Impact of Velocity Space Distribution on Hybrid Kinetic-MHD Simulation of the $(1, 1)$ Internal Kink Mode

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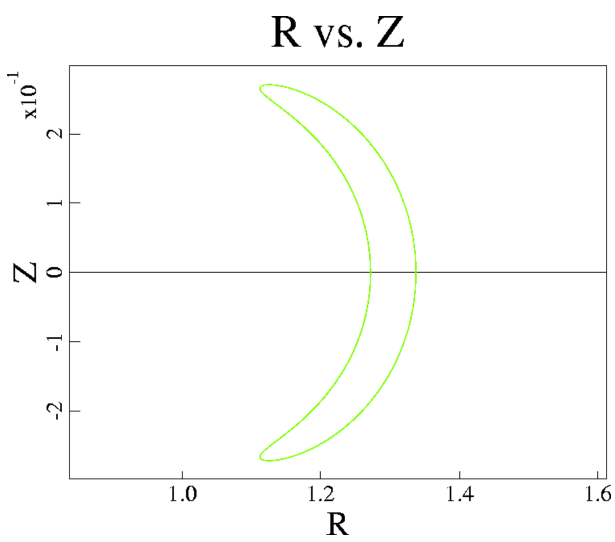
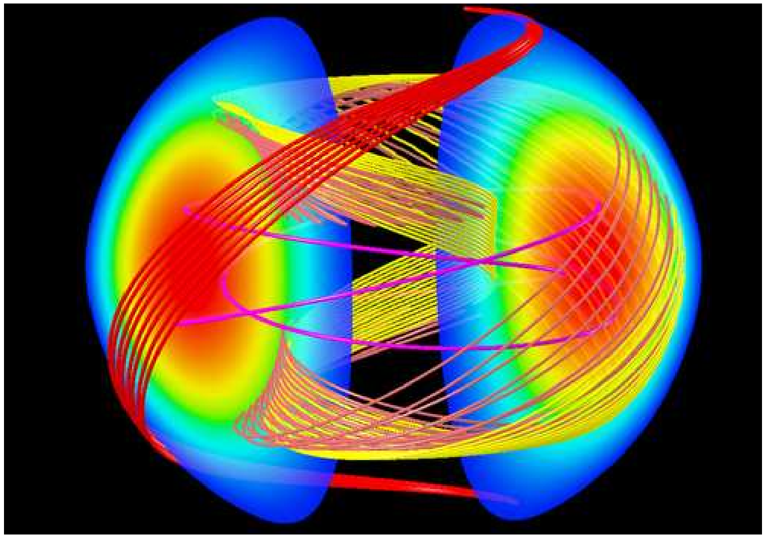
Hybrid Kinetic-MHD - Building Bridges

- capture kinetic effects lost in MHD approximation
- lost kinetic effects can significantly effect MHD instabilities[a]
  - high energy tail of plasma executes non-trivial orbits - large gyro-orbits, banana-orbits, potatoes





  - implement δf PIC in FE MHD code (NIMROD)

[a] W. Park, et al, "Three-dimensional hybrid gyrokinetic- magnetohydrodynamic simulation", *Physics of Fluids B*, **4**, 1992

Numeric representation of NIMROD fields

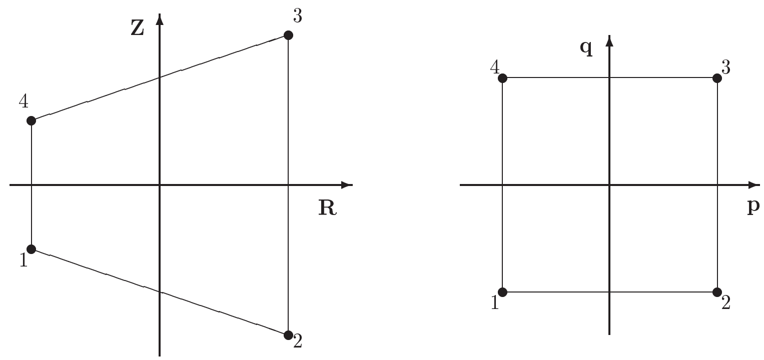
- the perturbed NIMROD fields are in FE-Fourier representation

