Benchhmark of continuum kinetics in NIMROD with NEO
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1 First-order DKE in the (ξ,s) velocity variables

Hazeltine's form for the drift kinetic equation (ϵ, μ) :

$$\partial_t f + (\mathbf{v}_{||} + v_D) \cdot \nabla f + \left(\mu \frac{\partial B}{\partial t} + e(\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E}\right) \partial_\epsilon f = C.$$

Using $\xi = v_{||}/v$ and $s = v/v_0$ yields

$$\partial_t f + (\mathbf{v}_{||} + v_D) \cdot \nabla f - (\mathbf{v}_{||} + v_D) \cdot \left[\frac{1-\xi^2}{2\xi} \nabla \ln B \partial_\xi + s \nabla \ln v_0 \partial_s\right] f$$
$$+ \left(\frac{e}{2e_0 s^2} (\mathbf{v}_{||} + \mathbf{v}_D) \cdot \mathbf{E}\right) (s \partial_s f + 2g(\xi) \partial_\xi f) \; = \; C$$

with general form for drift

$$v_D = \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \frac{e_0 s^2}{eB} \left[\mathbf{b} \times \left(\left(1-\xi^2\right) \nabla \ln B + 2\xi^2 \kappa - \frac{v_0 s \xi}{e_0 B} \nabla \times \mathbf{E}\right) + (1-\xi^2) \frac{\mu_0 \mathbf{J}_{||}}{B}\right].$$

2 Coulomb collision operator written in moment form.

- Full, linearized Coulomb collision operator taken from Ji and Held, PoP (2006) :

$$
\begin{aligned}
C^{ab} &= C(f_{1a}, f_{0b}) + C(f_{0a}, f_{1b}) \\
&= \sum_{lk} \frac{f_a^{(0)}}{\sigma_k^l} P_l(v_{||}/v) \left(\nu_{ab}^{lk,0} M_{||a}^{lk}(\mathbf{r}, t) + \nu_{ab}^{0,lk} M_{||b}^{lk}(\mathbf{r}, t) \right)
\end{aligned}
$$

where $f_a^{(0)}$ is Maxwellian, $\nu_{ab}^{lk,0}$ and $\nu_{ab}^{0,lk}$'s are speed dependent collision frequency and

$$
n_a M_{||a}^{lk} = \frac{l!}{(2l-1)!!} v_{Ta}^{l+2k} \int d\mathbf{v} L_k^{l+1/2}(s^2) s^l P_l(v_{||}/v) F_a
$$

3 Several pitch angle bases implemented.

- In each pitch-angle cell expand f as :

$$f(\mathbf{r},t,\xi,s) \;=\; \sum_i f_i(\mathbf{r},t,s)\phi_i(x)$$

- Modal (built from Legendre polynomials) and nodal (Lagrange and Gauss-Lobatto-Legendre) bases implemented.

- Pitch-angle coefficients, f_i, computed on speed grid determined by quadrature on semi-infinite ($s\epsilon[0,\infty)$) or finite ($s\epsilon[0,s_{max}]$) domains.

4 Speed grid determined by nonclassical quadrature schemes.

- Quadrature in $s_a$to compute moments of $f_a = \sum_l F_{la}(s_a)\phi_l(v_{||}/v)$:

$$
\begin{aligned}
n_a M_{||a}^{lk} &= \frac{l!}{(2l-1)!!} v_{Ta}^{l+2k} \int_0^\infty ds s^{2l+1} L_k^{l+1/2}(s^2) \sum_{l'} P_{ll'} F_{l'a}(s) \\
&= \frac{l!}{(2l-1)!!} v_{Ta}^{l+2k} \sum_j w_j s_j^{2l+1} L_k^{l+1/2}(s_j^2) \sum_{l'} P_{ll'} F_{l'a}(s_j)
\end{aligned}
$$

- For cases with thermodynamic drives proportional to Maxwellian: $w(s) = s^\alpha exp(-s^2)$ on $s\epsilon[0,\infty)$.

- For slowing down distributions of hot particles: $w(s) = s^\alpha/(1+s^3)$ on $s\epsilon[0, s_{max}]$.

5 Order DKE for neoclassical transport calculations.

Order $v_D << v_{||}$ and assume weak (relative to Dreicer) electric field:

$$\partial_t f_0 + \mathbf{v}_{||} \cdot \nabla f_0 - \mathbf{v}_{||} \cdot \left[\frac{1-\xi^2}{2\xi} \nabla \ln B \partial_\xi + s \nabla \ln v_0 \partial_s \right] f_0 \;=\; C(f_0)$$

