Electron parallel closures for arbitrary ion charge number

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Fluid equations and closures

Maxwellian moment $(n_a, \mathbf{V}_a, T_a)$ equations

$$d_t n_a + n_a \nabla \cdot \mathbf{V}_a = 0 \, (d_t \equiv \partial_t + \mathbf{V}_a \cdot \nabla)$$

$$\tfrac{3}{2} n_a d_t T_a + n_a T_a \nabla \cdot \mathbf{V}_a + \nabla \cdot \mathbf{h}_a + \nabla \mathbf{V}_a : \boldsymbol{\pi}_a = Q_a$$

$$m_a n_a d_t \mathbf{V}_a - n_a q_a (\mathbf{E} + \mathbf{V}_a \times \mathbf{B}) + \nabla p_a + \nabla \cdot \boldsymbol{\pi}_a = \mathbf{R}_a$$

General moment equations $D\mathsf{n} + \Omega \mathbf{b} \breve{\times} \mathsf{n} = C\mathsf{n}$ ($\mathsf{n}^{lk} \to v^{l+2k}$ moment)

$$d_t \mathbf{h} + \Omega \mathbf{b} \times \mathbf{h} + \tfrac{7}{5}(\nabla \cdot \mathbf{V})\mathbf{h} + \tfrac{7}{5}\mathbf{h} \cdot (\nabla \mathbf{V}) + \tfrac{2}{5}(\nabla \mathbf{V}) \cdot \mathbf{h} + \frac{5p}{2m}\nabla T$$
$$+ \frac{T}{m}\nabla \cdot \boldsymbol{\pi} + \frac{7}{2}\frac{\nabla T}{m} \cdot \boldsymbol{\pi} - \mathbf{a} \cdot \boldsymbol{\pi} + \nabla \cdot \boldsymbol{\theta} + \tfrac{1}{3}\nabla \mathsf{u}^{02} + \nabla \mathbf{V} : \mathsf{u}^{30}$$
$$= C^1_{10}\mathbf{V}_{\mathrm{ei}} + C^1_{11}\mathbf{h} + C^1_{12}\mathbf{r} + \cdots \text{ (}\mathbf{h}\text{ heat flux } \mathsf{n}^{11}\text{)}$$

$$d_t \mathbf{r} + \Omega \mathbf{b} \times \mathbf{r} + \cdots = C^1_{10}\mathbf{V}_{\mathrm{ei}} + C^1_{21}\mathbf{h} + C^1_{22}\mathbf{r} + \cdots \quad \text{(}\mathbf{r}\text{ heat heat flux } \mathsf{n}^{12}\text{)}$$

$$d_t \boldsymbol{\pi} + \Omega \mathbf{b} \breve{\times} \boldsymbol{\pi} + (\nabla \cdot \mathbf{V})\boldsymbol{\pi} + 2\overline{\boldsymbol{\pi} \cdot (\nabla \mathbf{V})} + p\mathsf{W} + \tfrac{4}{5}\overline{\nabla \mathbf{h}} + \nabla \cdot \mathsf{u}^{30}$$
$$= C^2_{00}\boldsymbol{\pi} + C^2_{01}\boldsymbol{\theta} + \cdots \text{ (}\boldsymbol{\pi}\text{ viscosity } \mathsf{n}^{20}\text{)}$$

$$d_t \boldsymbol{\theta} + \Omega \mathbf{b} \breve{\times} \boldsymbol{\pi} + \cdots = C^2_{10}\boldsymbol{\pi} + C^2_{11}\boldsymbol{\theta} + \cdots \text{ (}\boldsymbol{\theta}\text{ heat viscosity } \mathsf{n}^{21}\text{)}$$

where $\mathbf{a} = \frac{q}{m}(\mathbf{E} + \mathbf{V} \times \mathbf{B}) - d_t \mathbf{V}$ and $\mathsf{W} = \nabla \mathbf{V} + (\nabla \mathbf{V})^{\mathrm{T}} - \frac{2}{3}\nabla \cdot \mathbf{V}\mathsf{I}$

