Incorporation and Verification of a Fluid Model for Runaway Electrons in NIMROD

A. P. Sainterme, C. R. Sovinec, Ge Wang

University of Wisconsin - Madison

Nimrod Team Mtg. February 8, 2022

Overview

The inclusion of runaway electron physics in the extended MHD code NIMROD aims to complement ongoing modelling efforts.

- Much of the theory and modelling of runaway electrons deals with individual particle orbits or kinetic effects
- Using a reduced fluid model coupled with extended MHD aims to capture the interplay between the transport of runaways and the evolution of the MHD

Other extended MHD codes have had success implementing this type of reduced model.

- Cai and Fu use a fluid model in the M3D code to consider runaways effect on the resistive internal kink.[1]
- Bandaru, et al use a fluid model including sourcing in JOREK - benchmarked with GO code.[2]
- Zhao, et al have applied the model in M3D-C1 calculations to investigate the effect of runaways on low order tearing modes.[3]

[1] H. Cai and G. Fu, Nuclear Fusion **55**, 22001 (2015).
[2] V. Bandaru et al., Physical Review E **99**, 1–11 (2019).
[3] C. Zhao et al., Nuclear Fusion **60**, 10.1088/1741-4326/ab96f4 (2020).

The reduced model evolves a beam-like runaway population density with volumetric sources.

Continuity equation for runaway electron population:

$$\frac{\partial n_r}{\partial t} + \boldsymbol{\nabla} \cdot (n_r \boldsymbol{v_r}) = S_D(\mathcal{E}_{||}) + S_A(E_{||}) \tag{1}$$

Where n_r is the number density of runaways and

$$\mathcal{E}_{||} \equiv \frac{E_{||}}{E_D}, \quad \boldsymbol{v_r} = c_r \hat{\mathbf{b}} + \frac{\boldsymbol{E} \times \boldsymbol{B}}{B^2}, \quad |c_r| = \text{const.} \ggg v_{th,e} \ggg v_A$$

Numerically, a least squares finite element method is used to solve the continuity equation for the runaway species

The source terms model the Dreicer and avalanche.

The model for the Dreicer source S_D is that presented by Connor and Hastie[4]

$$S_D = (n_e - n_r)^2 \frac{e^4 \ln \Lambda}{4\pi\epsilon_0^2 m_e^2 v_{th}^3} \mathcal{E}_{||}^{-\frac{3}{16}(1+Z_{eff})} \exp\left\{ -\frac{1}{4\mathcal{E}_{||}} - \sqrt{\frac{1+Z_{eff}}{\mathcal{E}_{||}}} \right\}$$
$$\times \exp\left\{ -\frac{T_e}{mc^2} \left(\frac{1}{8\mathcal{E}_{||}^2} + \frac{2}{3}\sqrt{\frac{1+Z_{eff}}{\mathcal{E}_{||}^3}} \right) \right\}$$

The avalanche source S_A is given by Rosenbluth and Putvinski[5]:

$$S_A = \frac{n_r}{\tau}\sqrt{\frac{\pi a}{3(Z+5)}} \left(\frac{E_{||}}{E_c} - 1 \right) \left(1 - \frac{E_c}{E_{||}} + \frac{4\pi(Z+1)^2}{3a(Z+5)(E_{||}^2/E_c^2 + 4/a^2 - 1)} \right)^{-1/2}$$

$$E_c = \frac{n_e e^3 \ln \Lambda}{4\pi\epsilon_0^2 mc^2}, \quad a(\varepsilon) = (1 + 1.46\sqrt{\varepsilon} + 1.72\varepsilon)^{-1}, \quad \tau = \frac{mc \ln \Lambda}{eE_c}$$

[4] J. W. Connor and R. J. Hastie, Nuclear Fusion **15**, 415–423 (1975)
[5] M. N. Rosenbluth and S. V. Putvinski, Nuclear Fusion **37**, 1355–1362 (1997)