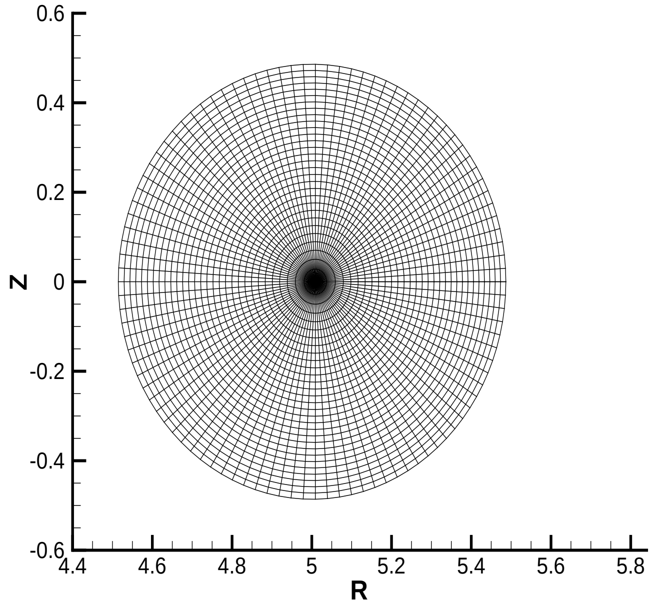
This equation satisfied by stationary Maxwellian with flux functions n and T.
To next order :

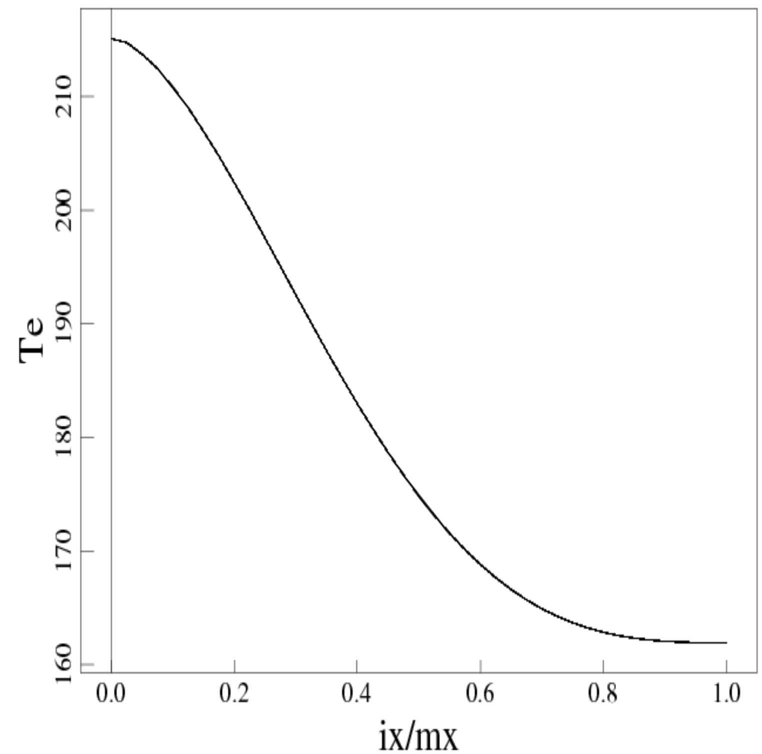
$$\begin{aligned} \partial_t f_1 + \mathbf{v}_{||} \cdot \nabla f_1 - \left(\mathbf{v}_{||} \cdot \nabla \ln B \right) \frac{1-\xi^2}{2\xi} \partial_\xi f_1 \;&=\; \\ -\mathbf{v}_D \cdot \nabla f_0 + s v_D \cdot \nabla \ln v_0 \partial_s f_0 - \frac{e}{2\epsilon_0 s} \mathbf{v}_{||} \cdot \left(\mathbf{E}^A - \nabla \phi_1 \right) \partial_s f_0 \;&+\; C^{aa} + C^{ab} \end{aligned}$$

Using $g = f_1 - (e\phi_1/T_0) f_0$yields (compare with Eq. 23 of Belli and Candy, 51 PPCF 2009):