Express $\mathbf{h}_a(\mathsf{n}^{11}_a)$, $\boldsymbol{\pi}_a(\mathsf{n}^{20}_a)$, Q_a, $\mathbf{R}_a$ in terms of $n_a, \mathbf{V}_a, T_a$

Braginskii for high collisionality

$$\mathbf{R}_{\mathrm{e}} = -\mathbf{R}_{\mathrm{i}} = -(\alpha)(\mathbf{V}_{\mathrm{ei}}) - (\beta)(\nabla T_{\mathrm{e}}), \quad \mathbf{h}_a = (\beta)(\mathbf{V}_{\mathrm{ei}}) - (\kappa)(\nabla T_a)$$

$$\boldsymbol{\pi}_a = (\eta)(\mathsf{W}_a)$$

Parallel closures for low (arbitrary) collisionality

Integral parallel closures [Ji and Held, PoP 21, 122116 (2014),signs corrected]

$$\boxed{n_{AB}(\eta) = \int d\eta' K_{AB}(\eta-\eta') g_B(\eta')}$$ where $A, B = h, R, \pi$ and $d\eta = \frac{d\ell}{\lambda_C}$

$$h_\parallel(\eta) = -\frac{1}{2} T v_T \int d\eta' K_{hh} \frac{n}{T}\frac{dT}{d\eta'} + T v_T \int d\eta' Z K_{hR} n \frac{V_{ei\parallel}}{v_T} - T v_T \int d\eta' K_{h\pi} (\frac{3}{4} n \tau_{ee} W_\parallel)$$

$$R_\parallel(\eta) = -\frac{mn}{\tau_{ei}} V_{ei\parallel} + \frac{m v_T}{\tau_{ei}} \int d\eta' [-K_{Rh} \frac{n}{2T}\frac{dT}{d\eta'} + Z K_{RR} n \frac{V_{ei\parallel}}{v_T} - K_{R\pi} (\frac{3}{4} n \tau_{ee} W_\parallel)]$$

