Recent results from low $S$ resistive MHD simulations of the HIT-SI experiment

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• NIMROD’s implicit advection algorithm has reduced the computation time by a factor of 10, which allows us to run more injector cycles in a reasonable amount of time.

• Results from low $S$ resistive MHD simulations show production of $n=0$ magnetic energy, toroidal flux and current during sustainment.

• The growth in $n=0$ occurs after 2 flat-top injector cycles.

• No separatrix formation observed during sustainment, consistent with Taylor state calculations for a current amplification factor $\frac{I_{\text{inj}}}{I_{\text{tor}}} \leq 1$.

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1G. Marklin, C. Hansen calculations
1. Highlights

2. The HIT-SI Experiment

3. Computational Model
   - Injector fields
   - NIMROD implementation

4. Results from Resistive MHD Simulations
Helicity Injected Torus with Steady Inductive Helicity Injection (HIT-SI) is a current drive experiment

The spheromak plasma is formed and sustained by Steady Inductive Helicity Injection (SIHI) provided by two semi-toroidal injectors.

- 2 helicity injectors (X and Y) inject flux and current at a constant rate and inductively into the tank.
- The injectors have \( n=\text{odd} \) toroidal symmetry and spatially rotated 90° for minimal coupling.
- Insulating layer guarantees inductive drive by preventing arcing to the flux conserver.
- Closed flux device.
Helicity injectors are not directly modeled in NIMROD

- Injectors have a non-axisymmetric geometry.
- SIHI is implemented by applying flux and current B.C. at the appropriate locations on the annular regions of the HIT-SI tank.
The insulating layer is simulated as a highly resistive edge layer

The resistive layer is approximately five orders of magnitude more resistive than the plasma: \( \frac{\eta_{\text{edge}}}{\eta} \sim 10^5 \)

Finite Element Mesh

- 1 mm thickness
- Resistivity profile is laid directly on the quad. points of the finite elements → prevents overshoot
- Resistive layer covers the injector mouths:
  - turns the injectors into a current source and prevents line-tying.
  - creates an electric field on the annulus consistent with the locations of the voltage gaps in the experiment.
The injectors are modeled as a force-free, large aspect ratio RFP

- Axisymmetry $\rightarrow$ MATLAB mimetic operator GS. solver $^1$ solves $\triangle^* \psi + \lambda_{inj}^2 \psi = 0$
- The cross-section is tailored to yield an eigenvalue of $\lambda_{inj} = 30 \text{ m}^{-1}$ which corresponds to a solution with the reversal surface at the wall (spheromak mode).

Injector half-torus used in MATLAB for B.C.

Actual HIT-SI injector (courtesy of J. Rogers)

$^1$Originally developed by G. Marklin, and modified by CA and CCK for quad-element meshes
In NIMROD $\mathbf{B} \cdot \mathbf{\hat{n}}$ and $\mathbf{E}_\parallel$ must be prescribed on the annular surfaces to apply SIHI B.C.

- A radial electric field $E_R$ is applied on each annulus to induce the injector flux($B_z$). Applying Faraday’s law to $B_z$ with $E_\phi = 0$ yields
  $$E_R = \frac{R}{in} \dot{B}_z = \frac{\omega_{inj} R}{n} B_{inj} \quad (1)$$

- A call is made at the end of the magnetic field advance to overwrite $\mathbf{B} \cdot \mathbf{\hat{n}} \mid_{\text{annulus}} = 0$ and update $\mathbf{B}$ according to flux B.C.