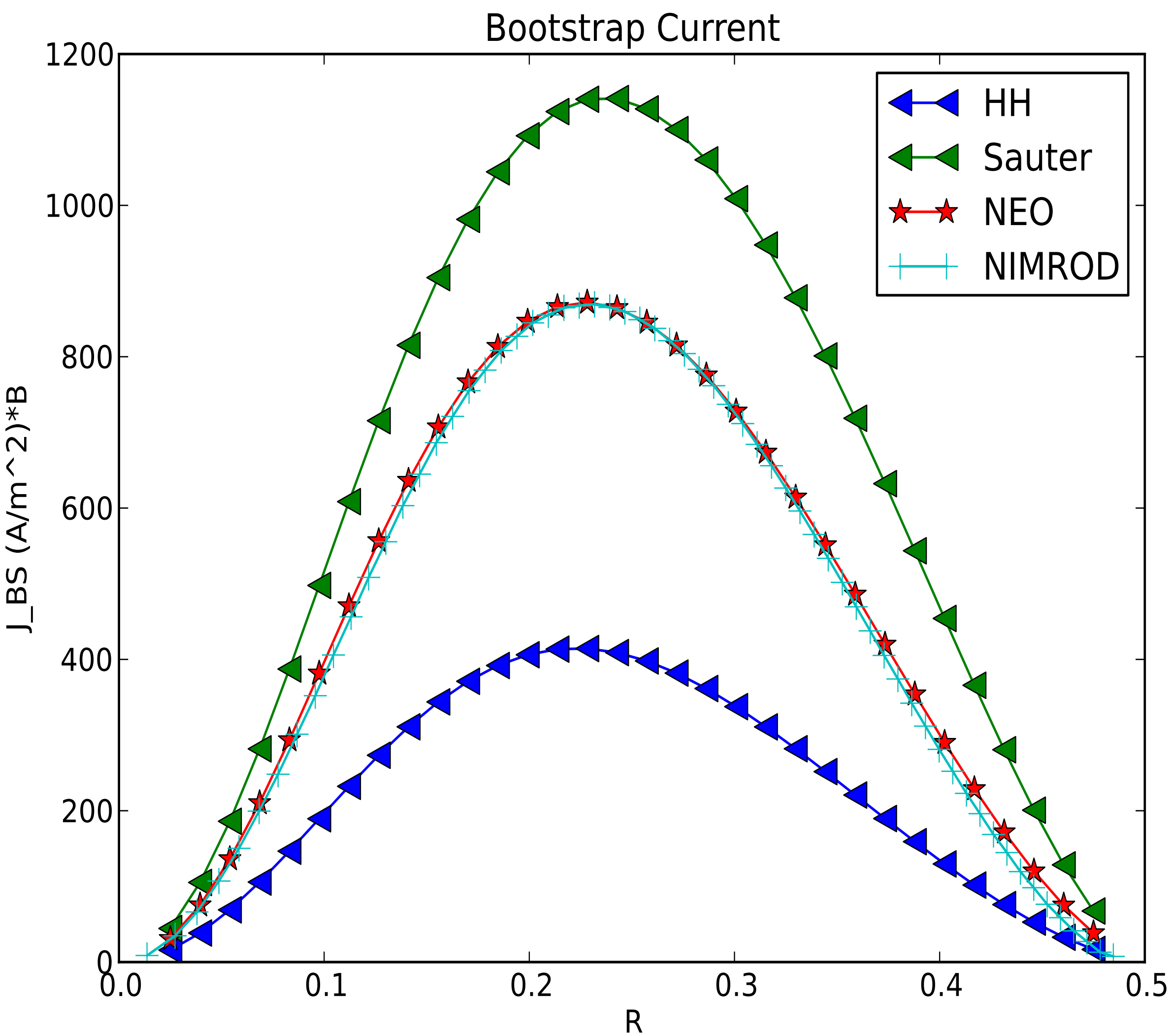
$$\begin{aligned} \partial_t g + \mathbf{v}_{||} \cdot \nabla g - \mathbf{v}_{||} \cdot \nabla \ln B \partial_\xi g \quad &= \\ -\mathbf{v}_D \cdot \nabla f_0 + s v_D \cdot \nabla \ln v_0 \partial_s f_0 \quad &+ \\ C^{aa} + C^{ab} - \frac{e}{2\epsilon_0 s} \mathbf{v}_{||} \cdot \mathbf{E}^A \partial_s f_0 \;& + (e f_0/T_0) \partial_t \phi_1 \end{aligned}$$