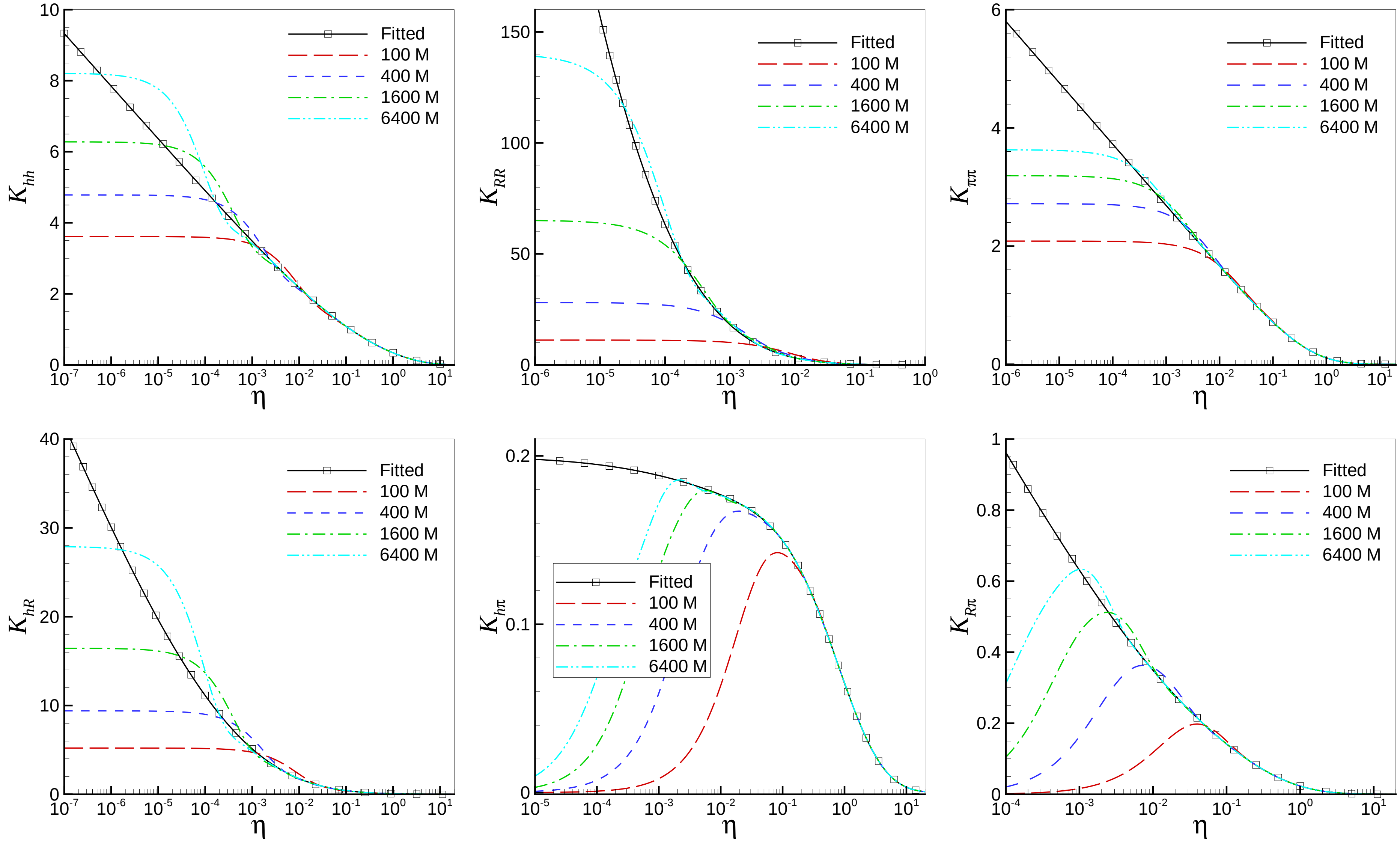
$$\pi_\parallel(\eta) = -T \int d\eta' K_{\pi h} \frac{n}{T}\frac{dT}{d\eta'} + 2T \int d\eta' Z K_{\pi R} n \frac{V_{ei\parallel}}{v_T} - T \int d\eta' K_{\pi\pi} (\frac{3}{4} n \tau_{ee} W_\parallel)$$

