field:

$$\partial_t f_0 + \mathbf{v}_{||} \cdot \nabla f_0 - \mathbf{v}_{||} \cdot \left[\frac{1-\xi^2}{2\xi} \nabla \ln B \partial_\xi + s \nabla \ln v_0 \partial_s \right] f_0 \;=\; C(f_0)$$

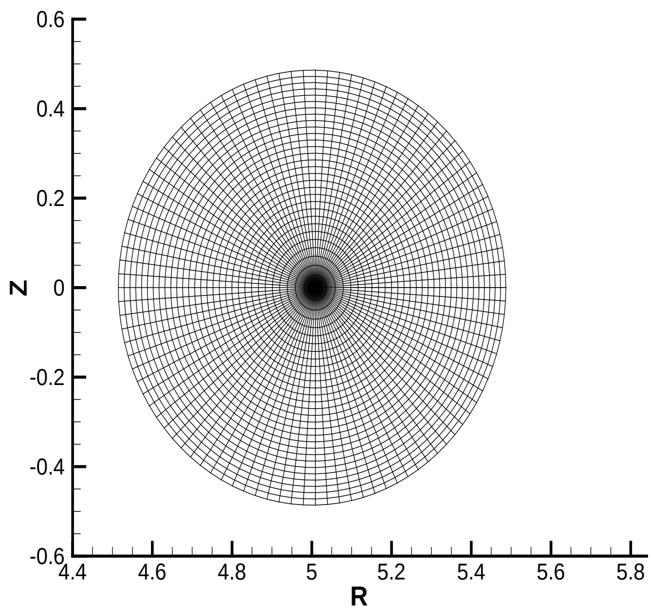
which is satisfied by stationary Maxwellian with flux functions n and T.
To next order :

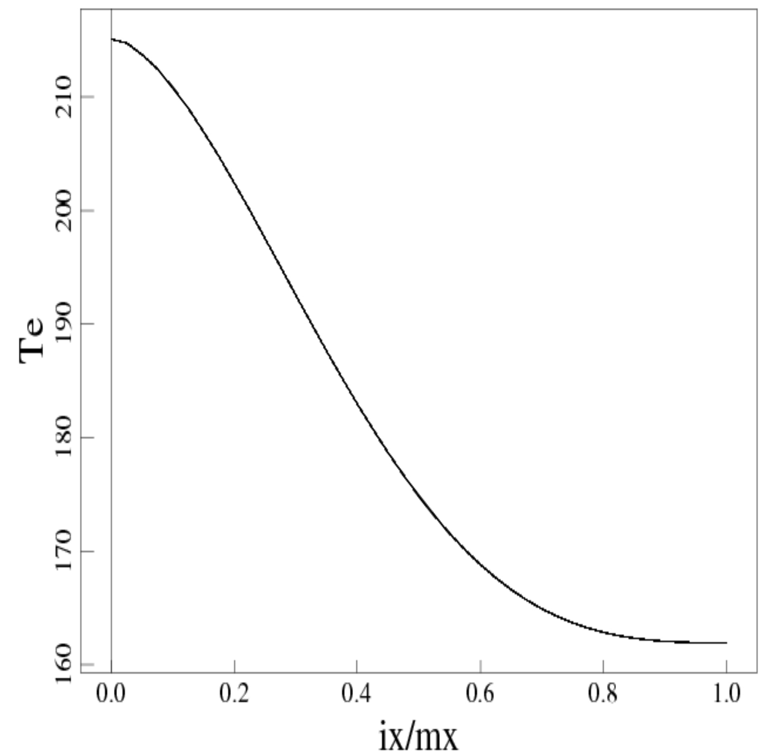
$$\begin{aligned} \partial_t f_1 + \mathbf{v}_{||} \cdot \nabla f_1 - \left(\mathbf{v}_{||} \cdot \nabla \ln B \right) \frac{1-\xi^2}{2\xi} \partial_\xi f_1 \;&= \\ -\mathbf{v}_D \cdot \nabla f_0 + s v_D \cdot \nabla \ln v_0 \partial_s f_0 - \frac{e}{2\epsilon_0 s} \mathbf{v}_{||} \cdot \left(\mathbf{E}^A - \nabla \phi_1 \right) \partial_s f_0 \;&+\; C^{aa} + C^{ab} \end{aligned}$$