6 First test: high-aspect ratio Grad-Shafranov equilibrium

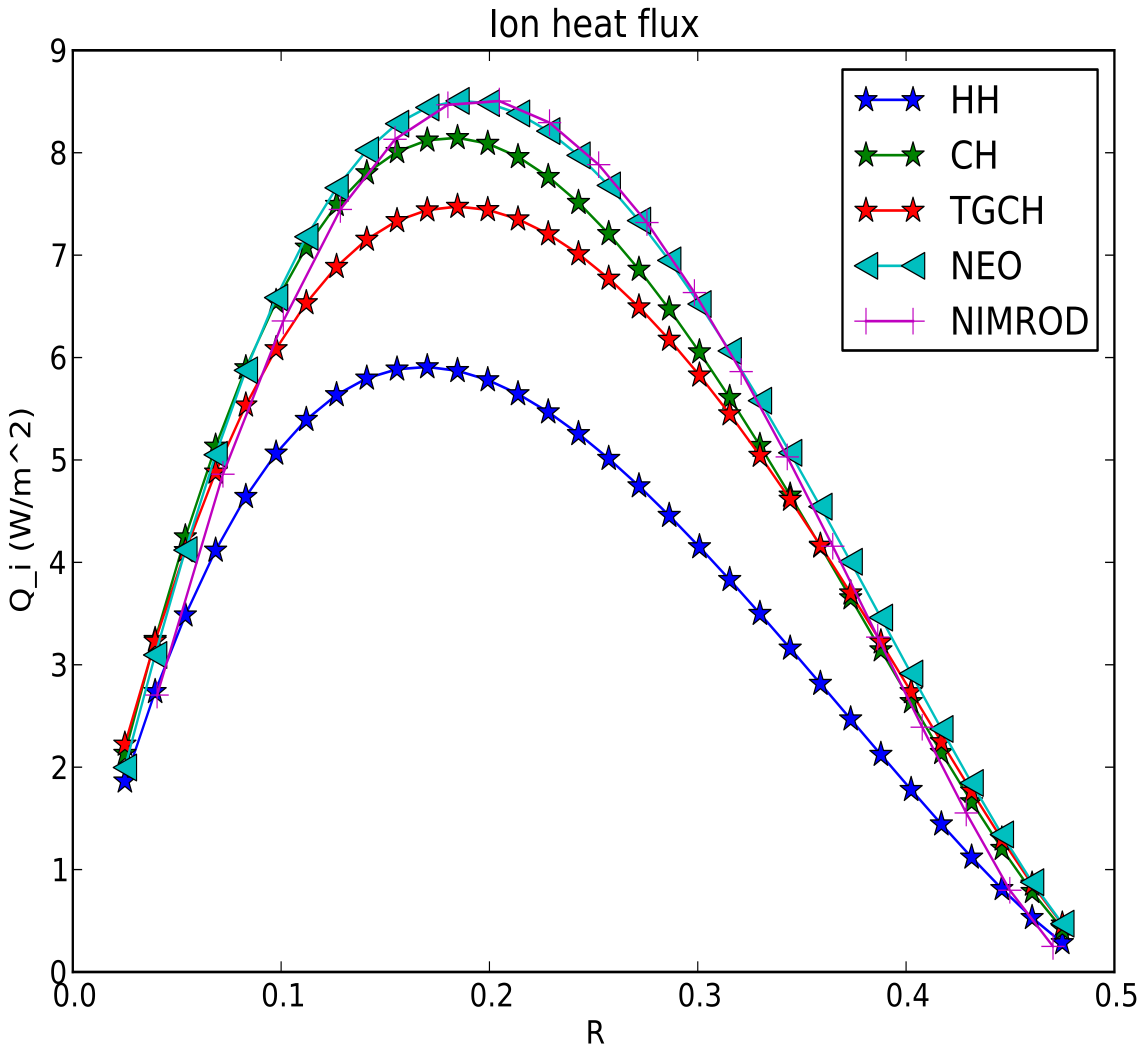