- Fitted kernel functions $(Z = 1)$

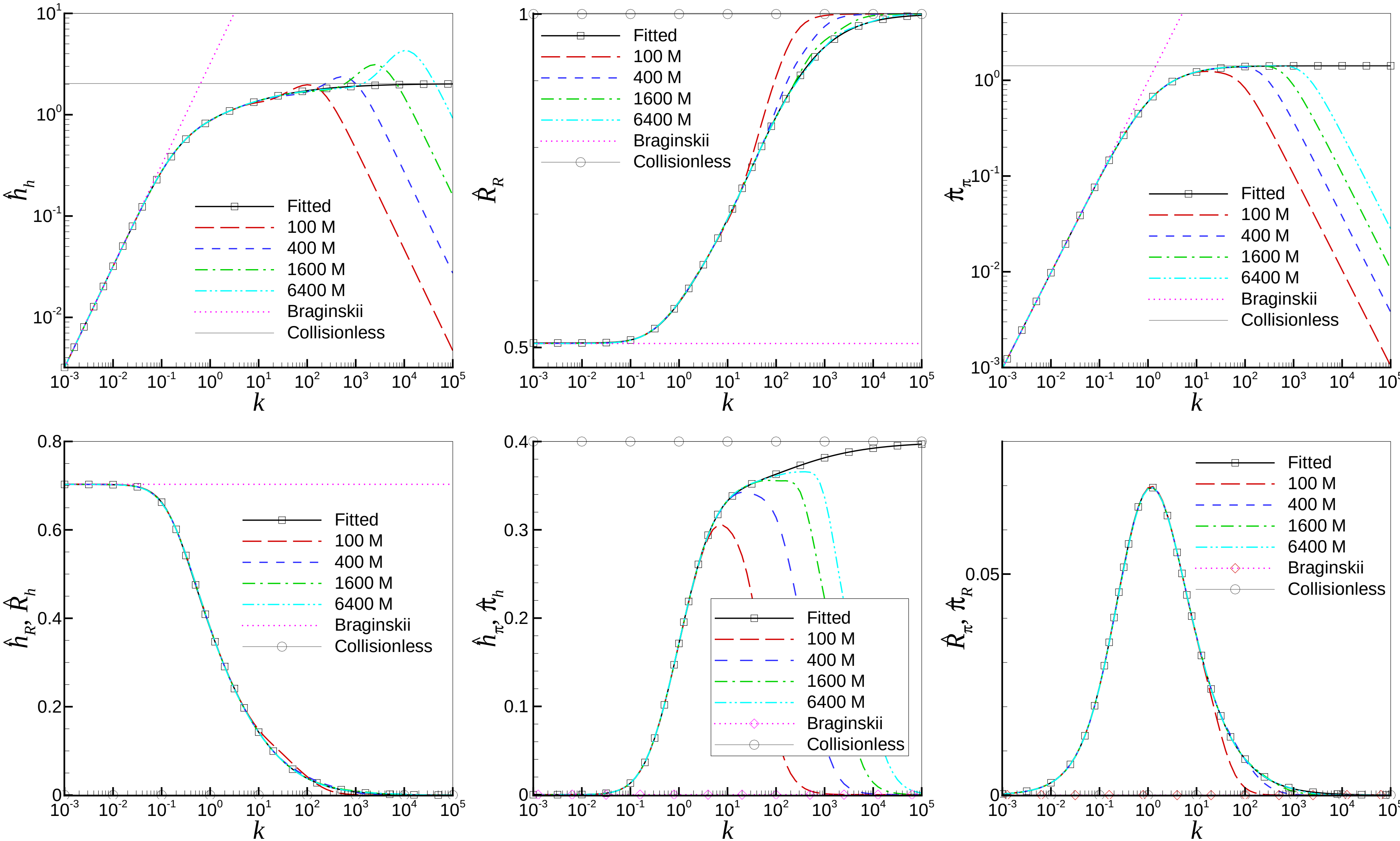
$$K_{AB}(\eta) = -[d + a\exp(-b\eta^c)]\ln[1 - \alpha\exp(-\beta\eta^\gamma)]$$

	a	b	c	d	α	β	γ
K_{hh}	-5.32	0.170	0.646	6.87	1	2.02	0.417
K_{hR}	6.37	5.12	0.160	0.100	1	1	0.583
$K_{h\pi}$	-0.229	2.26	0.594	0.363	0.775	1.49	0.478
K_{RR}	245	8.06	0.147	0.432	1	3.40	0.347
$K_{R\pi}$	-0.226	3.21	0.678	0.696	1	3.40	0.347
$K_{\pi\pi}$	0.724	0.932	0.654	0.195	1	1.60	0.491

Fitted kernel functions for $Z = 1$ (6400 M + collisionless)



Closures for sinusoidal drives



Extending to $Z = 2, \cdots, 10$

- $K_{AB}(\eta) = -[d + a\exp(-b\eta^c)]\ln[1 - \alpha\exp(-\beta\eta^\gamma)]$
- Errors are less than 5 % in the convergent regime ($k \equiv \lambda_{\rm C}/|\nabla^{-1}| \lesssim 80$)
- For $\eta \ll 1$, kernels approach the collisionless asymptote [Ji, Held, and Jhang, PoP 20, 082121 (2013)]