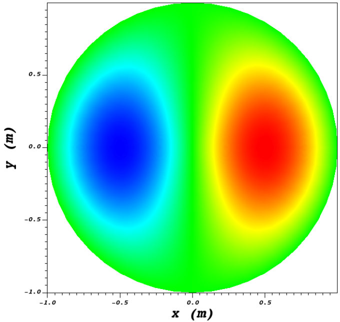
The runaway electrons couple to the MHD evolution via a modified Ohm's law.

$$\mathbf{E} = -\boldsymbol{V} \times \boldsymbol{B} + \eta \left(\boldsymbol{J} + q_e n_r c \hat{\boldsymbol{b}} \right) \tag{2}$$

- This simple Ohm's law is valid for sufficiently small $\frac{n_r}{n_e}$, where n_e is the total number density of electrons(thermal + runaway).
- The momentum evolution assumes the runaways have negligible inertia as well.
- This model is similar the model employed in Bandaru[6]and Matsuyama[7].

[7]V. Bandaru et al., Physical Review E **99**, 1–11 (2019)
[7]A. Matsuyama et al., Nuclear Fusion **57**, 10.1088/1741-4326/aa6867 (2017)

The advection of REs along field lines was verified.



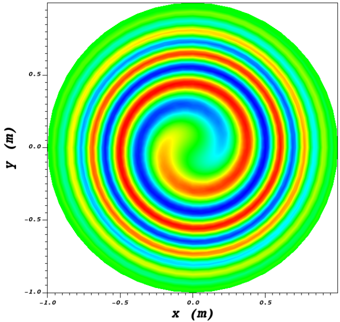


Figure: Evolution of an asymmetric initial runaway profile. The spiraling effect is expected for advection along sheared magnetic field lines with fixed velocity.

This spiraling effect is similar to the result obtained in JOREK[8]

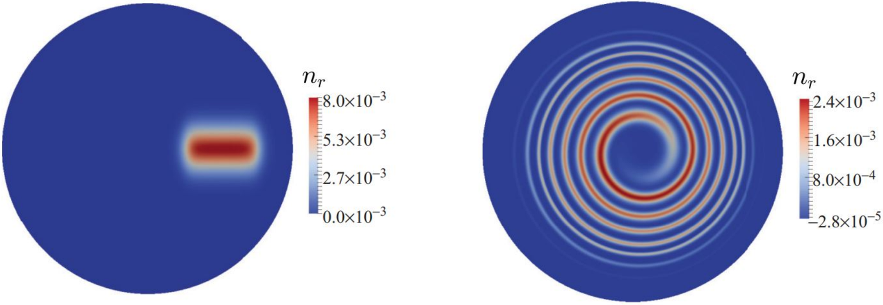


FIG. 1. Runaway electron number density n_r at time $t = 0$.

(b)

Figure: Adapted from Bandaru showing the spiraling of the runaway number density evolving with advection only

[8] V. Bandaru et al., Physical Review E **99**, 1–11 (2019).

NIMROD results were compared to JOREK results for runaway production following an artificial thermal quench in a large aspect ratio, circular tokamak.

Initial Parameters

$$|c_r| = 2.998 \times 10^5 \, \mathrm{m/s},$$
$$n_0 = 1.0 \times 10^{20} \, \mathrm{m}^{-3},$$
$$I_p \approx 0.76 \, \mathrm{MA},$$
$$\eta_0 = 1 \times 10^{-7} \, \Omega - \mathrm{m}$$
$$\eta = \eta_0 (T_0/T_e)^{3/2}$$
$$T_0 = 1.7 \, \mathrm{keV}$$

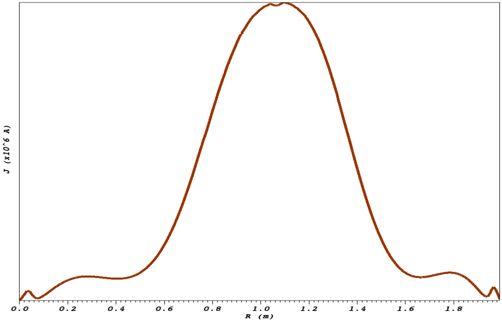


Figure: Equilibrium toroidal Current Density profile reconstructed in NIMROD for JOREK Benchmark case

Comparing the evolution of the total and runaway current in NIMROD shows agreement with JOREK.