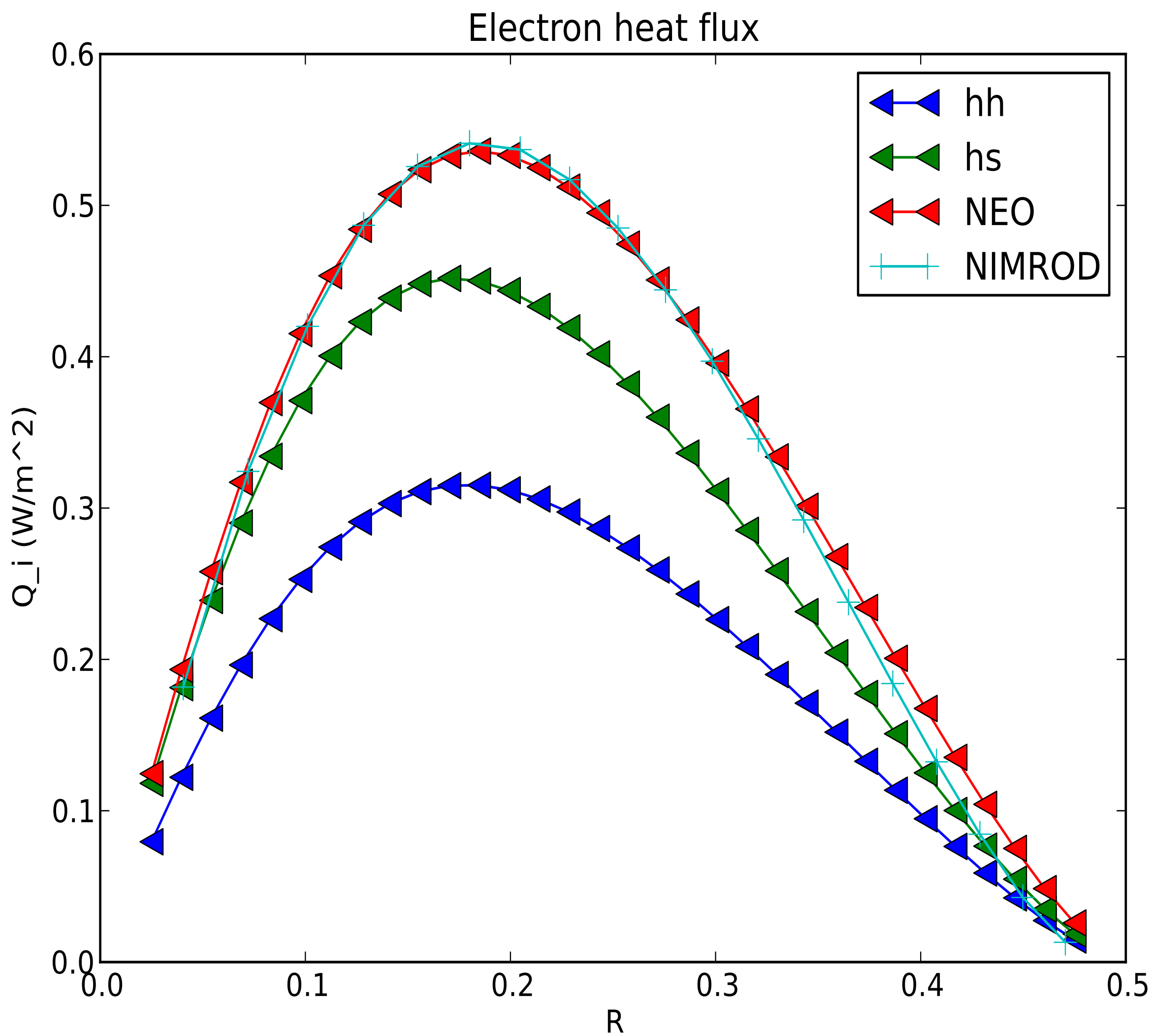
7 Comparison of Bootstrap Currents



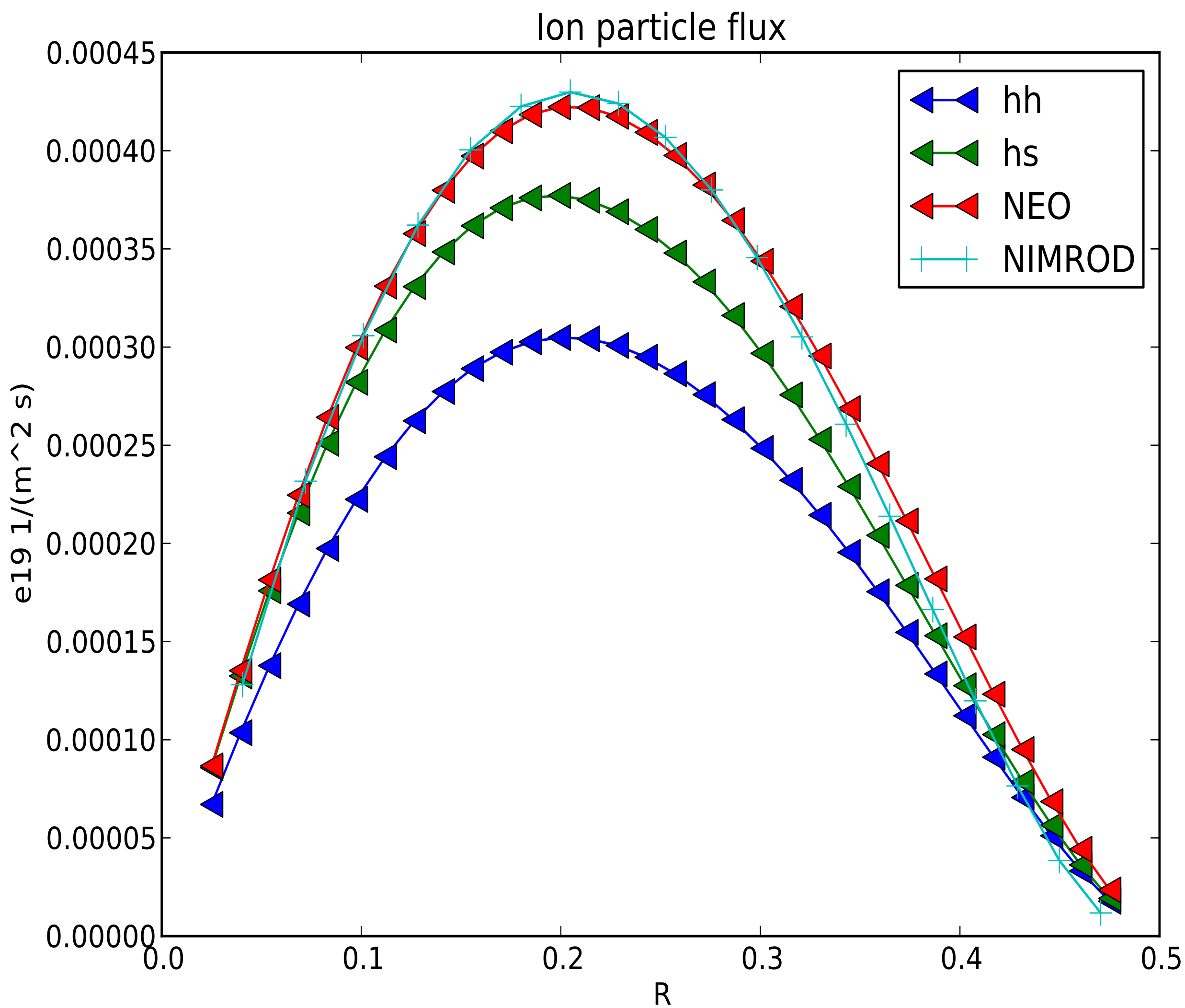