Using $g = f_1 - (e\phi_1/T_0) f_0$yields (compare with Eq. 23 of Belli and Candy, 51 PPCF 2009):

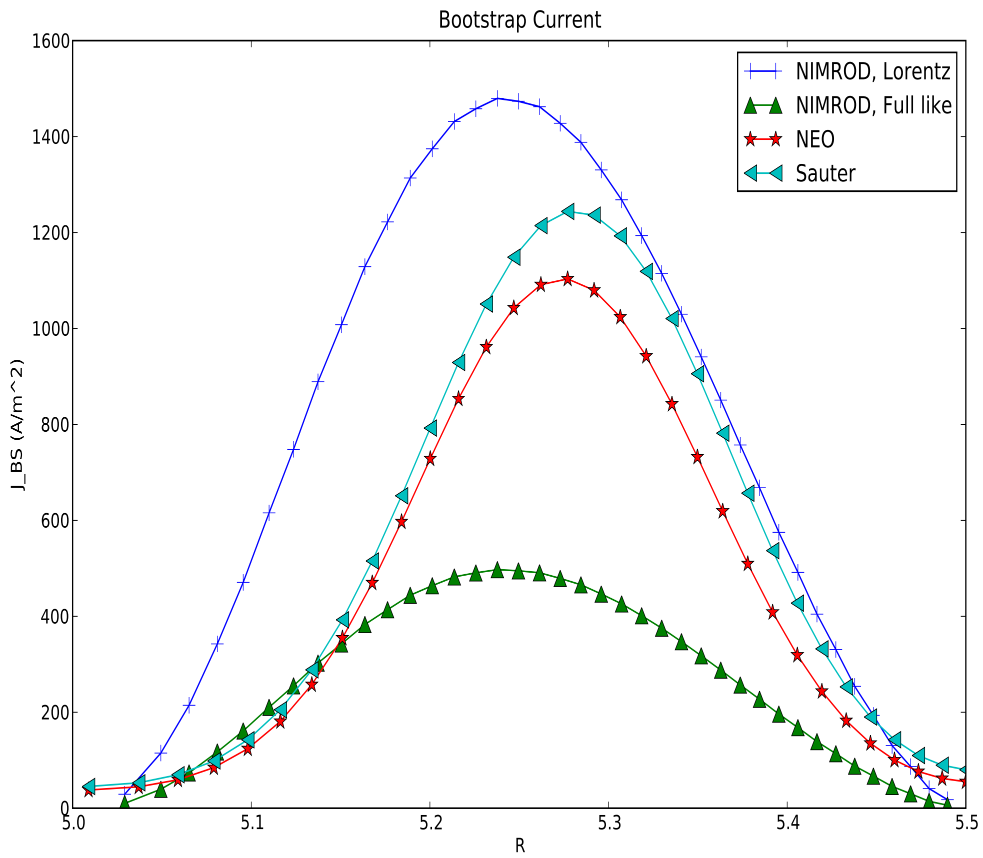
$$\begin{aligned} \partial_t g + \mathbf{v}_{||} \cdot \nabla g - \mathbf{v}_{||} \cdot \nabla \ln B \partial_\xi g \quad &= \\ -\mathbf{v}_D \cdot \nabla f_0 + s v_D \cdot \nabla \ln v_0 \partial_s f_0 \quad &+ \\ C^{aa} + C^{ab} - \frac{e}{2\epsilon_0 s} \mathbf{v}_{||} \cdot \mathbf{E}^A \partial f_0 \;\; &+ (e f_0/T_0) \partial_t \phi_1 \end{aligned}$$

