Modeling Magnetized Sheath Boundary Conditions

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1 Vertical Displacement Events

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A VDE occurs when the plasma moves uncontrollably away from its equilibrium position into the containment vessel.

Below is a series of snapshots (in Alfvén times) of the pressure (color) and poloidal flux (black lines) during a VDE in NIMROD.
Halo currents begin to flow when VDEs occur.

- Halo currents are currents that occur when the plasma begins moving into the wall. The currents are outside the confined plasma, and the current then flows through the confinement vessel.

- The poloidal vacuum vessel currents interact with the large toroidal magnetic fields in tokamaks to cause large wall forces.

- Asymmetries in the current can cause asymmetric VDEs. These can cause horizontal forces on the vacuum vessel and are of concern for ITER and JET.\(^1\)

\(^1\)Rocella, Rocella, et al., Nucl. Fus. 56, 106010 (2016)
NIMROD solves the non-linear MHD equations.

\[
\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{V}) = \nabla \cdot (D_n \nabla n - D_h \nabla \nabla^2 n)
\]

Continuity with diffusive numerical fluxes

\[
m_n \left( \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) \mathbf{V} = \mathbf{J} \times \mathbf{B} - 2 \nabla (nT) - \nabla \cdot \vec{\Pi}
\]

Flow evolution

\[
\frac{3}{2} n \left( \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \right) T = -n T \nabla \cdot \mathbf{V} - \nabla \cdot \mathbf{q}
\]

Temperature evolution

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times (\eta \mathbf{J} - \mathbf{V} \times \mathbf{B}) + \kappa_B \nabla \nabla \cdot \mathbf{B}
\]

Faraday's/Ohm's Law with numerical error control

\[
\mu_0 \mathbf{J} = \nabla \times \mathbf{B}
\]

low-\(\omega\) Ampere's law

\[
\vec{\Pi} = -\rho \nu \vec{\mathbf{W}} - \rho \nu || (\hat{\mathbf{b}} \hat{\mathbf{b}} : \vec{\mathbf{W}})(\hat{\mathbf{b}} \hat{\mathbf{b}} - \frac{1}{3})
\]

\[
\vec{\mathbf{W}} = \nabla \mathbf{V} + (\nabla \mathbf{V})^T - \frac{2}{3} \mathbf{I} (\nabla \cdot \mathbf{V})
\]
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Sensitivity to the boundary conditions suggests that boundary modeling is important for VDE physics.

- Computations of a VDE with and without insulating boundary conditions on temperature is shown below.
- The computation with Dirichlet conditions on $T$ loses approximately 20% of its thermal energy over the first $1400\tau_A$.

Evolution of plasma current is sensitive to boundary conditions on $T$.

Contours of $T$ with $J$ vectors overlaid at $t = 1410$ with Dirichlet (left) and insulating (right).
More detailed modeling of sheath physics provides a set of boundary conditions that can be put in an MHD form.

- A sheath boundary condition model has been successfully developed for a fluid turbulence code.\(^2\)
- The boundary conditions are formulated as being at the entrance to the magnetic presheath. They include a Chodura-Bohm velocity boundary condition.
  - The edge can be divided into presheath, magnetic presheath (MPS), and sheath regions.

Figure of edge from Stangeby’s *The Plasma Boundary of Magnetic Fusion Device* (2000). Note not to scale.

Finding the MPS boundary condition method reduces to finding the determinant of a matrix.

- We rewrite the continuity (first row), ion momentum (second row), and electron momentum equations (third row) into matrix form $Mx = S$ where $S$ represent sources that vanish as we get into the magnetic presheath. $V$ is the ion flow velocity, with the geometry shown below assuming adiabatic ions.

- An ordering is imposed on the equations where derivatives along the wall are assumed to be order $\epsilon = \rho_s/L \ll 1$ with $\rho_s$ the ion sound speed Larmor radius.

$$
\begin{bmatrix}
\hat{n} \cdot V & n \sin \alpha \\
\gamma T_i \sin \alpha & nm_i \hat{n} \cdot V \\
T_e \sin \alpha & 0
\end{bmatrix}
\begin{bmatrix}
\hat{n} \cdot \nabla n \\
\hat{n} \cdot \nabla V_z \\
\hat{n} \cdot \nabla \phi
\end{bmatrix} = S
$$

- $\det M = 0$ then yields a relation that allows us to solve for $V$. 
The boundary equations are then adapted for full MHD.