8 Comparison of Ion Radial Heat Fluxes



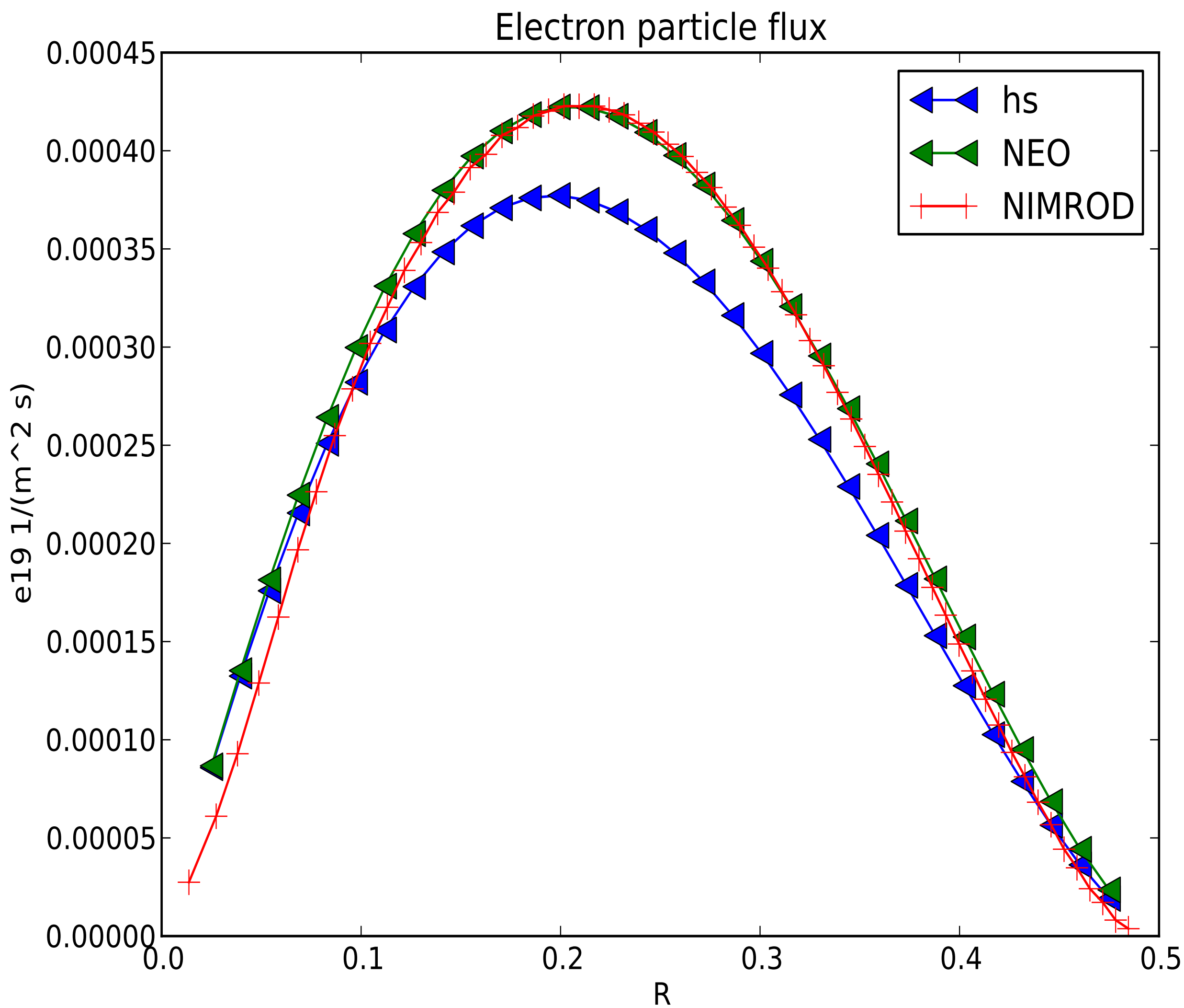
9 Comparison of Electron Radial Heat Fluxes