$$\delta A(\mathbf{x},t) = \sum_j A_{j,0}(t)\alpha_{j,0} + \sum_j \sum_n (A_{j,n}(t)\alpha_{j,n} + c.c.)$$

where

$$\alpha_{j,n} = N_j(p,q)\exp(in\phi)$$

(p,q) are logical coordinates



- structured finite elements
- PIC becomes nontrivial in FE

PIC in FEM

- gather/scatter require logical coordinates
- FE representation for (R, Z) need to be inverted

$$R = \sum_j R_j N_j(p, q), \quad Z = \sum_j Z_j N_j(p, q),$$

- Use Newton method to solve for (p, q) given (R, Z)

$$\left\{ \begin{array}{c} p^{k+1} \\ q^{k+1} \end{array} \right\} = \left\{ \begin{array}{c} p^{k} \\ q^{k} \end{array} \right\} + A(p_k, q_k) \left\{ \begin{array}{c} R - R^{k} \\ Z - Z^{k} \end{array} \right\}$$

- Jacobian gives gradient of shape functions for free
- sorting is also required for parallelization
- efficient load balancing is tricky
- current implementation is limited to linear elements for PIC
  - high order elements for fluid variables

The Hybrid Kinetic-MHD Equations[a]

- in the limit $n_h \ll n_0$, $\beta_h \sim \beta_0$, quasi neutrality, only modification of MHD equations is addition of the hot particle pressure tensor in the momentum equation:

$$\rho\left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U}\cdot\nabla\mathbf{U}\right) = \mathbf{J}\times\mathbf{B} - \nabla p_b - \nabla\cdot\underline{\mathbf{p}}_h$$

  the subscripts b, h denote the bulk plasma and hot particles

- the steady state equation

$$\mathbf{J}_0\times\mathbf{B}_0 = \nabla p_0 = \nabla p_{b0} + \nabla p_{h0}$$

- evolved momentum equation is $(\mathbf{U}_s = 0)$

$$\rho_s\frac{\partial\delta\mathbf{U}}{\partial t} = \mathbf{J}_s\times\delta\mathbf{B} + \delta\mathbf{J}\times\mathbf{B}_s - \nabla\cdot\delta\underline{\mathbf{p}}_b - \nabla\cdot\delta\underline{\mathbf{p}}_h$$

[a] C.Z.Cheng,'A Kinetic MHD Model for Low Frequency Phenomena',*J. Geophys. Rev* **96**, 1991

Deposition of $\delta\underline{\mathbf{p}}_h$ onto Finite Element grid

- assume CGL-like form $\delta\underline{\mathbf{p}}_h = \begin{pmatrix} p_\perp & 0 & 0 \\ 0 & p_\perp & 0 \\ 0 & 0 & p_\parallel \end{pmatrix}$

- evaluate pressure moment at a position $\mathbf{x}$ is

$$\begin{aligned} \delta p(\mathbf{x}) &= \int m(v - V_h)^2 \delta f(\mathbf{x}, \mathbf{v}) d^3 v \\ &= \sum_{i=1}^{N} m(v_i - V_h)^2 g_0 w_i \delta^3(x - x_i) \end{aligned}$$

  where sum is over the particles, m mass of the particle, $g_0 w_i$ is the perturbed phase density, and V_h is the hot flow velocity

- discretize to the poloidal plane by performing a Fourier transform,[a]

$$\delta p_n(R, Z) = \sum_{i=1}^{N} m(v_i - V_h)^2 g_0 w_i \delta(R - R_i)\delta(Z - Z_i) e^{-in\phi_i}$$

[a] for large n runs, a conventional ϕ deposition with a **FFT** is performed

- express lhs $\delta p_n(R,Z)$ in the finite element basis,

$$\delta p_n(R,Z) = \sum_k \delta p_n^k N^k(\eta,\xi)$$

where sum k is over the basis functions and (η,ξ) are functions of (R,Z)

- project onto the finite element space by casting in weak form:

$$\int N^l \sum_k \delta p_n^k N^k d^3x = \int N^l \sum_{i=1}^{N} m(v_i - V_h)^2 g_0 w_i \delta(R-R_i)\delta(Z-Z_i) e^{-in\phi_i} d^3x$$

$$\underline{\mathbf{M}}\delta p_n^k = \sum_{l\in k}\sum_{i=1}^{N} m(v_i - V_h)^2 g_0 w_i e^{-in\phi_i} N^l(\eta_i,\xi_i)$$

