Update on Two-Fluid Internal Kink Calculations

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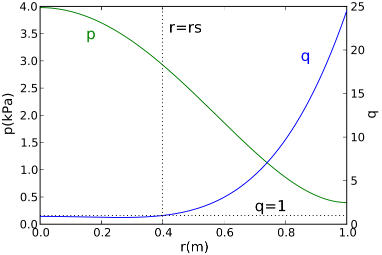
A verification effort is underway as the first step towards nonlinear two-fluid sawtooth simulations.

- Two-fluid effects introduce drifts which can stabilize the sawtooth when the drift frequency is sufficiently large compared to the growth rate.
  - Marginal stability occurs roughly when $\omega_* \gtrsim 2\gamma_0$. [Ara et al., 1978]
- Finite electron compressibility allows the electrons and ions to decouple.[Zakharov and Rogers, 1992].
  - The decoupling increases the linear growth rates.
  - The semi-collisional growth rate scales as $\gamma \sim \rho_s^{4/7} S^{-1/7}$.
  - The collisionless growth rate scales as $\gamma \sim \rho_s^{2/3} d_e^{1/3}$.
- Two-fluid modeling of the sawtooth requires an accurate representation of both the drift stabilization and the enhanced growth rate.

Calculations are performed in a screw pinch that is $n = 1$ ideal kink unstable.

Equilibrium profiles



Equilibrium Parameters:

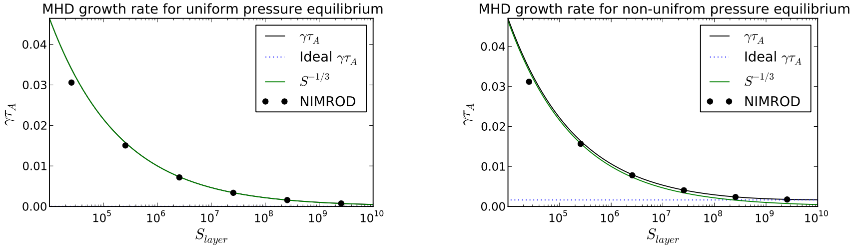
q_0	0.9
$q(a)$	24.5
B_0	1T
R/a	30
β_0	1%

- Equilibria are generated by specifying the pressure and safety factor.
- Two equilibria are studied: one with a uniform pressure, and the other with a nonuniform pressure profile.
  - This allows for the study of two-fluid modifications to the kink with and without drift effects.
- A strongly sheared profile is needed to ensure thin inertial and resistive layers.

Trigonometric elements are used to avoid small scale numerical instabilities.

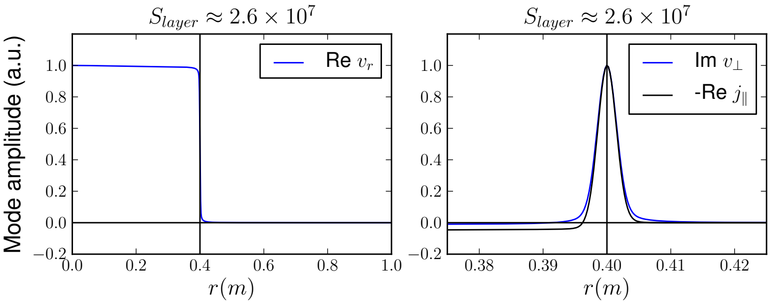
- The trig-elements represent the radial direction using the standard 1-D elements, but they represent the logical y-direction using Fourier cardinal functions.
- A numerical mode develops in two-fluid calculations when the standard 2-D elements are used.
  - The numerical mode only appears when $\nabla p_0 \neq 0$.
  - The numerical mode exhibits mesh scale oscillations in the logical y-direction (θ or z) but has finite width in r.
  - Calculations that use a uniform T_0 are the least problematic.
  - This mode might be a numerical drift wave...

Resistive MHD calculations capture the transition from resistive to ideal kink.



- The resistive interchange scaling $\gamma\tau_A = S^{-1/3}$ is an excellent approximation for the uniform pressure equilibrium with $S < 10^{10}$.
  - Here the ideal drive is weak: $\gamma_{ideal}\tau_A = 4.2 \times 10^{-5}$
- The pressure gradient is the dominant source of free energy for the nonuniform pressure equilibrium.
  - Ideal behavior is recovered for $S \gtrsim 10^9$.
  - Here the ideal growth rate is $\gamma_{ideal}\tau_A = 1.6 \times 10^{-3}$.
- At small S the validity of the analytic theory breaks down due to a finite layer width.

