

Parallel electron heat flow closure in inhomogeneous magnetic field

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Outline

1 Motivation

- A general analytical and computational framework for calculating the electron heat flux along magnetic field lines in plasmas of arbitrary collisionality
- Comparison between the previous theory and the new theory
- Solving the equation

2 Results

- Main Results
- Conclusions

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Close temperature evolution equation

with kinetically derived q

- Species evolution equations and closure moments for five moment model

$$\partial_t n + \nabla \cdot n \mathbf{V} = 0 \rightarrow \text{Density}$$

$$mn(\partial_t \mathbf{V} + \mathbf{V} \cdot \nabla \mathbf{V}) = en(\mathbf{E} + \frac{1}{c} \mathbf{V} \times \mathbf{B}) - \nabla(nT) - \nabla \cdot \Pi + \mathbf{R} \rightarrow \text{Flow}$$

$$\frac{3}{2}n(\partial_t T + \mathbf{V} \cdot \nabla T) = -nT \nabla \cdot \mathbf{V} - \Pi : \nabla \mathbf{V} - \nabla \cdot \mathbf{q} + Q \rightarrow$$

Temperature

$$\mathbf{q} \equiv \int d^3 v' \frac{1}{2} m v'^2 \mathbf{v}' f \rightarrow \text{Heat flow} \quad \Pi \equiv \int d^3 v' m [\mathbf{v}' \mathbf{v}' - \frac{v'^2}{3} \mathbf{I}] f \rightarrow$$

Stress tensor

- Gyrophased-averaged drift kinetic equation

$$\frac{\partial f}{\partial t} + v_{\parallel} \hat{b} \cdot \nabla f + a_{\parallel} \cdot \nabla_v f = C(f)$$

- Distribution function is written as a superposition of dynamic Maxwellian f_M and a kinetic distortion F

$$f = f_M + F = \frac{n(x,t)}{(\frac{2\pi T}{m})^{3/2}} \exp(-\frac{m}{2T} (\mathbf{v} - \mathbf{u})^2) + F$$

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Novel treatment of $|B|$ term in the kinetic equation

- The equation we are trying to solve

$$\partial_t \mathbf{F} + \mathbf{v}_{\parallel} \hat{\mathbf{b}} \cdot \nabla \mathbf{F} - \langle C(\mathbf{F} + f_M) \rangle = L_1^{\frac{3}{2}} v (\nabla_{\parallel} \ln T) f_M - f_M L_1^{\frac{1}{2}} (\partial_t \ln T)$$

- $[M_t \partial_t + M_{\parallel} v \partial_L + M_B v \partial_L \ln B + C] \mathbf{F} =$

$$\delta_{n1} L_1^{\frac{3}{2}} v (\nabla_{\parallel} \ln T) f_M - \delta_{n0} L_1^{\frac{1}{2}} (\partial_t \ln T) f_m$$

- Expanding the kinetic distortion as $\mathbf{F} = \sum_n F_n(v, \mathbf{x}, t) P_n \left(\frac{v_{\parallel}(\mathbf{x})}{v} \right)$

$$\text{here } \frac{v_{\parallel}(\mathbf{x})}{v} = \pm \sqrt{1 - \frac{\mu B(\mathbf{x})}{w}}$$

- To lowest order, the magnetic moment μ and kinetic energy w of the particles are conserved

$$\mu = \frac{mv_{\perp}^2}{2B} \text{ and } w = \frac{mv^2}{2} = \frac{mv_{\parallel}^2}{2} + \mu B$$

- The collision operator $C_a^{(1)} = -\frac{v_{La}}{2} \sum_n n(n+1) P_n F_n$

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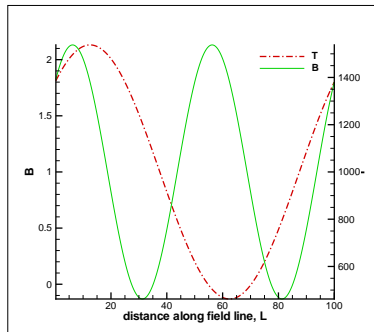
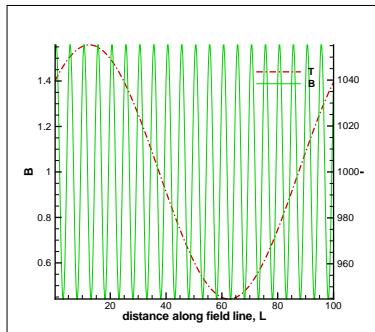
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Comparison between the previous theory and the new theory