$\underline{\mathbf{M}}$ is the finite element mass matrix

- gather is done using the same shape functions

$$A(\mathbf{x}_i) = \sum_l A^l N^l(\eta_i,\xi_i)$$

for some field quantity A

The δf PIC method [a] [b]

- PIC is a Lagrangian simulation of phase space $f(\mathbf{x}, \mathbf{v})$
- PIC evolves the $f(\mathbf{x}(\mathbf{t}), \mathbf{v}(\mathbf{t}))$
- spatial grid is not inherently necessary, but very convenient!
- in principle, $f(\mathbf{x}(\mathbf{t}), \mathbf{v}(\mathbf{t}))$ contains everything
- typically PIC is noisy, can't beat $1/\sqrt{N}$
- δf PIC reduces the discrete particle noise associated with conventional PIC
- Vlasov Equation

$$\frac{\partial f(\mathbf{z})}{\partial t} + \dot{\mathbf{z}} \cdot \frac{\partial f(\mathbf{z})}{\partial \mathbf{z}} = 0$$

$\mathbf{z}$ is the phase coordinate

[a] S. E. Parker and W. W. Lee, 'A fully nonlinear characteristic method for gyro-kinetic simulation', *Physics of Fluids B*, **5**, 1993

[b] G. Hu and J. A. Krommes, "Generalized weighting scheme for δf particle simulation method", *Physics of Plasmas*, **1**, 1994

- split phase space distribution into steady state and evolving perturbation:

$$f = f_{eq}(\mathbf{z}) + \delta f(\mathbf{z}, t)$$

- δf evolves along the characteristics $\dot{\mathbf{z}}$ (control variates MC[c])

$$\dot{\delta f} = -\tilde{\mathbf{z}} \cdot \frac{\partial f_{eq}}{\partial \mathbf{z}}$$

using $\mathbf{z} = \mathbf{z}_{eq} + \tilde{\mathbf{z}}$ and $\dot{\mathbf{z}}_{eq} \cdot \frac{\partial f_{eq}}{\partial \mathbf{z}} = 0$

- the drift kinetic equations of motion are used as the particle characteristics

$$\dot{\mathbf{x}} = v_{\|}\hat{\mathbf{b}} + \frac{m}{eB^4}\left(u^2 + \frac{v_{\perp}^2}{2}\right)\left(\mathbf{B} \times \nabla \frac{B^2}{2}\right) + \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \frac{\mu_0 m v_{\|}^2}{eB^2}\mathbf{J}_{\perp}$$

$$m\dot{v}_{\|} = -\hat{\mathbf{b}} \cdot (\mu \nabla B - e\mathbf{E})$$

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[c] A. Y. Aydemir, "A unified MC interpretation of particle simulations...", *Physics of Plasmas*, **1**, 1994

Slowing Down Distribution for Hot Particles

- for the slowing down distribution function

$$f_{eq} = \frac{P_0 \exp(\frac{P_\zeta}{\psi_0})}{\varepsilon^{3/2} + \varepsilon_0^{3/2}}$$

where $P_\zeta = g\rho_\parallel - \psi$ is the canonical toroidal momentum and ε is the energy, ψ_0 is the total flux, and ε_c is the critical slowing down energy

$$\begin{aligned}\dot{\delta f} &= f_{eq}\left\{\frac{mg}{e\psi_0 B^3}\left[\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\delta\mathbf{B}\cdot\nabla B - \mu_0 v_\parallel \mathbf{J}\cdot\mathbf{E}\right]\right. \\ &\left. + \frac{\delta\mathbf{v}\cdot\nabla\psi_p}{\psi_0} + \frac{3}{2}\frac{e\varepsilon^{1/2}}{\varepsilon^{3/2}+\varepsilon_0^{3/2}}\mathbf{v}_D\cdot\mathbf{E}\right\}\end{aligned}$$

where

$$\begin{aligned}\mathbf{v}_D &= \frac{m}{eB^3}\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)(\mathbf{B}\times\nabla B) + \frac{\mu_0 m v_\parallel^2}{eB^2}\mathbf{J}_\perp \\ \delta\mathbf{v} &= \frac{\mathbf{E}\times\mathbf{B}}{B^2} + \mathbf{v}_\parallel\cdot\frac{\delta\mathbf{B}}{B}\end{aligned}$$

