3D MHD Simulations of Spheromak Compression

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Outline

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Raise nT with compression

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dump.00000
C=5 and tilt instability (APSfinal_14)
Comparison of C=2 with analytic modeling, APSfinal_14

Further Work

Summary

Motivation: raise nT with compression

- The physics of compression proceeds from the ideal gas law, usually stated as $P\ .\ V = N\ .\ R\ .\ T$ where N is the total number of molecules, and R is gas constant, T is temperature.
- For processes that are adiabatic, the change of pressure goes as follows (γ=5/3):

$$P_0\ .\ V_0^{\gamma} = P_f\ .\ V_f^{\gamma}.$$

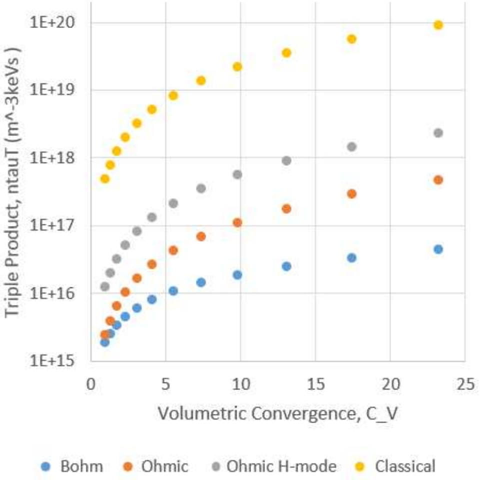
- Considering a self-similar compression ($C = a_0\ /\ a_f$), the following adiabatic scaling relations are derived.

Motivation: raise nT with compression

- If particles are conserved during compression, then the density increases as the volume decreases: $n_f = n_0 \ . \ C^3$.
- If the compression is adiabatic, then $PV^{5/3} = \text{const}$. So $P_0 {V_0}^{\gamma} = P_f {V_f}^{\gamma}$ and since $V_f = V_0 C^3$ then: $P_f = P_0 \ . \ C^5$ and since $P \sim n.T$ then: $T_f = T_0 \ . \ C^2$.
- If magnetic flux (=BA) is conserved during a compression, then the magnetic field strength scales with the change of the cross-sectional area: $B_f = B_0 \ . \ C^2$
- Finally, the ratio of the plasma pressure to the magnetic pressure is represented by the parameter $\beta = P \ / \ P_B \sim C^5 \ / \ C^4$: $\beta_f = \beta_0 \ . \ C$

Context

- Historical: S1, ATC, TUMAN3M
- Current: ARPA-E, MTF, private sector fusion development including FRCs, Spheromaks and tokamaks
- Basic idea is to get to high $n\tau T$ with adiabatic (i.e. faster than τ_E) compression.



Method: 3D MHD code NIMROD

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0 \tag{1}$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = \mathbf{J} \times \mathbf{B} - \nabla P - \nabla \cdot \overleftrightarrow{\Pi} \tag{2}$$

$$\frac{n_s}{\gamma - 1} \left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) T_s = -P_s \nabla \cdot \mathbf{v}_s - \Pi_s : \nabla \cdot \mathbf{v}_s - \nabla \mathbf{q}_s + Q \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \tag{4}$$

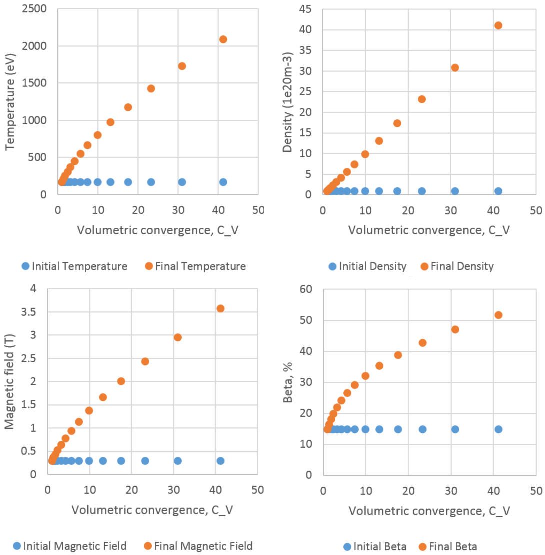
$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \eta J + \frac{1}{ne}(\mathbf{J} \times \mathbf{B} - \nabla P_e) \tag{5}$$