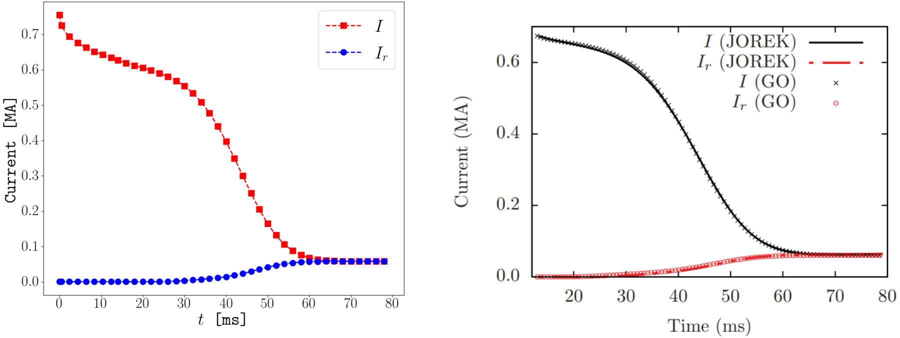


Figure: Total plasma and runaway current evolution from NIMROD simulation(left) and JOREK/GO(right)

A modified numerical scheme was implemented to ensure consistent treatment of the source terms and allow numerical diffusion.

The advection of the runaway density is handled with the least squares projection: find g such that

$$\int dV \left\{ \Delta n_r + \Delta t \boldsymbol{\nabla} \cdot \left(\boldsymbol{v}_r \frac{\Delta n_r}{2} \right) \right\} \times \left\{ g + \Delta t \boldsymbol{\nabla} \cdot \left(\boldsymbol{v}_r \frac{g}{2} \right) \right\}$$

$$= - \int dV \Delta t \boldsymbol{\nabla} \cdot \left(n_r^k \boldsymbol{v}_r \right) \times \left\{ g + \Delta t \boldsymbol{\nabla} \cdot \left(\boldsymbol{v}_r \frac{g}{2} \right) \right\}$$

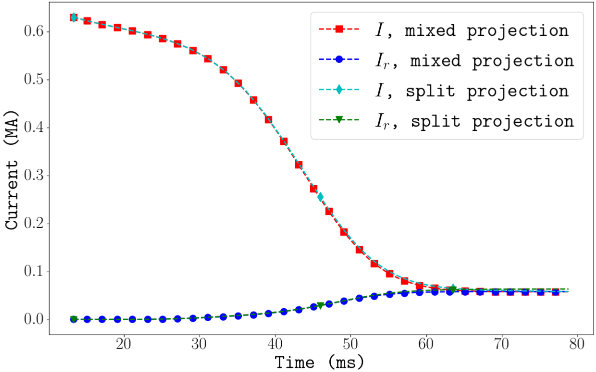
For any g in the space of admissible test functions for the finite element space. Here, n_r^k is the value of the runaway density at time $t = t_k$

The source and diffusion terms use a standard Galerkin projection.

$$\int dV \left\{ \Delta n_r g + \Delta t D \boldsymbol{\nabla} \left(\frac{\Delta n_r}{2} \right) \boldsymbol{\nabla} g \right\} = \Delta t \int dV \left\{ (S_D + S_A)\, g - \Delta t D \boldsymbol{\nabla} n_r^{k\prime} \cdot \boldsymbol{\nabla} g \right.$$

Where $n_r^{k\prime} = n_r^k + \Delta n_r$, This time-split scheme allows the use of numerical diffusion, D.

The results for the thermal quench benchmark agree using either projection.



Summary: The implementation of the reduced model in NIMROD is able to reproduce results observed in JOREK.

- NIMROD results for advection agrees with results obtained with JOREK - verifying consistency of the least-squares algorithm.
- Results from the thermal quench case also agree, suggesting the source term representation & MHD coupling iterations are effective.

Future work

- Complete benchmarking with other codes on cases involving a thermal quench and temperature-dependent resistivity using more realistic equilibria
- Apply the model to 3D disruption simulations
- Include additional runaway electron physics, additional sources/sinks