Non-axisymmetric effects in strongly driven Coaxial Helicity Injection in simulations in the NSTX geometry

Bick Hooper, Woodruff Scientific
(Subcontract with LLNL)
Carl Sovinec, Univ. of Wisconsin

NIMROD Team Meeting
Savannah, GA — November 14
**Field-line-following instability during simulations of CHI**

- Seen in resistive MHD simulations (NIMROD)
- Forms on the surface of flux-bubbles during injection

- The mode had little effect on injection in weakly-driven ($I_{inj} = 2-10$ kA), low temperature (T $< 25$ eV) plasmas used to compare with experimental results.

- Studied here in simulated, strongly-driven ($I_{inj} \approx 15$ kA) plasmas with reduced impurity radiation and reduced cross-field thermal conductivity:
  - Higher T ($\sim 50-100$ eV) in the current channel; lower ($\sim 5$ eV) outside
  - The mode is stronger and affects the evolution and closure of the flux-bubble.
Axisymmetric \((n=0)\), strongly-driven simulations with high \(T\) (in current channel), low thermal conductivity — X-point forms during injection
— X-point is in cold (5 eV) plasma outside current channel
— Plasma divides into two “lobes”

\[ t = 1.3 \text{ ms after start of injection} \]

\[ t = 3.0 \text{ ms after start of injection} \]

A single, well-defined flux bubble does not form
Non-axisymmetric \((n=0, 1, 2)\) simulations: The axisymmetric poloidal flux distribution is more like that in low-temperature simulations

Poloidal flux

![Poloidal flux plot](image-url)
Non-axisymmetric \((n=0, 1, 2)\) simulations with high \(T\) (in current channel), low thermal conductivity
— Symmetry-breaking mode prevents X-point formation seen in axisymmetric simulations

The jagged structures with \(n = 1\) and \(n = 1, 2\) result from relaxation oscillations of the \(n=1\) mode

Time not extended beyond 9 ms due to long run time requirements at high resolution and numerical problems. At lower resolution \(I_{\text{tor}}\) drops to 40% at 14 ms
Non-axisymmetric (n=0, 1, 2) simulations with high T (in current channel), low thermal conductivity (cont.)

— Plasma evolution (toroidal current, internal energy)

The jagged, internal energy structures are due to the instability. During dips in n=0,1 & n=0,1, 2 internal energy (at ≈ 8 ms) the mode is low-level and not bursting
Kinetic and magnetic energies show a bursting characteristic correlated with the injection current.

During the dips (at ≈ 8 ms) the mode is low-level and not bursting.
The mode consists of eddies in velocity and magnetic field. The real parts are shown.

VR-VZ: n=1

BR-BZ: n=1

NSTX29/n=1/201500
(t = 7.75 ms)
The instability broadens the $n=0$ current flow

- Current is generated outside the flux-bubble where the field lines are bent by the expanding bubble

n = 0 only

n = 0 & 1
Linear analysis — used to identify the instability driving force

Two approaches have been used to determine the linear-mode characteristics

• Linear characteristics have been determined by using NIMROD
  – Starting from a non-linear axisymmetric simulation, a linear mode is excited from noise
  – The code parameters (e.g. viscosity, resistivity) and results (e.g. flow) were varied to determine the sensitivity to simulation parameters

• Simple slab models were used to examine the physics driving the mode

Conclusion: The mode is primarily an ideal, current-driven instability
NIMROD — The linearized $n=1$ mode has a rapid growth rate ($t_{1/e} \approx 1.5 \mu s$)
The n=1 mode grows without the n=0 flow velocity

Option in the code: The n=0 velocity field can be set to zero in the linear study

- Mode grows when the n=0 flow is zeroed
  \[ v_{\text{max}} \approx 10^4 \text{ m/s} (<< v_A, c_s) \text{ away from the injection slot} \]

- Growth rate — reduced from \(6.45 \times 10^5 \text{ s}^{-1}\) to \(2.88 \times 10^5 \text{ s}^{-1}\)