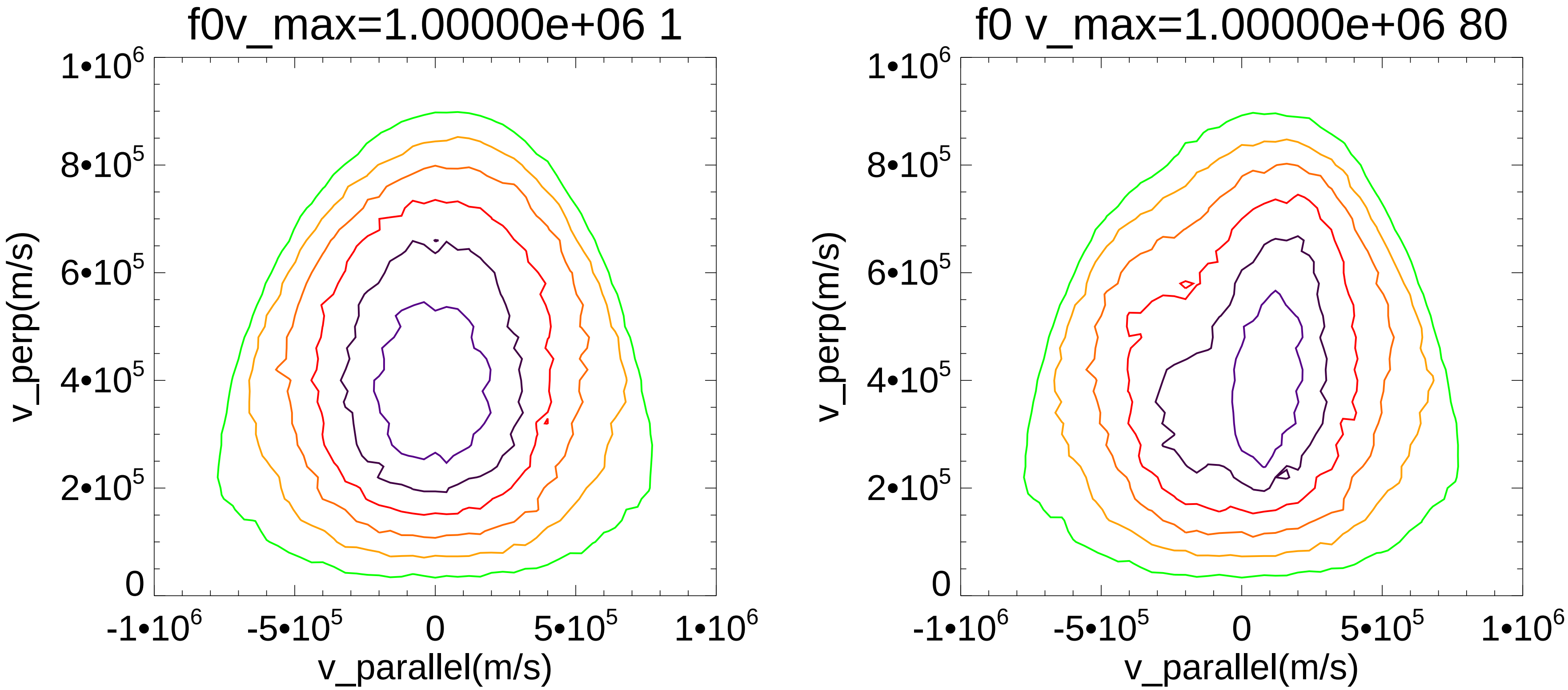
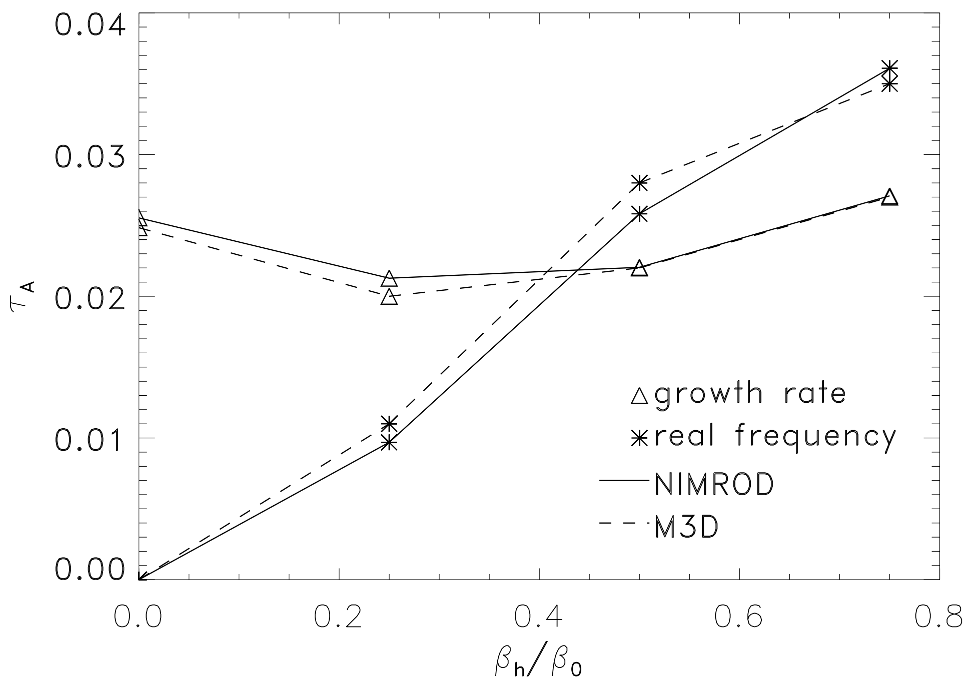