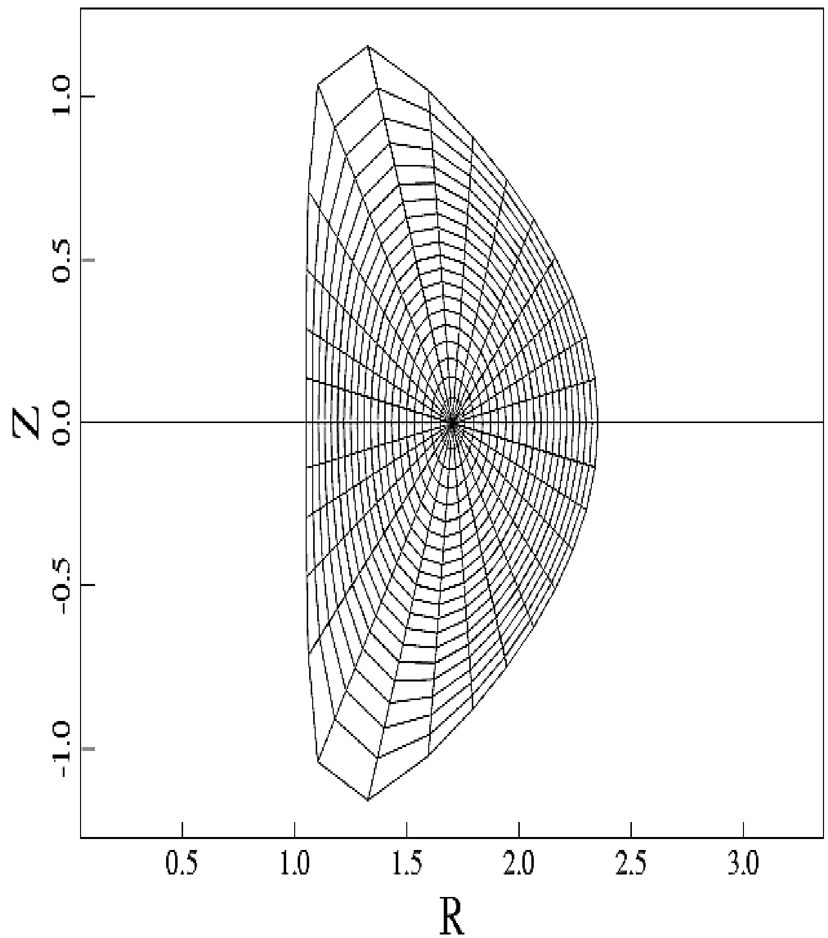
10 Comparison of Ion Radial Particle Fluxes