Also: The mode structure lies on the outer edge of the n=0 flow field

- This is where the n=0 component of current is strongest

Conclusions:

- The mode does not need the n=0 velocity field to grow, although it does contribute to the mode growth rate

- The mode is primarily current driven
Linear-instability calculation: MHD input parameters (e.g. viscosity, resistivity) can be changed to evaluate the sensitivity of the linear mode

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Base value</th>
<th>Test value</th>
<th>Growth rate ($s^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base calculation*</td>
<td></td>
<td></td>
<td>6.45x10^5</td>
</tr>
<tr>
<td>Kinematic viscosity</td>
<td>150 m^2/s</td>
<td>15 m^2/s</td>
<td>8.79x10^5</td>
</tr>
<tr>
<td>Resistivity (magnetic diffusion)</td>
<td>$411/T_e^{3/2}$ m^2/s</td>
<td>$1x10^4/T_e^{3/2}$ m^2/s</td>
<td>9.48x10^5</td>
</tr>
<tr>
<td>Particle diffusivity (holds n ≈ const)</td>
<td>$10^5$ m^2/s</td>
<td>10 m^2/s</td>
<td>5.30x10^5</td>
</tr>
</tbody>
</table>

*All n=0 plasma quantities are constant (slides 5-8) except as listed above

Conclusions:
- Instability growth is insensitive to the numerical parameters used in the calculation
- Dissipation is not needed — The mode is primarily ideal
The linear mode structure is similar to the non-linear — eddies in velocity and magnetic field are generated by the instability.
Analytic Modeling: Instability drive

Ideal MHD Energy Principle — term proportional to $j_{||}$

$$
\delta W_F = -\frac{1}{2} \int_P d\mathbf{r} J_{||} \left( \left( \xi_\bot^* \times \mathbf{b} \right) \cdot \mathbf{B}_{\bot} \right)
$$

$$
= \frac{1}{2} \int_P d\mathbf{r} J_{||} \hat{b} \cdot \left( \xi_\bot^* \times \mathbf{B}_{\bot} \right)
$$

The term in parenthesis is proportional to the negative of the electric field:

$$
\tilde{E} = -\frac{\partial \tilde{\xi}}{\partial t} \times \mathbf{B}_{\bot}
$$

This term is negative when the volume-averaged $J_{||} \hat{b} \cdot \tilde{E}$ is positive —— In the direction to extract energy from the current
**Slab Model — example of current sheet**

Assume current is force-free and constant in the slab.
Model the downward current along the inside leg of the flux-bubble ($j_y < 0$)

\[
B_z = B_{z_0} \quad x < -a \\
B_z = B_{z_0} - \mu_0 j_y (x + a) \quad -a \leq x \leq a \\
B_z = B_{z_0} - \mu_0 j_y (2a) \quad x > -\Delta/2
\]

\[
\frac{\partial B_y}{\partial x} = \frac{B_{z_0}}{B_{y_0}} \mu_0 j_y
\]

\[
B_y = \sqrt{-2\mu_0 j_y B_{z_0} (2a)} \quad x < -a \\
B_y = \sqrt{2\mu_0 j_y B_{z_0} (x - a)} \quad -a \leq x \leq a \\
B_y = 0 \quad x > a
\]
Slab Model — Equilibrium and perturbed equations

Equilibrium

\[ \rho_0 = \text{const}. \]
\[ v_0 = 0 \quad \text{Neglect flow} \]
\[ B_0 = B_{0z} \hat{z} + B_{0y}(x) \hat{y} \quad B_{0y} \ll B_{0z} \]
\[ j_0 = j_{0z}(x) \hat{z} + j_{0y}(x) \hat{y} \quad j_0 \cdot B_0 = 0 \]