- An electrostatic $\mathbf{E}$ applied on each annulus gives rise to a tangential $\mathbf{B}$ through ($\dot{\mathbf{B}} = \int d\mathbf{S} \times \mathbf{E} + \cdots$) with a net curl along $\mathbf{\hat{z}}$. This term drives the injector current.
Toroidal profiles ($\phi$) of the injected fields with 11 (5 injector–odd) modes

- $\phi$ dependence of $B_{z_{inj}}$
- $\phi$ dependence of applied $E_{\phi}$
- $\phi$ dependence of applied $E_{R}$
- $\phi$ dependence of $E_{\text{faraday}}$

**Graphs:**
- **$\phi$ dependence of $B_{z_{inj}}$:**
  - Amplitude range: $-3000$ to $3000$
  - $\phi$ range: $0$ to $400$
- **$\phi$ dependence of applied $E_{\phi}$:**
  - Amplitude range: $-1 \times 10^0$ to $0.8$
  - $\phi$ range: $0$ to $400$
- **$\phi$ dependence of applied $E_{R}$:**
  - Amplitude range: $-3000$ to $3000$
  - $\phi$ range: $0$ to $400$
- **$\phi$ dependence of $E_{\text{faraday}}$:**
  - Amplitude range: $-1000$ to $1500$
  - $\phi$ range: $0$ to $400$

**Legend:**
- odd modes only
- odd modes + $n=0$
Resistive MHD (rMHD) with uniform density and zero $\beta$

\[
\rho \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = \mathbf{J} \times \mathbf{B} - \nabla \cdot \Pi \quad \text{kin or iso_visc} \tag{2}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \frac{\eta}{\mu_0} \nabla^2 \mathbf{B} \tag{3}
\]

**Table:** Numerical parameters for low $S$ simulations

<table>
<thead>
<tr>
<th>Mesh (mx×my)</th>
<th>48x48</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poly_degree</td>
<td>3 , 4</td>
</tr>
<tr>
<td>Toroidal modes</td>
<td>11</td>
</tr>
<tr>
<td>mhdadv_alg</td>
<td>‘centered’</td>
</tr>
<tr>
<td>v_cfl</td>
<td>4.0</td>
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<tr>
<td>divbd</td>
<td>$2 \times 10^5$</td>
</tr>
</tbody>
</table>

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NIMROD simulations of HIT-SI
APS 2010-NIMROD
Physical parameters of the simulation and experiment closely match (assuming $T_i = T_e = 10$ eV)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Simulation</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injector flux (mWb)</td>
<td>0.47</td>
<td>0.6-1.4</td>
</tr>
<tr>
<td>Injector current (kA)</td>
<td>10.6</td>
<td>10-20</td>
</tr>
<tr>
<td>$n_e$ (m$^{-3}$)</td>
<td>$3 \times 10^{19}$</td>
<td>$3 \times 10^{19}$ (avg.)</td>
</tr>
<tr>
<td>Resistivity ($\Omega.m$)</td>
<td>$1.5 \times 10^{-5}$ (elecd=11.7)</td>
<td>$3.2 \times 10^{-5}$ ($\eta_{</td>
</tr>
<tr>
<td>Viscosity (kg/m/s)</td>
<td>$10^{-5}$ (kin_visc=100)</td>
<td>$2.6 \times 10^{-5}$</td>
</tr>
<tr>
<td>Edge resistivity</td>
<td>$8 \times 10^4 \eta_{pl}$</td>
<td>$\infty$</td>
</tr>
<tr>
<td>Edge thickness (mm)</td>
<td>1</td>
<td>$\sim 1$</td>
</tr>
<tr>
<td>$\tau_{inj}$ (ms)</td>
<td>0.2</td>
<td>0.172</td>
</tr>
<tr>
<td>$\tau_{L/R}$ (ms)</td>
<td>0.80</td>
<td>0.40 ($\eta_{</td>
</tr>
<tr>
<td>$\tau_A$ (ms)</td>
<td>0.017</td>
<td>0.025</td>
</tr>
<tr>
<td>$S^1$</td>
<td>45</td>
<td>16</td>
</tr>
<tr>
<td>$\lambda_{inj}$ (m$^{-1}$)</td>
<td>30</td>
<td>15-20</td>
</tr>
<tr>
<td>Pulse length (ms)</td>
<td>1-2</td>
<td>4-10</td>
</tr>
</tbody>
</table>

$^1$ S here is approximately 100 times smaller than $S^*$
Constant helicity injection is sustained for 6 injector periods from t=0.1 to 1.3 ms

Injectors are linearly ramped-up for 0.1 ms, then run at a constant amplitude (flat-top) for 6 cycles from t=0.1 to 1.3 ms, followed by a linear ramp-down for another 0.1 ms.