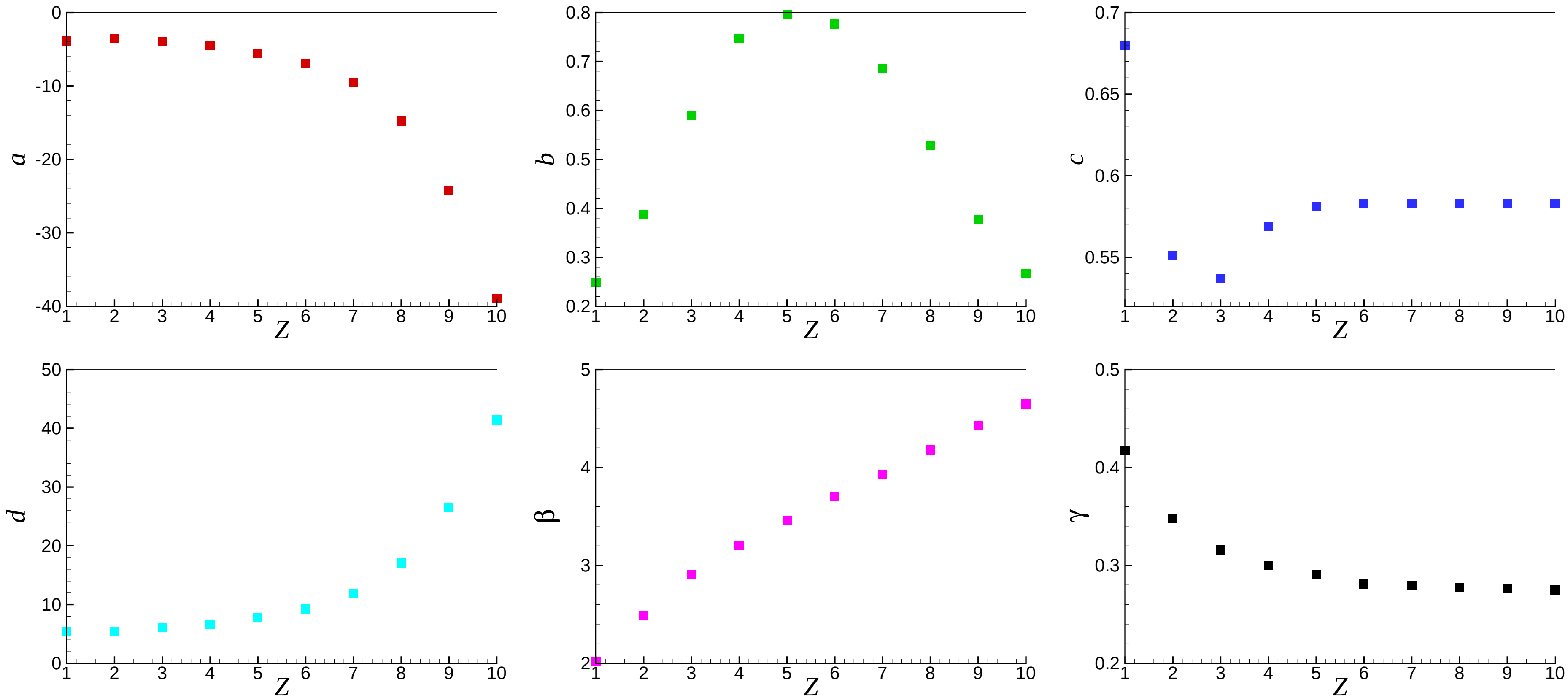
$$
\begin{aligned}
K_{hh}(\eta) &\approx -\frac{18}{5\pi^{3/2}}(\ln|\eta| + \text{const.}) \to a = \frac{18}{5\pi^{3/2}\gamma} - d \\
K_{h\pi}(\eta) &\approx \begin{cases} \frac{2}{5}, & \eta > 0 \\ -\frac{2}{5}, & \eta < 0 \end{cases} \to a = \frac{1}{5\ln(1-\alpha)} - d \\
K_{\pi\pi}(\eta) &\approx -\frac{4}{5\pi^{1/2}}(\ln|\eta| + \text{const.}) \to a = \frac{4}{5\pi^{1/2}\gamma} - d
\end{aligned}
$$