- In the zeroth order \((\epsilon = \rho_s/L)\) cold ion approximation, with \(V\) the ion velocity and \(c_s = \sqrt{(T_e + \gamma T_i)/m_i}\) the sound speed

\[
V_{wall} = \pm c_s \hat{b}, \quad \hat{n} \cdot \nabla T_e = 0
\]

\[
\frac{T_e}{nq} \hat{n} \cdot \nabla n = \hat{n} \cdot \nabla \phi = -\frac{m_i c_s}{q} \hat{n} \cdot \nabla (V \cdot \hat{b})
\]

\[
J_\parallel = qnc_s (1 - \exp[\Lambda - \eta])
\]

- Here \(\Lambda = \ln\left(\frac{m_i}{2\pi m_e}\right)\) and \(\eta = \frac{q}{T_e} (\phi_{wall} - \phi_{MPS})\) is the normalized potential relative to the wall.

- Strauss\(^3\) has considered sheath compatible boundary conditions and implemented a Neumann velocity boundary condition with effects similar to this Chodura-Bohm criterion.

\[^3\text{Strauss, Phys. Plasm. 21 (2014) 032506.}\]
An ion saturation current model is added to more realistically model the sheath near the wall.

- With the ion sheaths that are expected in tokamaks, the ion current going towards the wall should be limited to the ion saturation current ($\sim qnc_s$).
- The current model only applies a velocity boundary condition. Using a nonlinear resistivity helps prevent the current from becoming unphysically large near the edge.
- The nonlinear resistivity is of the form ($\alpha$ is a coefficient to cause drop-off from the wall, with $J_{cs}$ how large and quickly the resistivity increases)

$$
\eta = \eta_0 T^{-3/2} + F J_{cs1} \max \left\{ 0, 1 + \tanh \left( J_{cs2} \left( \frac{J_{\parallel}}{qnc_s} - 1 \right) \right) \right\}
$$

$$
F = \min \left\{ 1, \exp \left[ \alpha (R_{\min} - R) \right] + \exp \left[ \alpha (R - R_{\max}) \right] + \exp \left[ \alpha (Z_{\min} - Z) \right] + \exp \left[ \alpha (Z - Z_{\max}) \right] \right\}
$$
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The VDE calculations include a vacuum region outside of the first wall.

- The central region is described with fluid-based plasma modeling.
- The surrounding region is modeled as a vacuum.
- The two regions are coupled by the thin-wall relation:

\[
\frac{\partial (\mathbf{B} \cdot \mathbf{n})}{\partial t} = -\mathbf{n} \cdot \nabla \times \left( \frac{\eta_W}{\mu_0 \delta x} \mathbf{n} \times \delta \mathbf{B} \right)
\]

where \( \eta_W \) is the wall resistivity and \( \delta x \) is the wall thickness.

The plot above shows an example of how the resistive wall, central, and outer region may be situated.
An equilibrium with a forced VDE is used to investigate the effects of various boundary conditions.

- The equilibrium pressure with poloidal flux contours for the central plasma region is shown in the upper right.
- An example of the domains for resistive wall calculations is shown in the lower right.
- The VDE is created by turning off vertical stability coils and letting the induced currents decay on the resistive wall.
- We use a flat anisotropic thermal conductivity model with $k_\parallel/k_\perp = 10^3$.
- A Spitzer resistivity model is used where $\eta(T) = \eta_0 T^{-3/2}$. 

K.J. Bunkers, C.R. Sovinec VDEs
Putting in the Bohm-Chodura criterion on the top and bottom surfaces changes the current and internal energy evolution.

- The boundary condition $\mathbf{V} = \pm c_s \mathbf{\hat{b}}$ (chosen so $\mathbf{V}$ is pointing outward) is enforced only on the top and bottom surfaces.

- Both cases use an insulating single temperature boundary condition with an advective flux (based on flow velocity) for number density.
The flow velocity smoothly connects to the wall with the Bohm-Chodura criterion while the zero velocity condition shows stagnation near the wall.