Design point

- For C=3, and assuming adiabaticity we should expect to obtain the table on the right.
- University-scale experiment: fast compression ($<50\mu$s), small plasma ($<<1$m), like S1, possibly to be build at UMBC after we are done thinking.

Parameter	Initial	Final
Radius [m]	0.2	0.07
B [T]	0.25	0.63
Density[cm^{-3}]	3×10^{15}	2.7×10^{16}
Temperature	0.2keV	1.8keV
Beta	7%	21%
Confinement	$<10\mu$s	???

Parameter scaling with volumetric convergence, r_i=25cm

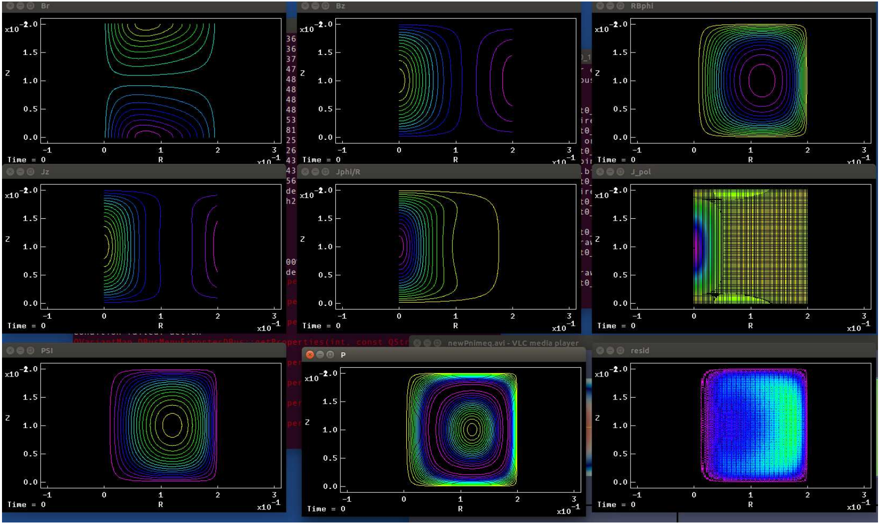


Simulation campaign

Except where noted, all cases run with mx=my=24, 24 blocks, lphi=2, continuity='full', ohms='mhd', p_model='isotropic', poly_degree=4, slu_dist

