Code Developments Supporting NTM Modeling

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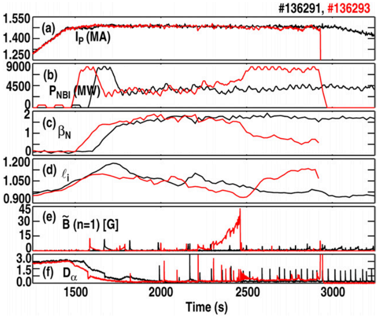
Outline

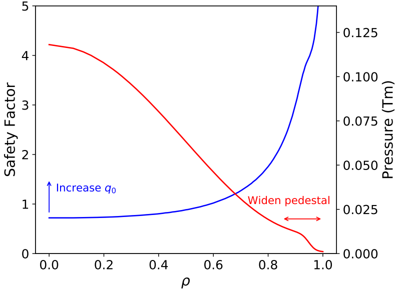
Motivation: Study NTM disputations using NIMROD

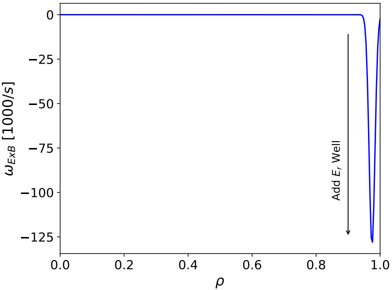
- Experiments contain dynamcis that we don't want to model
  - ELMs, Sawteeth
  - Control systems
  - Disruption mitigation systems
- Heuristic closures mimic dominant neoclassical effects
  - Self consistent treatment requires solving 5D DKE
  - Heuristic closures enable a more rapid exploration of disruption dynamics
  - Tradeoff between accuracy and computational costs



F Turco, NF 2010

Motivation: Model Equilibria are Created to Focus on NTMs





- Raise $q_0 > 1$ to avoid sawteeth
- Widen pedestal and add E_r well to stabilize edge modes
- Increase λ locally to increase Δ'
- Feedback system is needed to maintain shape

New Inputs Simplify Specification of Experimental Coils

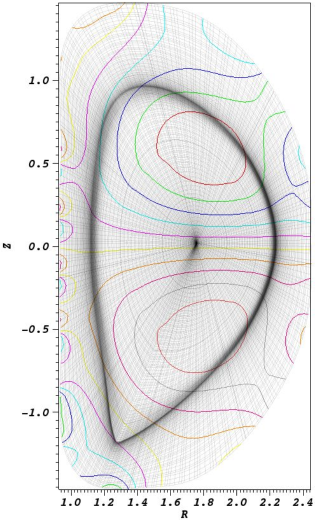
- Modeling coils with multiple filaments reduces ripple
- New input specifies coil shape
  - NIMEQ breaks coils into filaments

```
coiltype(i) = "parallel"
coilrz0(i,1) = r0
coilrz0(i,2) = z0
coilp(i, :) = <shape parameters>
```

- Predefined experimental coil sets

```
exp_coil_set = <experiment>
```

B_r



Inputs Specify External Currents Independently

- Shape controller determines optimal currents
- Number of independent currents is much smaller than the number of filaments used to model experimental coils
  - DIII-D: 18 independent currents but 5000+ filaments
- Coil specification points to a current
  - Multiple coils can point to the same current

```
coilextcindex(<coil index>) = <current index>
```

- Read coil currents from file

```
exp_curr_file = <file name>
exp_curr_type = <file type>
```

- Todo: Hardcode characteristic coil currents for experiments

Proportional-Derivative Feedback System Implemented for Shape Control

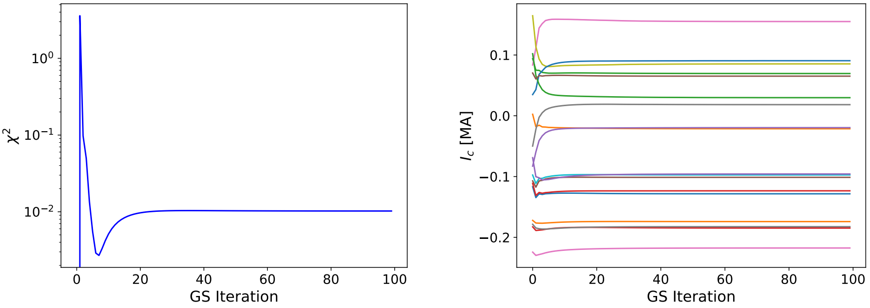
- External currents are updated using a PD controller:

$$\delta I_j^n = \alpha_p J_{ji}^{(k)\sim 1} e_i^n + \alpha_d J_{ji}^{(k)\sim 1} \left(e_i^n - e_j^{n-1} \right)$$

Shape control points:	$\{r_i\}$
Flux on control surface:	ψ_0
Square Error:	$e_i^2 = w_i \left(\psi\left(r_i\right) - \psi_0 \right)^2$
Jacobian:	$J_{ij} = \frac{\partial e_i}{\partial I_j} \equiv U \Sigma V^T$
SVD pseudo-inverse:[2]	$J^{(k)\sim 1} \equiv V \Sigma^k U^T$