  - ◇ In the collisionless limit ($k \to \infty$), closures approach exact values

- Linear interpolations for $Z \le Z_{\rm eff} \le Z + 1$

$$A_{Z_{\rm eff}} = (1 + Z - Z_{\rm eff})A_Z + (Z_{\rm eff} - Z)A_{Z+1} \text{ for } A = b, c, d, \beta, \gamma$$

Extending to $Z = 1, 2, \cdots, 10$ (K_{hh} only shown)



- Smooth change in Z
  - ◇ Accurate closures for non integer Z obtained by interpolation

Extending to $Z = 2, \cdots, 10$ (diagonal kernels)

K_{AB}	Z	1	2	3	4	5	6	7	8	9	10
	a	-3.85	-3.61	-4.02	-4.50	-5.52	-6.98	-9.59	-14.8	-24.2	-39.0
	b	0.248	0.387	0.590	0.746	0.796	0.776	0.686	0.528	0.377	0.267
	c	0.680	0.551	0.537	0.569	0.581	0.583	0.583	0.583	0.583	0.583
K_{hh}	d	5.40	5.47	6.07	6.66	7.74	9.28	11.9	17.1	26.5	41.4
	α	1	1	1	1	1	1	1	1	1	1
	β	2.02	2.49	2.91	3.20	3.46	3.70	3.93	4.18	4.43	4.65
	γ	0.417	0.348	0.316	0.300	0.291	0.281	0.279	0.277	0.276	0.275
	a	305	322	342	363	386	406	431	450	470	489
	b	8.30	8.67	8.90	9.09	9.23	9.32	9.40	9.49	9.52	9.54
	c	0.139	0.140	0.141	0.142	0.143	0.143	0.144	0.144	0.144	0.144
K_{RR}	d	0.362	0.459	0.576	0.686	0.830	0.972	1.14	1.30	1.47	1.67
	α	1	1	1	1	1	1	1	1	1	1
	β	3.24	4.11	4.75	5.23	5.68	6.06	6.39	6.71	6.97	7.24
	γ	0.349	0.314	0.290	0.272	0.258	0.248	0.237	0.232	0.225	0.219
	a	0.470	0.598	0.700	0.762	0.804	0.839	0.857	0.873	0.878	0.883
	b	1.06	1.19	1.31	1.45	1.59	1.72	1.85	1.97	2.08	2.18
	c	0.661	0.607	0.580	0.566	0.557	0.551	0.546	0.543	0.541	0.539
$K_{\pi\pi}$	d	0.357	0.275	0.207	0.166	0.139	0.118	0.106	0.096	0.091	0.087
	α	1	1	1	1	1	1	1	1	1	1
	β	1.66	1.97	2.17	2.34	2.49	2.61	2.74	2.85	2.97	3.08
	γ	0.546	0.517	0.498	0.487	0.479	0.472	0.469	0.466	0.465	0.465

Extending to $Z = 2, \cdots, 10$ (off-diagonal kernels)