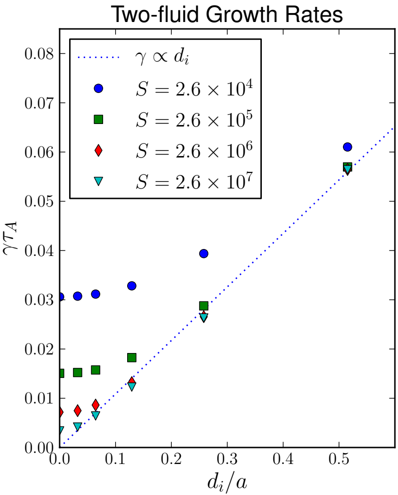
The radial velocity resembles the "top hot" trail function at large S.



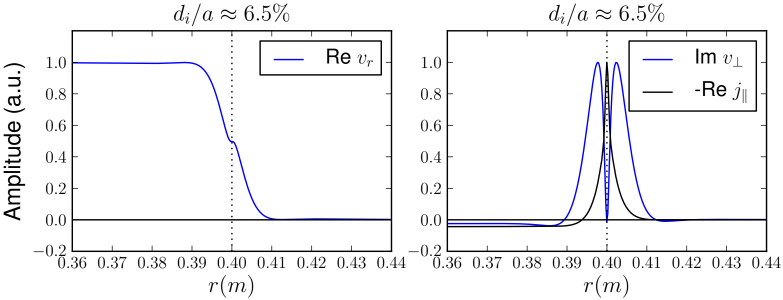
- Figures show the mode structure for the nonuniform pressure equilibrium at $S = 2.6 \times 10^7$.
- The momentum and current layers are of the same width.
  - Note that the x-axes use different scales in the two plots.

Two-fluid growth rates exhibit collisionless scaling at large d_i in the absence of drifts ($p' = 0$).

- The collisionless growth rate scales as $\gamma\tau_A \propto \rho_s^{2/3} d_e^{1/3}$.
  - Both ρ_s and d_e scale linearly with d_i, hence $\gamma\tau_A \propto d_i$.
  - At large d_i the growth rate is insensitive to S.
- Semi-collisional scaling is observed in calculations that neglect electron inertia.

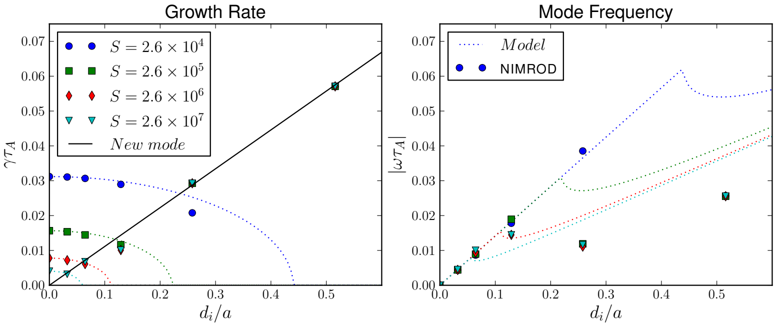


The separation of layer widths is observed at large d_i.



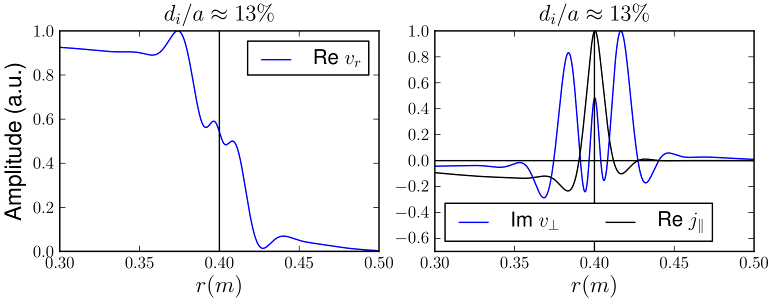
- The width of the flow layer scales with ρ_s.
  - The fluid treatment is valid provided $\rho_i < \rho_s$.
- The current layer width depends on both the resistive layer width and the electron skin depth.
  - In the collisionless limit the current layer width scales linearly with electron skin depth.
- Figures show the mode structure for $S = 2.6 \times 10^7$.

Drift effects reduce the linear growth rate at small d_i.



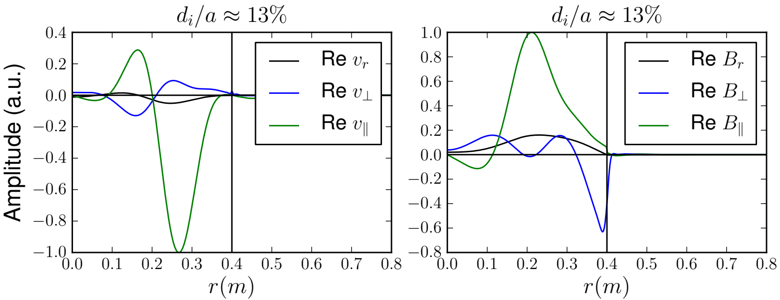
- The calculated growth rates are approximated by the model equation $\omega = \omega_{*i} + i\sqrt{\gamma_{MHD}^2 - \omega_{*i}^2/4}$.
- The nonuniform pressure equilibrium is used with $T_i = T_e$.
- The $1/1$ kink in not the dominant mode at large d_i.
  - The new mode is characterized by a large $v_{\|}$.
  - This modes scales linearly with d_i and is insensitive to S.