[2] J. D. Hanson NF 2009

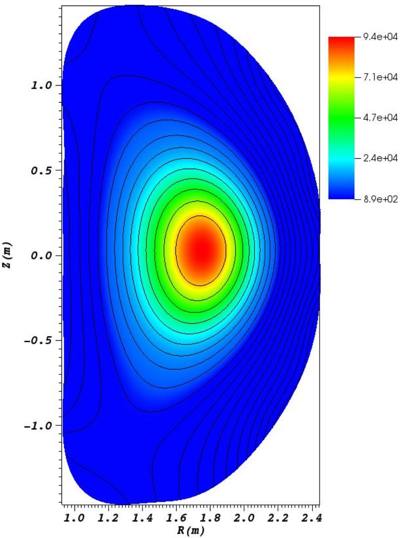
An Example Case Illustrates the Performance of the Feedback System

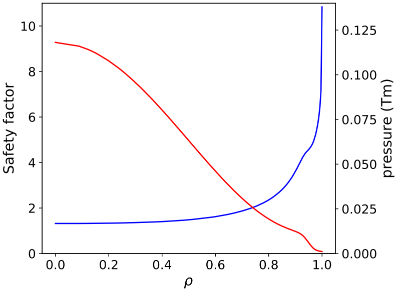


- 100 control points specify the $\hat{\psi} = 0.95$ surface
- Provides vertical and radial stability
- Performance is sensitive to the number of singular values
  - A cutoff tolerance of 10^{-4} works well

Modifications Enable Creation of ITER-Like Equilibria

Equilibrium Pressure(Pa)





I_p	0.958 MA
ϵ	0.33
κ	1.83
q_0	1.32
q_{95}	5.2
β_N	1.85
β_N/l_i	2.31

Numerical Implementation and Testing of Heuristic Closures

Neoclassical Effects Modify MHD Through the Stresses (and Heat Fluxes)

Continuity:	$\frac{\partial n}{\partial t} + \vec{v} \cdot \nabla n = n \nabla \cdot \vec{v}$
Momentum:	$\rho \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = \vec{J} \times \vec{B} - \nabla p - \nabla \cdot \vec{\vec{\Pi}}_{\nu} - \nabla \cdot \vec{\vec{\Pi}}_{i}$
Temperature:	$\frac{n}{\Gamma - 1} \left(\frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T \right) = -nT \nabla \cdot \vec{v} - \nabla \cdot \vec{q} + Q$
Faraday's Law:	$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$
Ohm's Law:	$E = -\vec{v} \times \vec{B} + \eta J - \frac{1}{ne} \nabla \cdot \vec{\vec{\Pi}}_{e}$

Heuristic Closures Model the Neoclassical Stresses

$$\nabla \cdot \vec{\vec{\Pi}}_i = n m_i \mu_i \left\langle B^2 \right\rangle \frac{\vec{v} \cdot \vec{e}_\Theta}{\left(\vec{B} \cdot \vec{e}_\Theta\right)^2} \vec{e}_\Theta$$

$$\nabla \cdot \vec{\vec{\Pi}}_e = -\frac{n m_e \mu_e}{ne} \left\langle B^2 \right\rangle \frac{\vec{J} \cdot \vec{e}_\Theta}{\left(\vec{B} \cdot \vec{e}_\Theta\right)^2} \vec{e}_\Theta$$

- Heuristic closures model dominant neoclassical effects[3]
  - Poloidal ion flow damping
  - Enhancement of polarization current
  - Bootstrap current

[3] Gianakon, POP 9, 2002

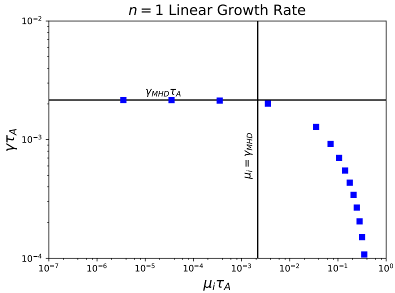
A linearized form of the stresses is implemented

$$\nabla \cdot \vec{\vec{\Pi}}_i = f n_0 m_i \mu_i \left\langle B_0^2 \right\rangle \frac{(\tilde{v} + \vec{v}_0) \cdot \vec{e}_\Theta}{\left(\vec{B}_0 \cdot \vec{e}_\Theta\right)^2} \vec{e}_\Theta$$

$$\nabla \cdot \vec{\vec{\Pi}}_e = -f \frac{m_e \mu_e}{e} \left\langle B_0^2 \right\rangle \frac{\left(\tilde{J} + \vec{J}_0\right) \cdot \vec{e}_\Theta}{\left(\vec{B}_0 \cdot \vec{e}_\Theta\right)^2} \vec{e}_\Theta$$

- Operators are self adjoint
- $\left\langle B_0^2 \right\rangle$ is calculated in the preprocessor
- $f_0 \rightarrow 0$ at $B_\Theta = 0$ to avoid division by zero
- $\hat{e}_\Theta = R\vec{B}_{p0}$

Neoclassical Enhancement of the Polarization Current is Used to Test the Ion Stress



S	10^5
Pr_m	1
$\nu_{visc}/\chi_\perp$	1
$k_\parallel/k_\perp$	10^8
μ_e	0

- The ion stress damps poloidal flows associated with tearing modes
- Damping increases inertia by a factor of $\left(1 + C\frac{\mu_i}{\gamma}\right)$
- Growth rate is reduced when $\mu_i \gtrsim \gamma_{MHD}$

Conclusions/Future Work

- Modifications to FGNIMEQ enable the generation of experimentally relevant equilibrium
  - New inputs simplify the specification of experimental coils
  - Feedback system enables shape control
  - Equilibria designed to focus on NTM physics
- Heuristic neoclassical stresses have been (re)implamented
  - Testing is in progresses
  - The ion stress enhances the polarization current
- Modifications will be used to investigate the mechanisms by which locked modes cause disruptions