Update and discussion of fluid ITG with NIMROD's model

E.C. Howell

University of Wisconsin-Madison

NIMROD Team Meeting November 6, 2010

A numerical mode is observed at large Hall parameter in linear two-fluid calculations of a periodic cylindrical spheromak.

- The mode only appears when the Hall term is included in Ohm's law.
- Resistive MHD calculations that include gyro-viscosity run smoothly.
- The numerical modes have coherent structure.
- The real frequency and growth rate of the modes depend on numerical parameters.
- Increasing numerical resolution does not help.
- Other people have observed odd behavior when using the two-fluid Ohm's law (Dalton Schnack-Giant Sawtooth, Ping Zhu-2fl peeling?)

Drift waves are a good test of the electron pressure term in Ohm's law

- The simple drift wave picture assumes that electrons have a pure Boltzmann response: $\tilde{n}_e = \frac{n_{e0} e}{T_e 0} \tilde{\phi}$
- The presence of a pressure gradient introduces stable drift waves: $\omega = k v_{*e}$.
- A variety of physical effects (resistivity, electron inertia, etc) can retard the electron response and cause instability.
- Phenomenologically these effects can be modeled by altering the Boltzmann relation: $\tilde{n}_e = \frac{n_{e0} e}{T_{e0}} \tilde{\phi}(1 + i\delta)$.
    - A slowly growing mode results even when $\delta \ll 1$.
- In NIMROD the Boltzmann electron response enters through the electron pressure term in Ohm's law with isothermal electrons.
- Inaccuracies in this term may result in a small numerical δ.
- The small δ effect is one of many possible culprits that may be responsible for instability.

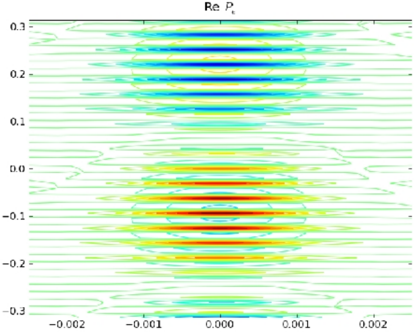
Calculations of the slab branch of the ITG are used to test NIMROD's ability to reproduce drift waves.

- A hyperbolic pressure profile is specified with uniform density.
- Coppi et al, Phys. Fluids 1967 give the slab ITG dispersion relation:
  $\omega^3 - \omega k_z^2 \frac{3T_i + T_e}{M} = -k_z^2 \frac{T_{e0}}{M} \frac{k_y T'_{i0}}{eB_0}$
- The above dispersion relation predicts:
  $\gamma \sim 1.4x10^5 s^{-1}$
  $\omega \sim 0.8x10^5 s^{-1}$
- When we derive the dispersion relation without the magnetization heat, we find a stable sound wave moving at the $E \times B$ drift.

- $B = 0.5T$
- $\tau_A \sim 1x10^{-8} s$
- $n = 1.0x10^{18} m^{-3}$
- $\beta = 1.6\%$
- $k_z = 0.1 m^{-1}$
- $k_y = 10 m^{-1}$
- $1/L_p = 10 m^{-1}$

A numerical mode associated with the electron pressure is observed in slab calculations

- γ and ω are on the same order of magnitude as predicted by Coppi et al, but the mode does not converge.
- The numerical mode behaves similarly to the one observed in the benchmark.
  - MHD and gyro-viscous calculations are stable.
  - The growth rate and frequency of the mode depend on δt.
  - The mode has a semi-coherent structure.



Coppi et al give a simple fluid derivation of the ITG.

- Uniform static magnetic field $\vec{B} = B_0 \hat{e}_Z$
- Electrostatic $\vec{E}$
- Uniform equilibrium number density
- Uniform equilibrium electron temperature
- Boltzmann electrons $\phi = \frac{T_{e0}}{n_{e0} e} n_e$
- Nonuniform equilibrium ion temperature that varies in x
- All other equilibrium quantities are allowed to vary in x.
- Perturbed quantities vary as $e^{i\omega t + ik_y y + ik_z z}$
- Low frequency modes $\omega << \Omega_i$
- Ion frame of reference $V_{i0} = 0$

Low frequency ordering is used for the perpendicular ion flow.

- The perpendicular ion momentum equation becomes:
  $\vec{v}_{i\perp} = \frac{\vec{E}\times\vec{B}}{B^2} - \frac{\nabla(nT_i)\times\vec{B}}{n_i e B^2} - \frac{1}{\Omega_i}\frac{D\vec{v}_i}{Dt} \times \hat{b}$.
- The ion polarization drift in neglected:
  $\vec{v}_{i\perp} = \frac{\vec{E}\times\vec{B}}{B^2} - \frac{\nabla(nT_i)\times\vec{B}}{n_i e B^2} + O(\frac{\omega}{\Omega_i})\vec{v}_{i\perp}$.
- Setting $V_{i0\perp} = 0$ determines $E_{0\perp}$:
  $\vec{E}_{0\perp} = \frac{T'_{i0}}{e}\hat{e}_x$.
- The linear perpendicular flow is:
  $\tilde{v}_{i\perp} = -\frac{ik_y}{eB}(e\tilde{\phi} + \tilde{T}_i + \frac{\tilde{n}}{n_0}T_{i0})\hat{e}_x$.
  - To lowest order $\tilde{v}_{iy} = 0$.

In solving the equation of state the authors neglect the linear ion diamagnetic flow.

- The linearized temperature evolution equation that appears is: $i\omega n_0 T_i + (\gamma_i - 1) n_0 T_{i0} i k_z v_z + (E_y/B_0) n_0 T'_{i0} = 0.$
- The advective term only includes the linear $E \times B$ drift.
- Neglecting the linear diamagnetic flows the dispersion relation is: $\omega^3 - \omega k_z^2 \frac{\gamma_i T_i + T_e}{M} = -k_z^2 \frac{T_{e0}}{M} \frac{k_y T'_{i0}}{eB_0}.$
- Instability results when $\omega_* = \frac{k_y T'_{i0}}{eB_0} \gg \omega.$
- In the above limit the diamagnetic flow advection term is greater than the partial time derivative of the temperature: $i\omega n_0 \tilde{T}_i + (\gamma_i - 1) n_0 T_{i0} i k_z \tilde{v}_z - i n_0 \omega_* e\tilde{\phi} - i n_0 \omega_* \tilde{T}_i - i n_0 \omega_* \frac{\tilde{n}}{n_0} T_{i0} = 0.$
- Keeping the full linear diamagnetic flow gives the stable dispersion relation: $\omega^2 - \omega_* \omega + k_z^2 C_s^2 = 0.$

A few thoughts for proceeding.

- A more rigorous derivation of the ITG mode is needed.
  - Gyro-viscosity cancellation partially cancels the polarization drift which is neglected do to small ordering.
  - If the cancellation is important, then higher order terms are needed.
  - Ultimately we want to know what is the simplest fluid model that will produce the ITG.
  - A more detail calculation that includes gyro-viscosity and magnetization heat flow yields: $(\omega + \omega_*)\omega^2 = k_z^2 \frac{T_i}{m_i}(4\omega - \frac{5}{2}\omega_*)$
- NIMROD relevant concerns?
  - The theory uses different pressure models for electrons and ions.
    - Currently NIMUW uses the same pressure model for both species.... easy fix.
  - NIMROD cannot explicitly specify an electro-static response.
    - Critical aspects of the ITG enter through the two-fluid Ohm's law.
  - The two fluid system leads to a very high order dispersion relation when we keep all contributions. It will be tough to get everything right numerically, spatially, and temporally.