Update on δf closure using stress moment

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Summary

- New δf PIC algorithm for kinetic closure of moment equations developed and tested
- Closure at low velocity moment
  - Stress closure (compute stress from δf), uses evolving background (Maxwellian)
- Stress closure also implements long Δt techniques (Implicit time differencing, orbit averaging)
  - Energy conservation for discrete system bounds δf
- Successful tests for uniform (1D waves) and stratified (2D G-mode)

IMP (Implicit Moment/Particle) stress closure algorithm – moments

- Additional fluid equations

$$\frac{Dn}{Dt} + n\nabla \bullet \mathbf{u} = 0$$

$$\frac{\partial P_e}{\partial t} + \mathbf{u}_e \bullet \nabla P_e + \Gamma_e P_e \nabla \bullet \mathbf{u}_e = 0$$

$$\mathbf{u}_e = \mathbf{u} - \frac{\mathbf{J}}{en}$$

$$Mn\frac{D\mathbf{u}}{Dt} = Mn\mathbf{g} + \mathbf{J} \times \mathbf{B} - \nabla P_e - \nabla \bullet \Pi_i$$

Apply Cloud in Cell rule

$$\frac{\dot{\tilde{h}}_i}{1-\tilde{h}_i} = \int d\mathbf{z}\, s_w(\mathbf{w}, \mathbf{w}_i)\, s(\mathbf{x}-\mathbf{x}_i)\left[\nabla\bullet\mathbf{u} - \frac{1}{\bar{v}_T^2}\left(\mathbf{ww}:\frac{\partial\mathbf{u}}{\partial\mathbf{x}} + \mathbf{w}\bullet\tilde{\mathbf{A}}\right)\right]$$

$$\frac{\dot{\tilde{h}}_i}{1-\tilde{h}_i} = \sum_g S(\mathbf{x}_g-\mathbf{x}_i)\left(\frac{\partial}{\partial \mathbf{x}_g}\bullet\mathbf{u} - \frac{\mathbf{w}_i\mathbf{w}_i}{\bar{v}_T^2}:\frac{\partial\mathbf{u}}{\partial\mathbf{x}_g}\right)$$

$$\tilde{\mathbf{A}}_g = -\sum_i W_i \tilde{g}_i \left(\mathbf{w}_i\mathbf{w}_i - \bar{v}_T^2 1\right)\bullet\frac{\partial}{\partial\mathbf{x}_g} S(\mathbf{x}_g-\mathbf{x}_i) \Big/ n\Delta_g$$

- Now <u>symmetric</u> and closure <u>eliminated</u> from weight advance

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Symmetry Leads to Energy Integral

$$\frac{d}{dt}\left\{-M\bar{v}_T^2\sum_i W_i\left[\log\left(1-\tilde{h}_i\right)+\tilde{h}_i\right]\right\} = -M\sum_g \Delta_g n_g \mathbf{u}_g \bullet \tilde{\mathbf{A}}_g$$

$$\approx \frac{d}{dt}\left(-\frac{M\bar{v}_T^2}{2}\sum_i W_i\left|\tilde{h}_i\right|^2\right)$$

$$\int d\mathbf{x}\left(\frac{Mnu^2}{2}+Mn\varphi+\frac{B^2}{2\mu_0}+\frac{P_e}{\Gamma_e-1}+M\bar{v}_T^2 n\log n\right)$$

$$-M\bar{v}_T^2\sum_i W_i\left[\log\left(1-\tilde{h}_i\right)+\tilde{h}_i\right] = \mathcal{E} = const.$$

Dimits (Sherwood 2007) has given similar result

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Symmetry Leads to Energy Integral

- r.m.s. of particle weights absolutely bounded
- Stability comparison theorem
  - Kinetic system more stable than iso *T* ion fluid system
  - But only for marginal mode at zero frequency