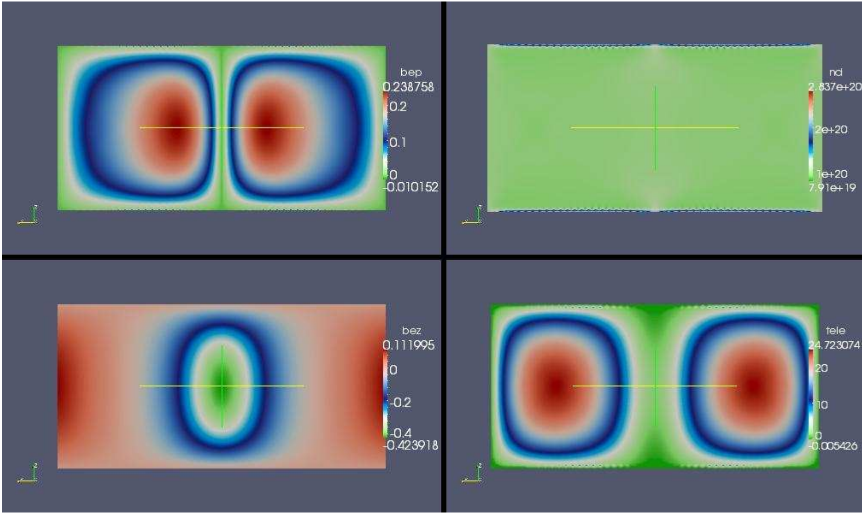
Name	IC	Notes
APSfinal_1	Bessel function	ve0 = 0
APSfinal_2	Bessel function	ve0 = 2e4
APSfinal_3	Bessel function	ve0 = 5e4
APSfinal_4	Bessel function	ve0 = 15
APSfinal_5	nimeq	ve0 = 0, mx doubled
APSfinal_6	nimeq	ve0 = 0, mx halved
APSfinal_7	nimeq	ve0 = 0, 1us run in
APSfinal_8	nimeq	ve0 = 0, 2us
APSfinal_9	nimeq	ve0 = 0, 3us
APSfinal_10	nimeq	ve0 = 0, 4us
APSfinal_11	nimeq	ve0 = 0, 5us
APSfinal_12	nimeq	ve0 = 0, 2us 40kV
APSfinal_13	nimeq	ve0 = 0, 2us, 80kV
APSfinal_14	nimeq	ve0 = 0, 2us, 160kV
APSfinal_15	nimeq	ve0 = 0, 2us, 320kV
APSfinal_16	nimeq	ve0 = 0, 80kV, Braginskii

Initial condition from nimeq, ./xdraw grad



0.2m radius wall.

APSfinal_14 dump.00000



dump.00000

Results: compression to C=5 and tilting (APSfinal_14)

- High voltages(160kV) on compression coils
- Run-in over 2us

For movies, see examples

Comparison of APSfinal_14 and analytic scaling

	Formula	Units	Initial	Final
r		m	0.2	0.1
h		m	0.2	0.2
C_R			1	2
Volume	$\pi r^2 h$	m^3	0.025128	0.006282
C_V			4	
Area	rh	m^2	0.04	0.02
C_A			2	

Particle con				
n	$\propto V_0/V_f$	m^{-3}	1e20	4e20
Flux con				
B	$\propto A_0/A_f$	T	0.25	0.5
Adiabaticity				
	$PV^{5/3} = \text{const}$			
	$P_0 V_0^\gamma = P_f V_f^\gamma$			
P	$(V_0/V_f)^\gamma . P_0$	Pa	1000	9986
T	$\propto P/n$	eV	66	165

Further work

- Simulate most meaningful experimental conditions.... theres another year to converge on design point. Much iterating to follow.

Summary

- NIMJOB updated with NIMPSI coil capability (E_{tang} at boundary)
- Adiabatic compression with nimeq IC investigated under different compression times/convergences, looks adiabatic to first order
- Not quite clear how confinement is affected during compression, more to follow.

APS posters

Session BP12: FRC, Spheromak, Mirror 9:30 AM Monday

- BP12.00039 Design Point for a Spheromak Compression Experiment S. Woodruff, C. A. Romero-Talamas, J. O'Bryan, J. E. Stuber
- BP12.00040 Numerical investigation and optimization of multi-pulse CHI spheromak performance J.B. O'Bryan, C.A. Romero-Talamas, S. Woodruff
- BP12.00041 Development of Synthetic Diagnostics for use in Validation D. Lemmon, S. Woodruff, J. E. Stuber, J. O'Bryan, C. A. Romero-Talamas
- BP12.00042 3D MHD Simulations of Spheromak Compression J. E. Stuber, S. Woodruff, J. O'Bryan, C. A. Romero-Talamas

Session YP12: Postdeadline 9:30 AM Friday

- YP12.00041 Additive manufacture (3d printing) of plasma diagnostic components and assemblies for fusion experiments P.E. Sieck, S. Woodruff, J. E. Stuber, C. A. Romero-Talamas, W. Rivera, S. You, A. Card
- YP12.00040 Design point and scenario development for a compact Spherical Tokamak with Double Null Divertor S. Woodruff, T. A. Casper, P. Buxton, M. Gryaznevich