Large oscillations in the mode structure are observed when the drift stabilization is significant.



- These oscillations are characteristic of drift stabilization.
- Figures show the mode structure for $S = 2.6 \times 10^5$.
- Here the growth rate is 25% smaller that the MHD growth rate.

The mode structure of the dominant mode at large d_i indicates that it is not a kink mode.



- The mode is concentrated inside the $q = 1$ surface, and it is characterized by a large parallel velocity.
  - The kink mode has a small parallel velocity.
- Figures show the mode structure for $S = 2.6 \times 10^6$.

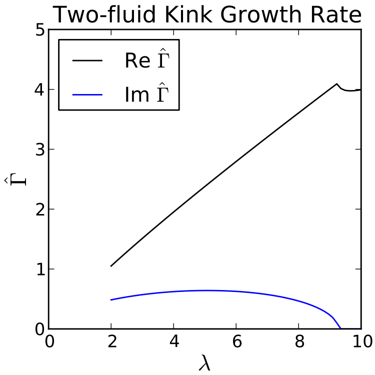
Zakharov's dispersion relation applies to 1/1 kink in the limit where the $\rho_s \gg \lambda_e, L_h$.

- Neglecting inertial layer corrections ($L_h = 0$), Zakharov's dispersion relation is:

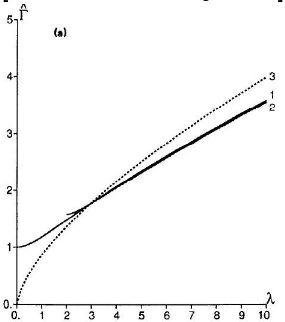
$$\frac{\pi}{2}\nu^4\left(1 - 2\nu^2\left[1 + \ln 2\right]\right) = \frac{\Gamma\left(1-\alpha\right)}{\Gamma\left(1+\alpha\right)}\left(\frac{\nu}{2\lambda}\right)^{2\alpha}$$

- ν is the normalized growth rate: $\nu \approx \frac{\gamma \tau_A}{q' \rho_s}$.
- λ is the ratio of the ion sound gyroradius to the current layer width: $\lambda \approx \rho_s/\lambda_e$.
- $\alpha = \sqrt{\frac{1}{4} + \nu^2}$.
- Formally the dispersion relation is valid in the limit that $\nu << 1$.
- Factors of $(\gamma - i\omega_*)$ in the definitions of ν and λ have been neglected for simplicity.

There is disagreement between my attempts to solve the dispersion relation and the published solution at small λ.



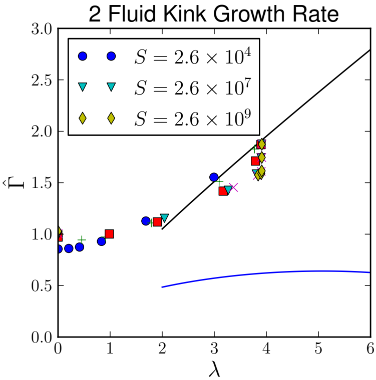
[Zakharov and Rogers,92]



- At small λ my solution of the dispersion relation has only complex roots. The published solution is purely real (line 2).
- Numerical solutions to the layer equation (line 1) are also real.
- The validity of the dispersion relation breaks down at small λ.
  - $\nu = \hat{\Gamma}/\lambda \sim 0.3 - 0.7$.

The comparison between NIMROD's calculated growth rates and solutions to the dispersion relation show promise.

- At high S and large d_i the numerically calculated growths agree with the theory.
- However, numerically calculated modes are purely growing.
- The validity of the dispersion relation is limited due the relatively large values of ν ($\nu \approx 0.4 - 0.5$).



NIMROD reproduces the two-fluid linear kink behavior in several regimes.

- Resistive MHD calculations correctly calculate the linear growth rate in both the inertial and resistive regimes.
  - Agreement between theory and calculations is limited by the validity of small layer approximation.
- In the absence of $\omega_* = 0$ effects, two-fluid calculations show an increased growth rate at large d_i characteristic of collisionless reconnection.
  - In this regime the electron stress tensor, which is not implemented in NIMROD, is important in many physical applications.
- The growth rate and real frequency are described by a simple model dispersion relation with finite ω_*.