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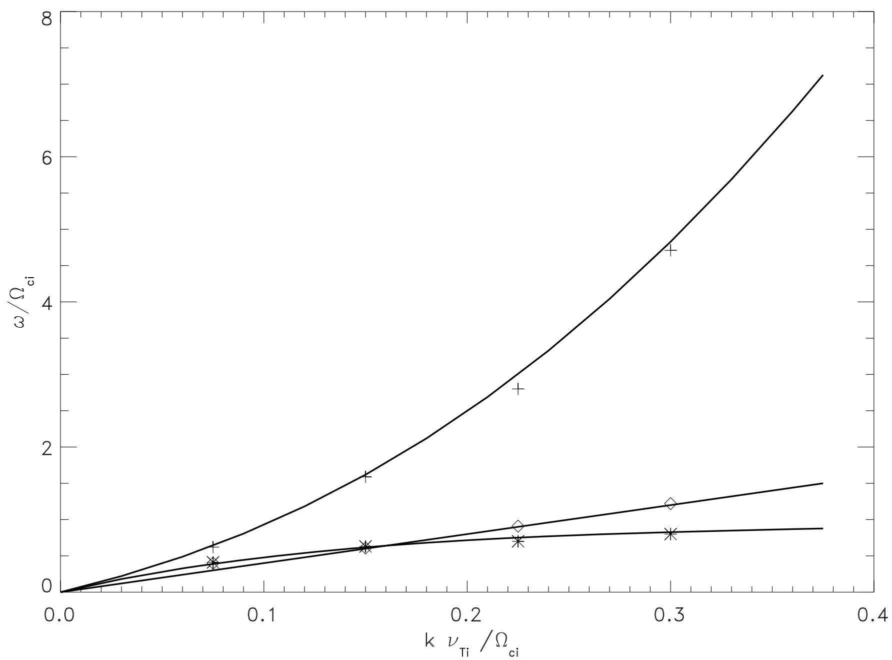
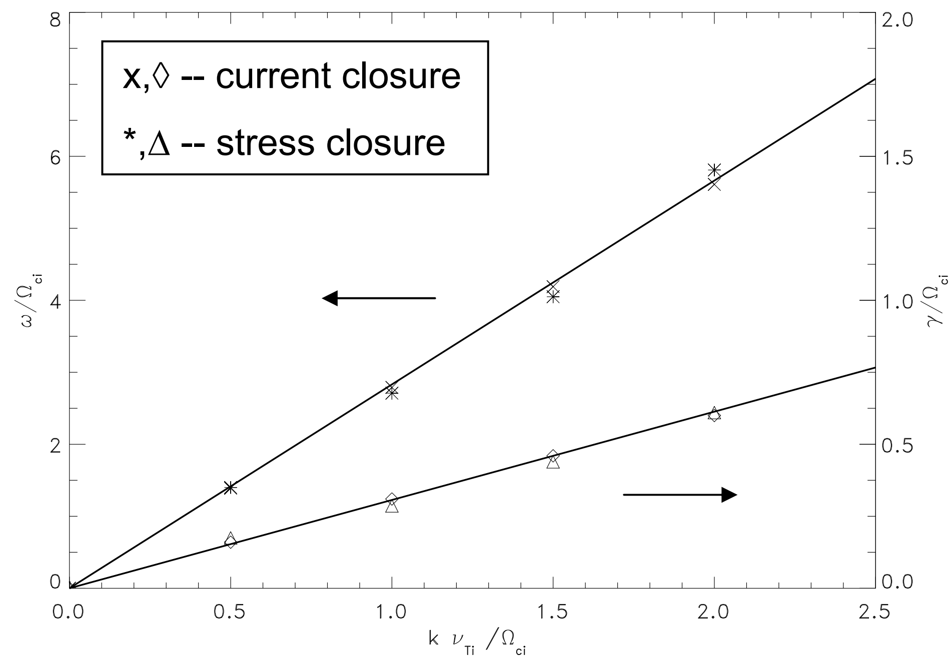
1D shows modes and Landau damping



3 waves
(whistler, ion-acoustic,shear)



Ion-acoustic dispersion
(real frequency, damping)

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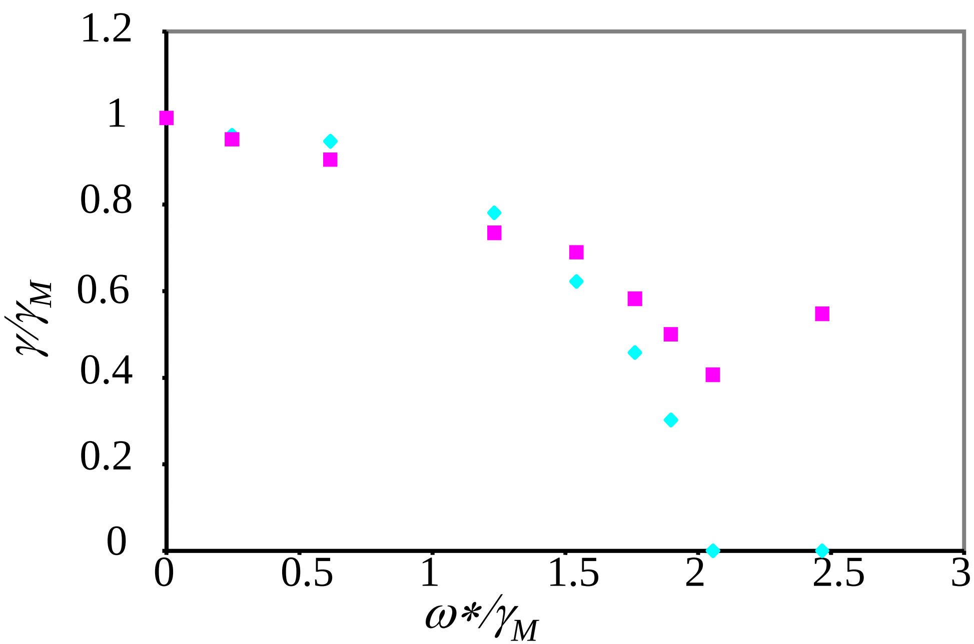
G-Mode Tests