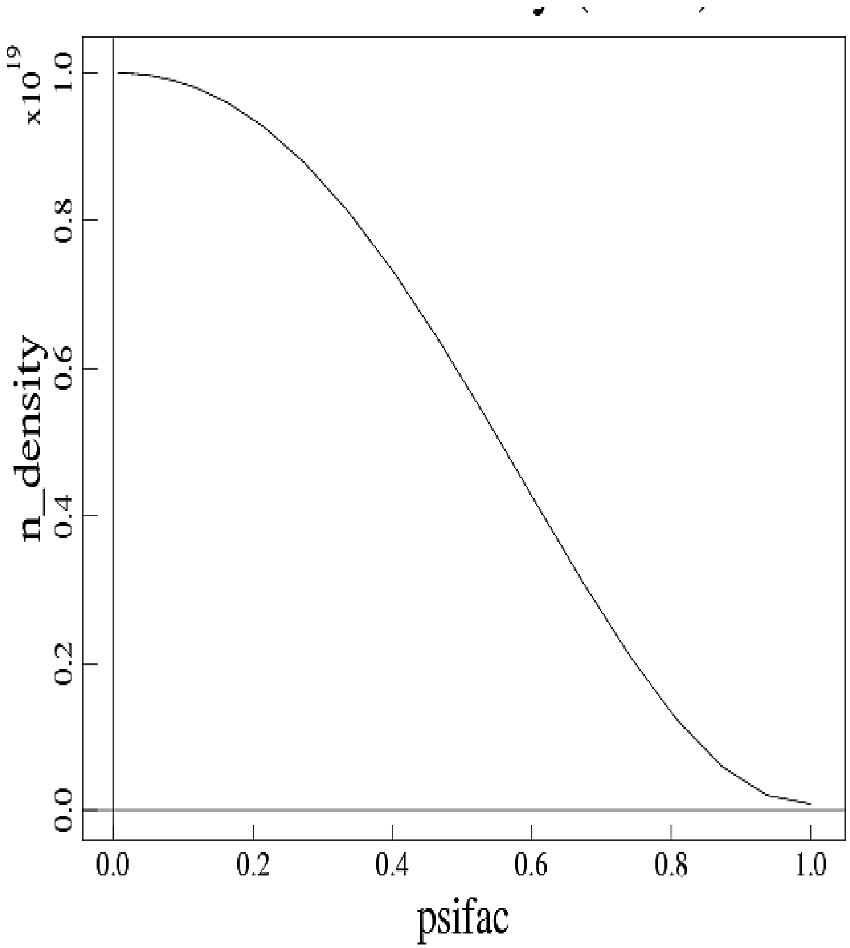


11 Comparison of Electron Radial Particle Fluxes

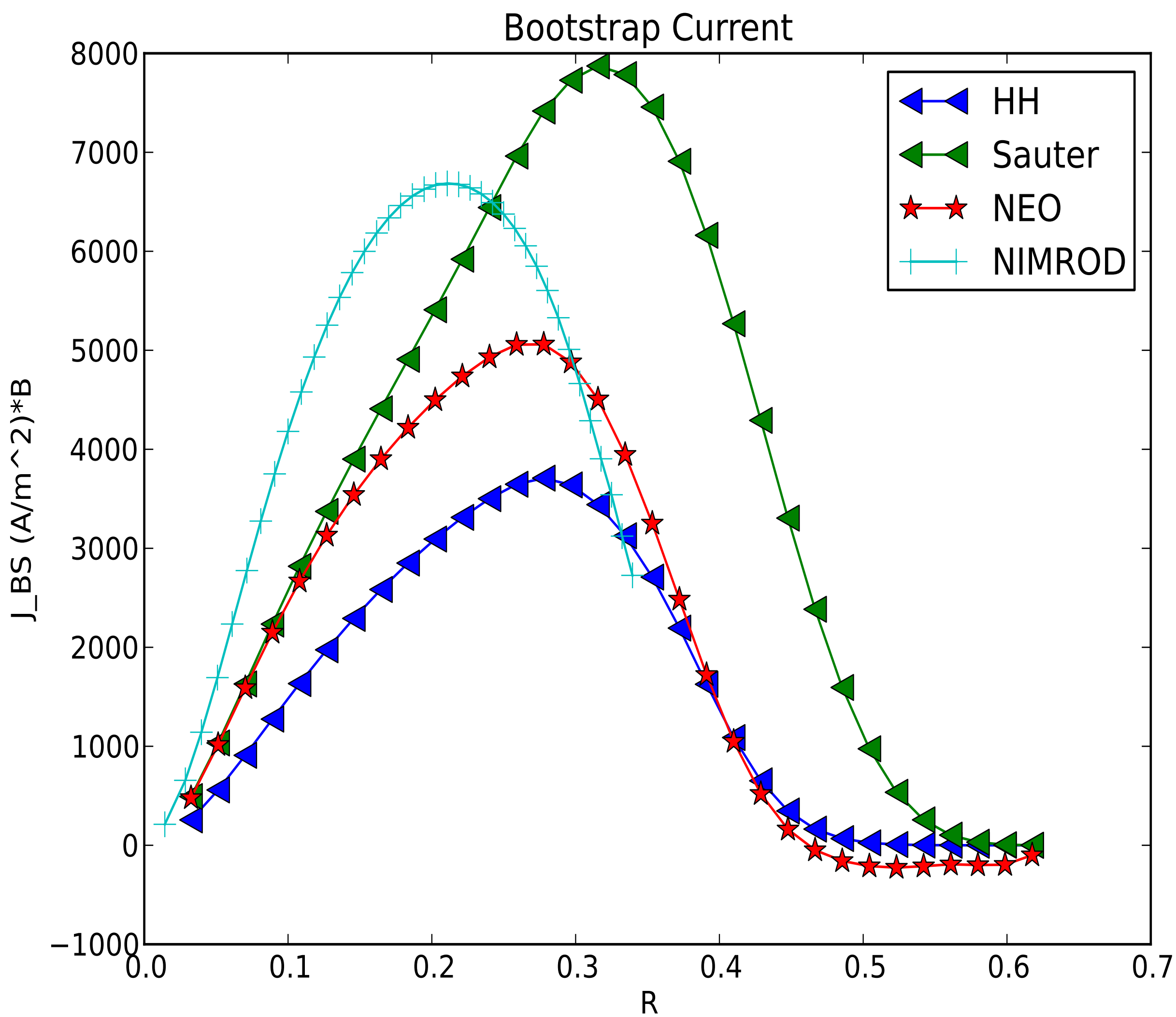


12 Next test: high-β, shaped equilibrium





13 Comparison of bootstrap currents for high-β case



14 Future Work

1. Parallelize over speed points to reduce memory constraints.

2. Improve preconditioning for collision terms in steady-state calculations.

3. Apply to dynamical problems with kinetic closures for NIMROD's fluid model.

4. Related work (at Utah State) on continuum solutions of kinetic equations:

   (a) Andy Spencer is developing a Fokker-Planck code using NIMROD's 2D finite-element/Fourier representation for velocity space (submitted JCP in July on test particle part).

   (b) Jeong-Young Ji's higher-order moment equations to be implemented in NIMROD.

   (c) Mukta Sharma's studies of heat flow along magnetic fields testing effects of particle trapping on transport.