Perturbation

\[ \delta \rho = 0 \]
\[ \nabla \cdot \delta \mathbf{v} = 0 \quad \text{incompressible} \]
\[ \rho_0 \frac{\partial \delta \mathbf{v}}{\partial t} = \delta \mathbf{j} \times B_0 + j_0 \times \delta \mathbf{B} \]
\[ \frac{\partial \delta \mathbf{B}}{\partial t} = \nabla \times (\delta \mathbf{v} \times B_0) \]
\[ \nabla \times \delta \mathbf{B} = \mu_0 \delta \mathbf{j} \]
After considerable algebra:

\[
\left( \omega^2 - \left( \frac{k \cdot B_0}{B_{0z}} \right)^2 \right) \xi = -\nu_A^2 \nabla \left[ \frac{B_{0y}}{B_{0z}} \left( -i \frac{k \cdot B_0}{B_{0z}} \xi_y + \frac{\partial}{\partial x} \left( \frac{B_{0y}}{B_{0z}} \right) \xi_x \right) - i \frac{k \cdot B_0}{B_{0z}} \xi_z \right]
\]

with

\[
\delta v = -i \omega \xi(x) \exp \left[ -i \left( \omega t - k_y y - k_z z \right) \right]
\]

\[
k_\parallel = \frac{k \cdot B_0}{B_{0z}} / B_{0z} = k_z + k_y \frac{B_{0y}(x)}{B_{0z}} \quad k_z \ll k_y, B_{0y} << B_{0z}
\]

\[
\nu_A^2 = \frac{B_{0z}^2}{\mu_0 \rho_0}
\]

Set

\[
\nabla \cdot \xi = 0 \quad \text{and} \quad \xi_z << \xi_x, \xi_y \quad \text{and eliminate} \quad \xi_x
\]

\[
\frac{\partial}{\partial x} \frac{1}{\Omega^2} \frac{\partial}{\partial x} \left( \Omega^2 \xi_y \right) - k_y^2 \xi_y = 0 \quad \text{with} \quad \Omega^2 = \omega^2 - k_\parallel^2 \nu_A^2
\]

Define \( \psi = \Omega^2 \xi_y \) so \( \Omega^2 \frac{\partial}{\partial x} \left\{ \frac{1}{\Omega^2} \frac{\partial \psi}{\partial x} \right\} - k_y^2 \psi = 0 \)
Slab Model — Solution

Apply continuity and jump conditions at $\tilde{x} = \pm 1$ to find the dispersion condition

\[ \tilde{\omega}^2 - \tilde{k}_z^2 = \left( \omega^2 - k_z^2 v_A^2 \right) \frac{B_{0z}}{\mu_0 j_0 z} \frac{1}{2k_z v_A^2} \]

In this simple model: Instability for all $\tilde{k}_y > a$ a minimum value where $\tilde{\omega}^2 = \tilde{k}_||$

- For strongly-driven injection the mode is clearly unstable
- The value of $j$ determines the minimum value of $k_y$ for instability

Linear simulations: inner leg

\[
\begin{align*}
    a & \approx 0.04 \text{ m}, \quad R \approx 0.4 \text{ m} \\
    \frac{\tilde{k}_z}{\mu_0 j_0 z} & \approx 0.1 \\
    B_{0z} & \approx 1.2 \text{ T} \\
    B_{0y} & \approx 0.15 \text{ T} \\
    \tilde{k}_y & = \frac{\tilde{k}_z}{B_{0z}} B_{0y} \approx 0.8 \\
    \tilde{k}_|| & \approx 0.2
\end{align*}
\]
Slab Model — Eigenmode → 2 solutions with off-set eddies

Parameters as in previous slide for $\tilde{k}_y = 0.8; \tilde{j} = 0.1$. 
Summary

At conditions where the plasma current is confined to a narrow channel at the surface of the expanding flux-bubble:

- A field-aligned instability is generated
- Linear analysis and simulations strongly suggest it is predominately an ideal, MHD current-driven mode

When the mode is low amplitude, it has little effect on the plasma evolution

At large amplitude the mode undergoes relaxation oscillations and significantly affects the evolution of the injected poloidal flux

- It prevents the formation of closed flux regions during injection
- It expands the current channel outside that due directly to the injection