K_{AB}	Z	1	2	3	4	5	6	7	8	9	10
	a	6.37	6.76	5.63	5.34	5.61	6.31	8.22	11.3	17.3	27.9
	b	5.12	5.72	6.09	6.53	6.85	7.06	7.31	7.51	7.61	7.71
	c	0.160	0.179	0.219	0.240	0.239	0.227	0.205	0.181	0.154	0.126
K_{hR}	d	0.100	0.187	0.339	0.440	0.465	0.457	0.411	0.374	0.325	0.278
	α	1	1	1	1	1	1	1	1	1	1
	β	1.00	1.73	2.50	2.96	3.19	3.33	3.37	3.39	3.37	3.34
	γ	0.583	0.465	0.387	0.346	0.332	0.326	0.327	0.327	0.328	0.329
	a	-.229	-.179	-.144	-.133	-.130	-.137	-.150	-.169	-.212	-.239
	b	2.26	3.08	3.72	4.35	4.72	4.94	5.05	5.12	5.15	5.38
	c	0.594	0.596	0.594	0.588	0.569	0.562	0.556	0.551	0.548	0.543
$K_{h\pi}$	d	0.363	0.280	0.240	0.225	0.210	0.220	0.241	0.269	0.308	0.334
	α	0.775	0.862	0.875	0.886	0.918	0.910	0.889	0.865	0.875	0.878
	β	1.49	1.69	1.81	1.97	2.12	2.32	2.53	2.76	3.03	3.23
	γ	0.478	0.460	0.454	0.442	0.432	0.415	0.399	0.380	0.362	0.351
	a	0.102	0.125	0.147	0.169	0.186	0.209	0.224	0.239	0.253	0.263
	b	0.528	0.724	0.898	1.06	1.22	1.30	1.51	1.61	1.77	1.91
	c	0.961	0.948	0.922	0.901	0.887	0.864	0.848	0.832	0.823	0.818
$K_{R\pi}$	d	0.198	0.212	0.225	0.230	0.231	0.225	0.220	0.213	0.207	0.202
	α	1	1	1	1	1	1	1	1	1	1
	β	2.45	3.06	3.52	3.87	4.15	4.38	4.57	4.73	4.88	5.02
	γ	0.408	0.370	0.347	0.332	0.322	0.313	0.307	0.303	0.299	0.294

Maximum percentage errors for convergent values $k \lesssim 80$

Z	K_{hh}	K_{hR}	$K_{h\pi}$	K_{RR}	$K_{R\pi}$	$K_{\pi\pi}$
1	1.0	0.6	0.6	0.7	1.0	0.5
1.5	2.4	3.2	2.7	4.0	3.2	1.6
2	2.8	0.9	1.0	0.7	0.9	0.8
2.5	3.0	4.4	1.8	2.4	1.5	1.1
3	4.9	1.9	0.7	0.7	0.6	0.6
3.5	4.3	2.3	1.1	1.5	0.9	1.0
4	4.8	4.3	0.8	0.3	0.3	0.4
4.5	4.4	4.1	1.2	0.8	0.9	0.5
5	4.7	4.4	0.8	0.2	0.5	0.4
5.5	4.2	3.7	0.8	0.3	0.8	0.4
6	4.6	3.9	0.8	0.5	0.5	0.4
6.5	3.1	1.0	0.8	0.3	0.6	0.2
7	3.1	0.8	1.0	0.4	0.7	0.5
7.5	2.8	1.5	1.4	0.2	0.5	0.4
8	3.0	0.9	2.0	0.2	0.7	0.4
8.5	3.7	4.0	2.3	0.3	0.9	0.4
9	3.4	1.8	2.5	0.2	0.9	0.5
9.5	3.4	3.1	2.6	0.3	0.7	0.3
10	3.4	3.2	2.7	0.8	0.9	0.3

Work in progress

- Ion parallel closures
  - ◇ Ion-electron collision operator included
    - ⋆ Temperature and mass ratios
- Electron parallel closures along an inhomogeneous magnetic field
  - ◇ Collisional to nearly collisionless regimes
    - ⋆ Moment solution in the Fourier series
    - ⋆ Convergence checked with increasing number of moments and Fourier modes
  - ◇ Collisionless limit: infinite number of moments required
    - ⋆ A kinetic equation with a Krook-type operator being solved
    - ⋆ $\nu \to 0$ limit solution
  - ◇ Fourier representation for arbitrary collisionality
- Closures and transport with neutrals
  - ◇ Charge exchange, ionization, and recombination operators