0.7 Test on high-aspect ratio Grad-Shafronov equilibrium

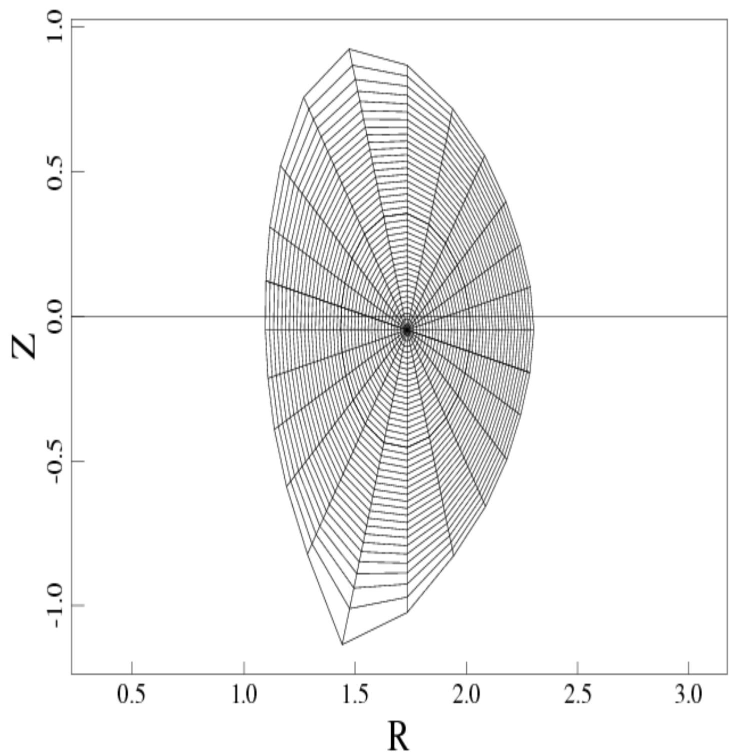


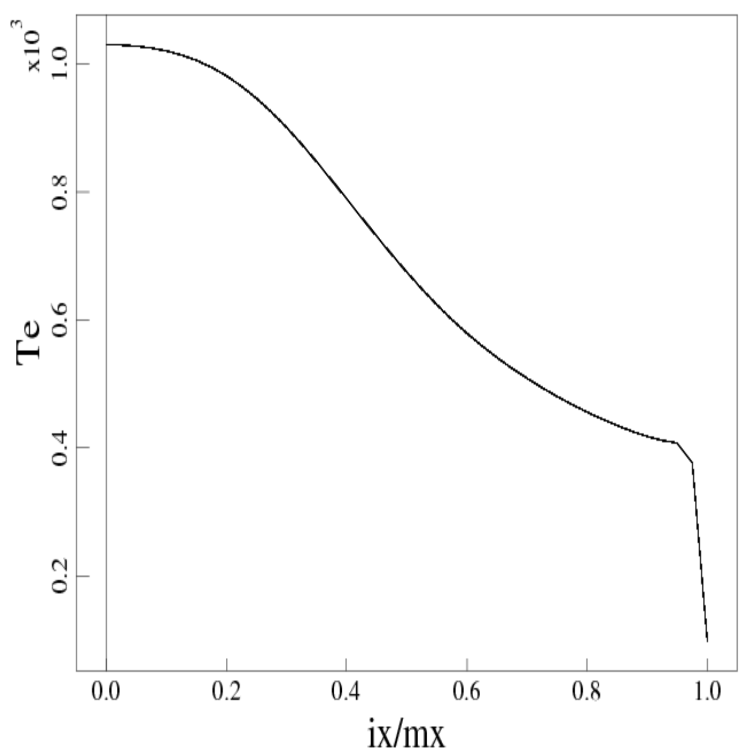


0.8 Preliminary Bootstrap Current Comparison



0.9 Test on DIII-D equilibrium 118897





0.10 Further disscussion

- C^{aa} -> NIMROD uses full linearized Coulomb operator; NEO uses various reduced forms with the best being the "re-normalized" form of Hirshman and Sigmar .

- C^{ab}-> NIMROD and NEO use $C^{ei} = L_{ee} + \nu_{ei}(v) m_e v_{||} V_{||i} f_{0e}/T_{0e}$. Ion/electron operator, $C^{ie} = -s\xi R_{||ei} f_{0i}/p_{0i}$, also implemented in NIMROD with simultaneous solve for both distribution functions. Here $R_{||ei} = m \int d\mathbf{v} v_{||} C^{ei}$.

- Works well on a workstation, but velocity space resolution limited. Debugging on Franklin.