The upper figure shows the boundary at $t \sim 550\tau_A$ for the $\mathbf{V} = \mathbf{0}$ boundary condition and the lower figure shows the boundary at $t \sim 550\tau_A$ for the Bohm-Chodura boundary condition with.
We have varied boundary conditions on density, temperature, and flow-velocity to determine physical sensitivities.

The case with two temperatures, Dirichlet $T_i$, insulating $T_e$, an advective flux on $n$, and the Bohm-Chodura velocity boundary condition is labeled $2T/naf/VBC$.

<table>
<thead>
<tr>
<th>Boundary conditions label</th>
<th>$T$</th>
<th>$n$</th>
<th>$V$</th>
</tr>
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<tr>
<td>1TI/naf/VBC</td>
<td>insulating</td>
<td>advective flux</td>
<td>Bohm-Chodura outward</td>
</tr>
<tr>
<td>1TD/naf/VBC</td>
<td>Dirichlet</td>
<td>advective flux</td>
<td>Bohm-Chodura outward</td>
</tr>
<tr>
<td>1TD/nD/V0</td>
<td>Dirichlet</td>
<td>Dirichlet</td>
<td>$E_{wall} \times B$</td>
</tr>
<tr>
<td>1TD/naf/V0</td>
<td>Dirichlet</td>
<td>advective flux</td>
<td>$E_{wall} \times B$</td>
</tr>
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</table>
Two-temperature modeling leads to current decay similar to the Dirichlet single-temperature case.

- The plot shows four single temperature computations and one two-temperature computation with labels as described above.
- The two-temperature calculation uses an insulating $T_e$ condition and Dirichlet $T_i$ condition.
  - The insulating $T_e$ condition comes from the MPS theory.
  - The Dirichlet $T_i$ condition is used assuming that ions are cooled as they accelerate.
An advective number density condition without any MPS effects leads to faster decay than a Dirichlet condition.

- The plots show $J$ at $t \approx 2400\tau_A$. The upper plot shows 1TD/nD/V0 calculation and the lower plot shows the 1TD/naf/V0.
The evolution of 1TD/nD/V0 and 1TD/naf/VBC are similar for much of the early tokamak decay.

- The plots show $J$ at $t \approx 2400\tau_A$. The upper plot shows 1TD/nD/V0 calculation and the lower plot shows the 1TD/naf/VBC.
The 1TD/nD/V0 calculation has its tokamak decay away faster than the 1TD/naf/VBC case.

The plots show $J$ at $t \approx 3700\tau_A$ and $t \approx 3800\tau_A$ for the upper and lower plots, respectively. The upper plot shows 1TD/nD/V0 calculation and the lower plot shows the 1TD/naf/VBC.
The two-temperature calculations show a much broader halo current layer, than the pure Dirichlet temperature calculation.

- The upper plot shows the 1TD/naf/VBC calculation and the lower plot shows the 2T/naf/VBC calculation results for $J$ at $t \approx 2400\tau_A$. 
The broadness appears to be driven by changes in the number density evolution and electron temperature.

- The upper plot shows the 1TD/naf/VBC calculation and the lower plot shows the 2T/naf/VBC calculation results at $t \approx 2400\tau_A$. 

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VDEs
Nonlinear resistivity calculations have been tested and do reduce the current to below the ion saturation current.

- The nonlinear resistivity (bottom) prevents $J_\parallel/(nc_s)$ from growing to large values and lowers $J/(nc_s)$ at $t = 300\tau_A$. Both are 2T/naf/VBC calculations.
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VDE calculations are sensitive to boundary conditions on temperature, number density, and velocity.

MPS boundary conditions have been partially implemented and show some differences in axisymmetric VDE evolution.

- Velocity and temperature boundary conditions are implemented.
- A nonlinear resistivity condition has been applied near the edge to limit current to the ion saturation current.
Future Work

- Implementing the electric potential along the wall and so completing the zeroth order MPS model for NIMROD.
- Using the nonlinear resistivity and determining its use in a full MPS model.
- Determine which parts of the MPS boundary conditions have the most significant effects.
- Calculations with more equilibria.
- Perform non-axisymmetric calculations with MPS boundary conditions.