Previous Theory	New Theory
Time dependent effects are ignored	Time dependent effects are included
Expansion of F in $C_n(\xi)$	Expansion of F in $P_n(\frac{v_{ }}{v})$
Multiple scale length expansion	No multiple scale length expansion
Parallel free streaming operator is treated with bounce average	Parallel free streaming operator treated as is
Boundary layer ignored near trapped/passing boundary	Trapped/passing boundary is treated more generally



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Solve the system of equations

- The equations we are trying to solve
- The equation at each s

$$\begin{aligned}
 & [\mathbf{M}_t \partial_t + \delta_{nn}] \mathbf{F} = \\
 & -\mathbf{M}_{\parallel} v \partial_L \mathbf{F} - \mathbf{M}_B v \partial_L \ln B \mathbf{F} - \delta_{n1} L_1^{\frac{3}{2}} v (\nabla_{\parallel} \ln T) f_M - \delta_{n0} f_M L_1^{\frac{1}{2}} (\partial_t \ln T) \\
 & \partial_t T = \frac{2}{3n} (S - \hat{b} \cdot \nabla \mathbf{q} + \mathbf{q} \hat{b} \cdot \nabla \ln B)
 \end{aligned}$$

Complex Fourier Series

$$\begin{aligned}
 \mathbf{F}_n &= F_{n0} + \sum_{k>0} (F_{nk} e^{ik\phi} + F_{nk}^* e^{-ik\phi}) \text{ where } \phi = 2\pi L / \Delta L s \\
 \partial_L \ln B &= \sum_{m>0} (B_m e^{im\phi} + B_m^* e^{-im\phi}) \text{ and} \\
 T &= T_0 + \sum_{k>0} (T_k e^{ik\phi} + T_k^* e^{-ik\phi})
 \end{aligned}$$

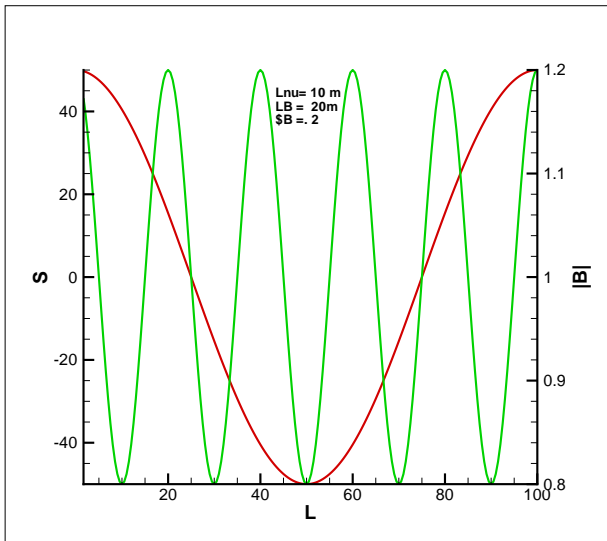
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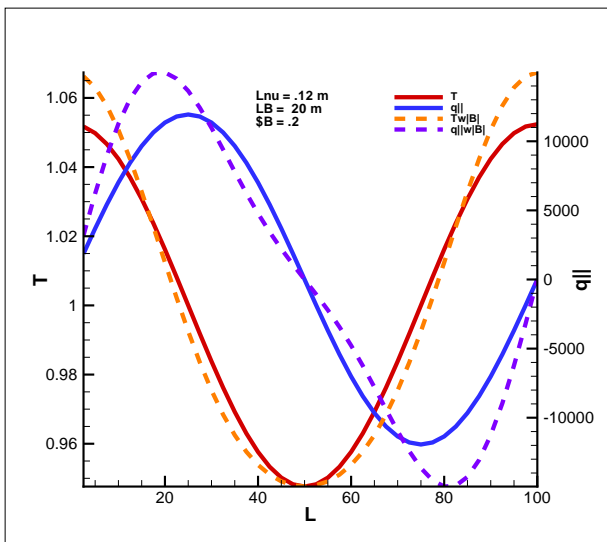
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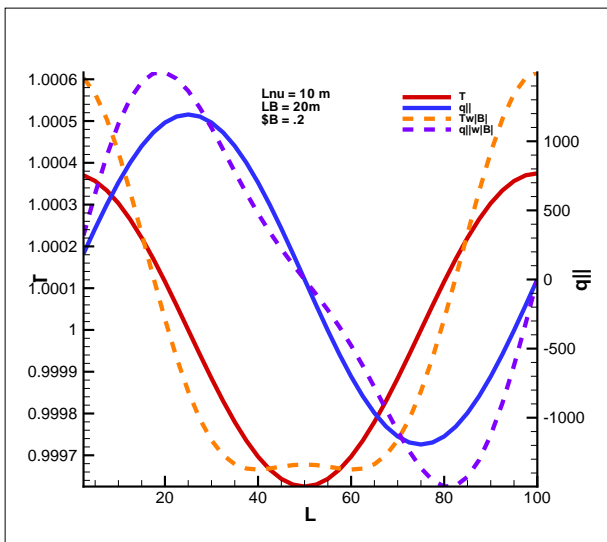
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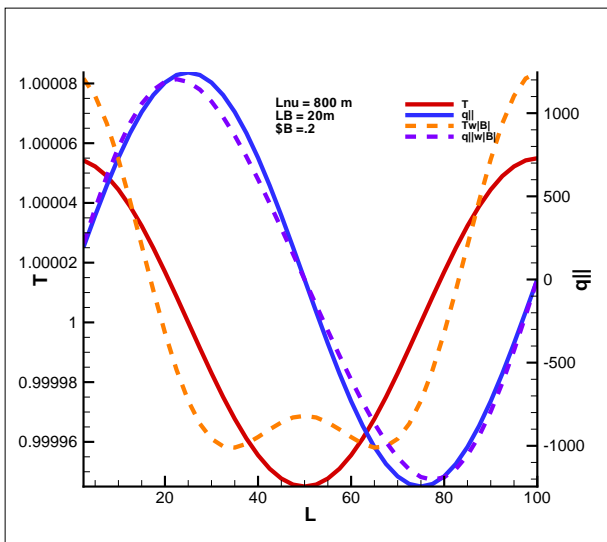
Collisional regime