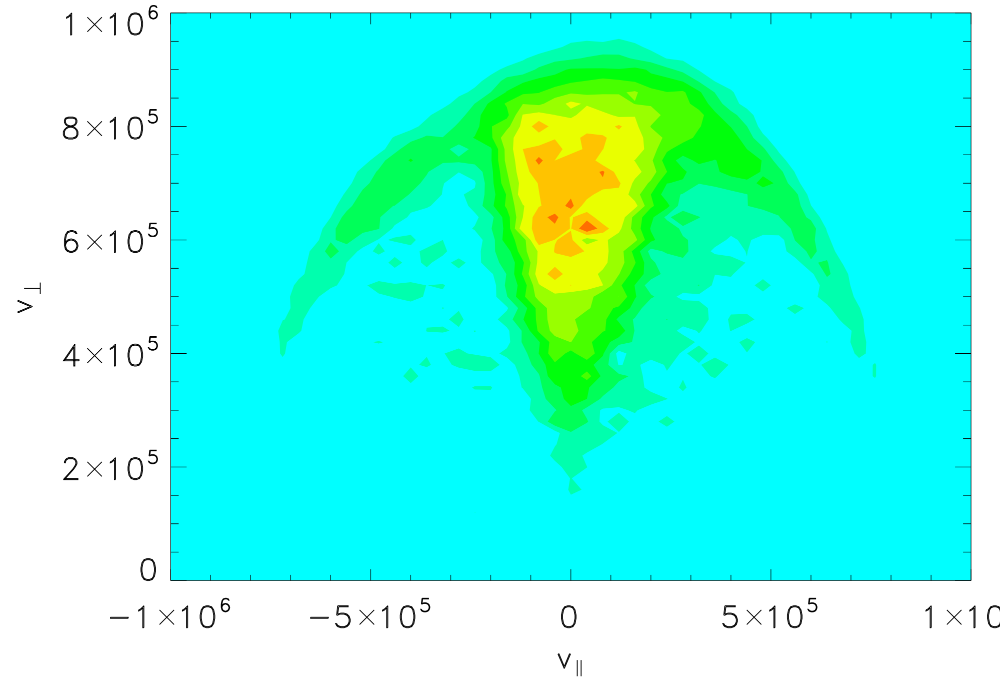