- Two series
  - Low β Hall stabilized – with and w/o closure
  - High β gyro-viscous stabilized (Hall turned off)
- Numerical parameters
  - N_x x N_y = 30 x 16
  - 9 – 25 particles/cell
  - Typically 100 particle steps/fluid step

Low-beta G-mode stabilized by Hall



Fluid only

Closure

- Low β
  - $\beta = 0.02$
  - B = 6.0 T
  - n = 2. x 10^{20} m^{-3}
  - g = 1. x 10^{12} m/s^2
  - L_n = 120 m
  - T = 8.94 keV
  - $\rho = 2.28$ mm
- $k\rho < 0.29$

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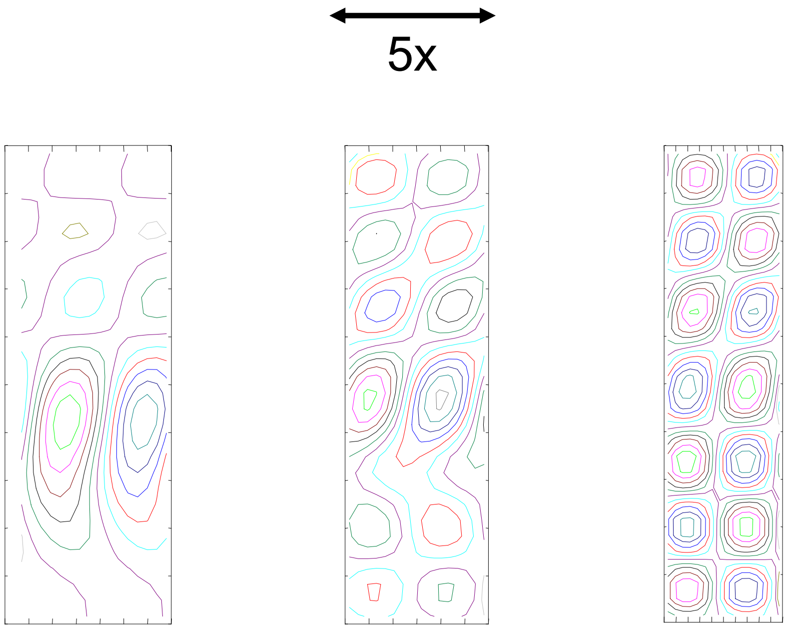
Mode transition near fluid stability



ω^*/γ_M = 1.76 2.06 2.47

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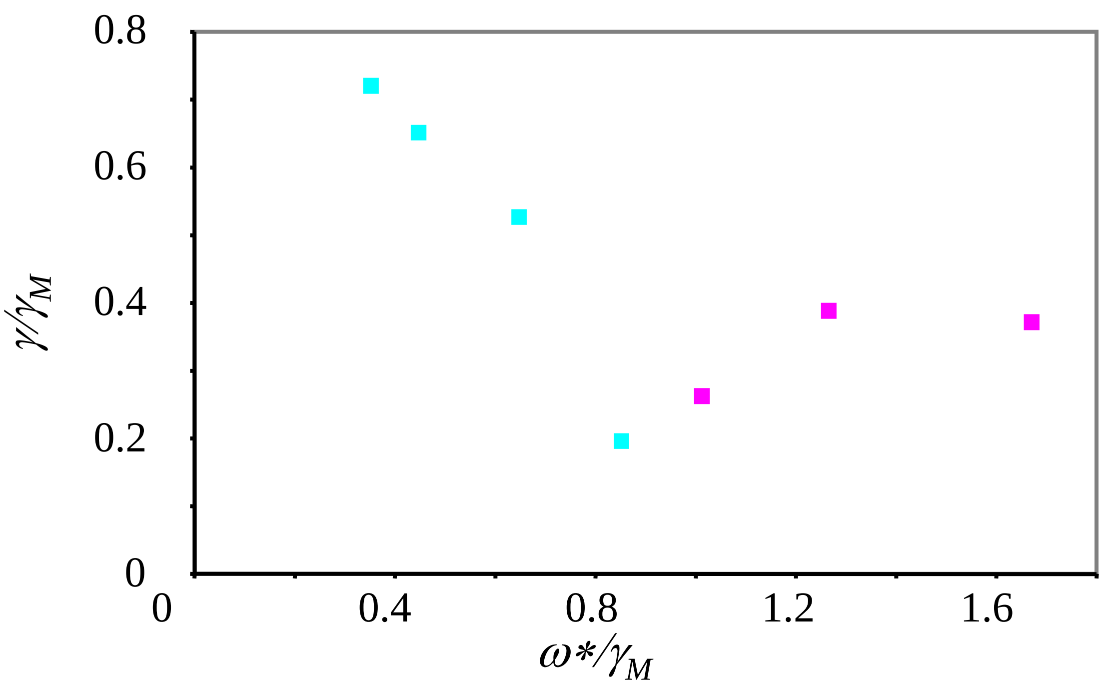
G-Mode Tests