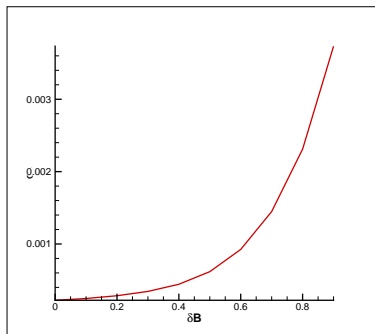


Moderate Collisional regime

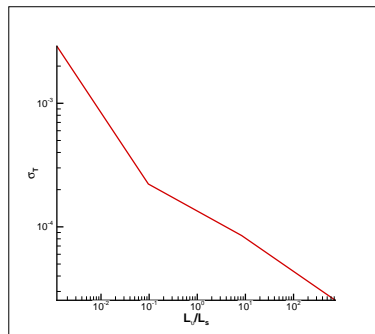


Collisionless regime





Change in T vs δB



Change in T vs Collisionality

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Summary

- In nearly collisionless plasmas the heat flow is reduced by the trapped electron fraction
- Change in temperature increases with increasing change in $|B|$ effects
- Change in temperature decreases as we go from collisional to less collisional regime
- Quantative means for calculating parallel electron heat flow along inhomogeneous magnetic fields
- Future work
 - Linearized full collision operator
 - Comparison between linear and non-linear effects

For Further Reading I

-  S. Chapman and T.G. Cowling
The Mathematical Theory of Nonuniform Gases 1939.
-  E.D. Held et.al *Phys. Plasmas.* 10,3933, 2003
-  J-Y Ji and E.D. Held *Phys. Plasmas.* 13,102,2006
-  E.D. Held et.al *Phys. Plasmas.* 11,2419, 2004
-  E.D. Held et.al *Phys. Plasmas.* 8,1171, 2001
-  J.G. Cordey *Nucl. Fusion.* 16,499,1976
-  F.L. Hinton and M.N. Rosenbluth *Phys. Fluids.* 16,836,1973
-  Z. Chang and J.D. callen *Phys. Fluids. B* 4,1167,1992