Injectors are shut-off at t=1.4 ms and the spheromak is left to decay for another 0.3 ms.
n=0 magnetic energy rises rapidly after 2 injector flat-top cycles ($t=0.5$ ms)

- Magnetic energy of the higher even modes also grow.
- $n=1$ magnetic energy drops as $n=0$ rises.

![Magnetic energies by toroidal mode S =50](image)
The linear growth ($\gamma\tau_A = 1.32$) of n=0 precedes the rapid build-up and saturates at $t=0.45$ ms, followed by non-linear growth during which n=0 magnetic energy is amplified 10-fold.

n=0 growth drives higher even modes. n=2 and 4 also go through a linear growth stage, but at a decreasing linear growth rate ($\gamma (2,3)\tau_A = (0.88, 0.86)$).
Kinetic energy spectrum exhibits increased odd mode activity during n=0 growth.

A marginally linear growth period is seen for the odd modes. In the aftermath of n=0 growth, kinetic energies of odd modes increase and even modes decrease, a trend opposite to the magnetic energy spectrum.
Plasma current ($I_{tor}$) reaches 9 kA

- Current amplification $\frac{I_{tor}}{I_{inj}} \sim 0.85$
- Peak $I_{tor} \approx 11$ kA occurs at injector shut-off ($t=1.4$ ms)
- Current amplification up to a factor of 2 is seen in the experiment.
Early profiles \((t=0.4 \text{ ms})\) exhibit a large \(n=1\) component, consistent with magnetic energy spectrum.

**Midplane \((Z=0)\) B profiles at \(\phi=0\)**

**Midplane \((Z=0)\) B profiles at \(\phi=180\)**

**Magnetic energies by toroidal mode \(S=50\)**
Profiles become mostly axisymmetric after rapid growth of n=0 (t=0.85 ms)
The $n=1$ distortion is still present at later times ($t=1.3\text{ms}$)

Midplane ($Z=0$) B profiles at $\phi=0$

Midplane ($Z=0$) B profiles at $\phi=180$

Magnetic energies by toroidal mode $S=50$

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No separatrix formation observed during sustainment (SIHI)

- Field lines are short and connect the injector mouths (t=0.825 ms)
- Puncture plots show the volume is filled with stochastic lines during sustainment

- The configuration begins to relax after injectors are shut-off (t=1.6 ms).
Nested flux surfaces do not form till the fluctuations $(\delta \equiv \sqrt{\frac{E_{n=1}}{E_{n=0}}})$ get really small.

$t=1.4 \text{ ms}, \delta = 0.22$

$t=1.5 \text{ ms}, \delta = 0.023$

$t=1.6 \text{ ms}, \delta = 0.004$

$t=1.7 \text{ ms}, \delta = 0.0017$
Emergence of $q \leq 1$ surfaces within 10 cm. of the magnetic axis (0.33 cm)

- A $q=1/2$ surface is shown here.
We see a rapid growth of $n=0$ magnetic energy after 2 injector cycles. Profiles become more axisymmetric, plasma current rises, but never exceeds injector current.

The onset of $n=0$ growth corresponds to one energy (helicity) e-folding time $\Rightarrow$ Helicity balance: $K(t) = K_{\text{inj}}(1 - e^{-2t/\tau_{L/R}})$

$n=1$ magnetic energy decreases during $n=0$ amplification.

No nested flux surfaces form during sustainment.

How is the energy transferred from $n=1$ to $n=0$? Local 3-D reconnection events (current layers)?
Future plans

- Check for toroidal and poloidal convergence (currently being analyzed)
- Perform an S scan [10-100] at $\lambda_{\text{inj}} = 30$ and also at a lower $\lambda_{\text{inj}}$ (15-25).
- Compare simulation results to experimental data and Taylor state calculations.
- Turn on the Hall term. Run finite $\beta$?