- High β
  - $\beta = 1.0$
  - B = 0.482 T
  - n = 5.78 x 10^{19} m^{-3}
  - g = 2.7 x 10^8 m/s^2
  - L_n = 10 m
  - T = 10 keV
  - γ/Ω_i = 2.25 x 10^{-4}
  - ρ = 2.99 cm
- $k\rho < 0.25$

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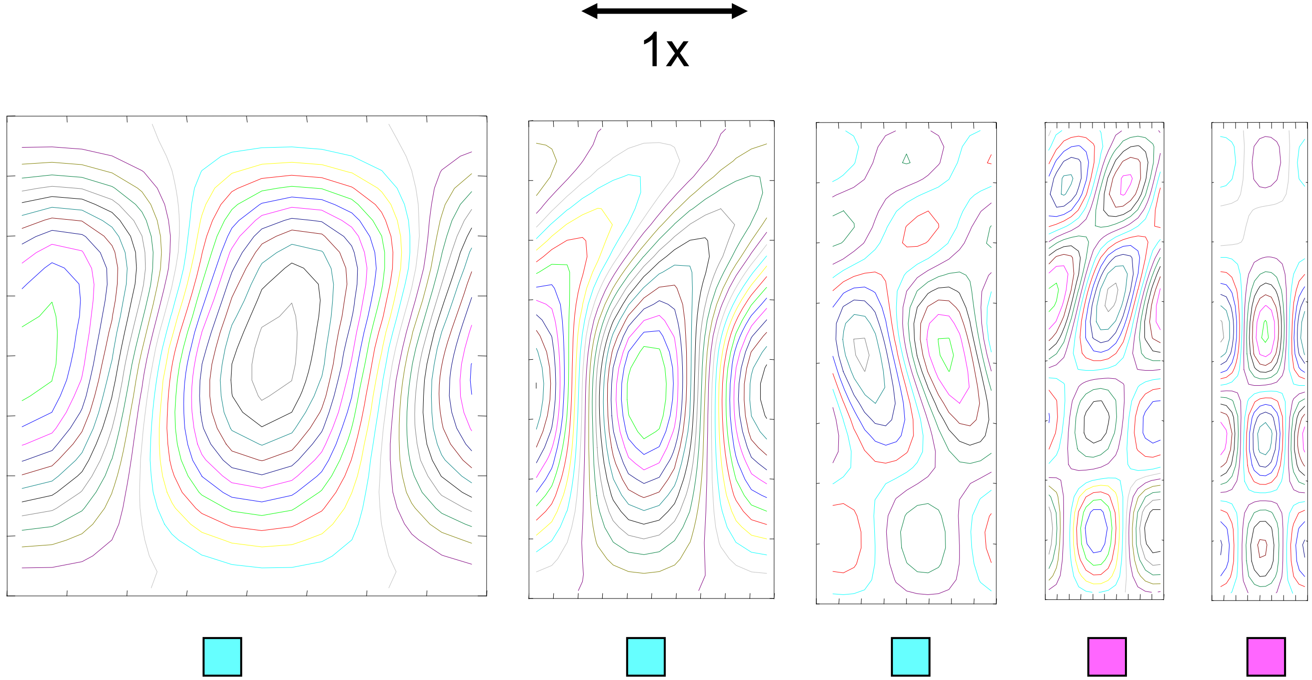
Mode transition near fluid stability



ω^*/γ_M = 0.35 0.65 0.85 1.27 1.67

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Some new work

- Temperature variation
  - Vlasov equation is linear, so superimpose number of uniform T solutions
  - Get same weight equation
  - $\Gamma(n) - \eta_i$
  - Heat flux carried by both particles (no df increase) and δT
  - Everything works (theoretically)
- Collisions
  - Apply previous work on evolving background (Brunner, Valeo & Krommes)
  - Binary collisions allowed