Figure 1: $f0$ at $v_{max} = 1.0 \times 10^6$m/s initial load and steady state

Benchmark with M3D





- ideal (1, 1) internal kink mode $\beta = 8\%$
- velocity shows most activity in the most energetic particles
- mostly in trapped region, also in extremes of passing particles

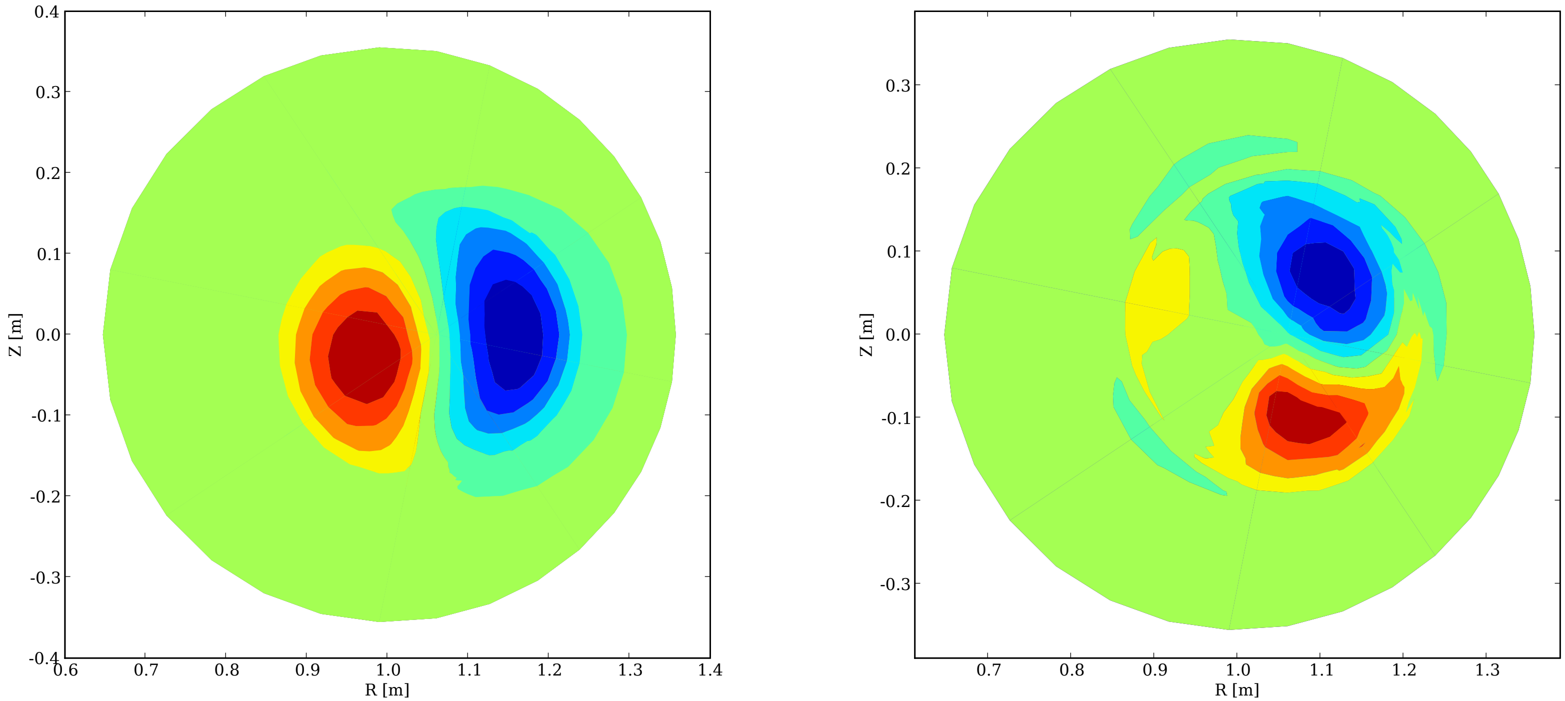


Figure 2: $n = 1$ $p_{h\perp}, \Delta p_h$

- simulations without anisotropy reproduce ideal MHD with γ within 10%
- no real frequency
- anisotropic pressure $\Delta p = p_{\|} - p_{\perp}$ is key to energetic effects

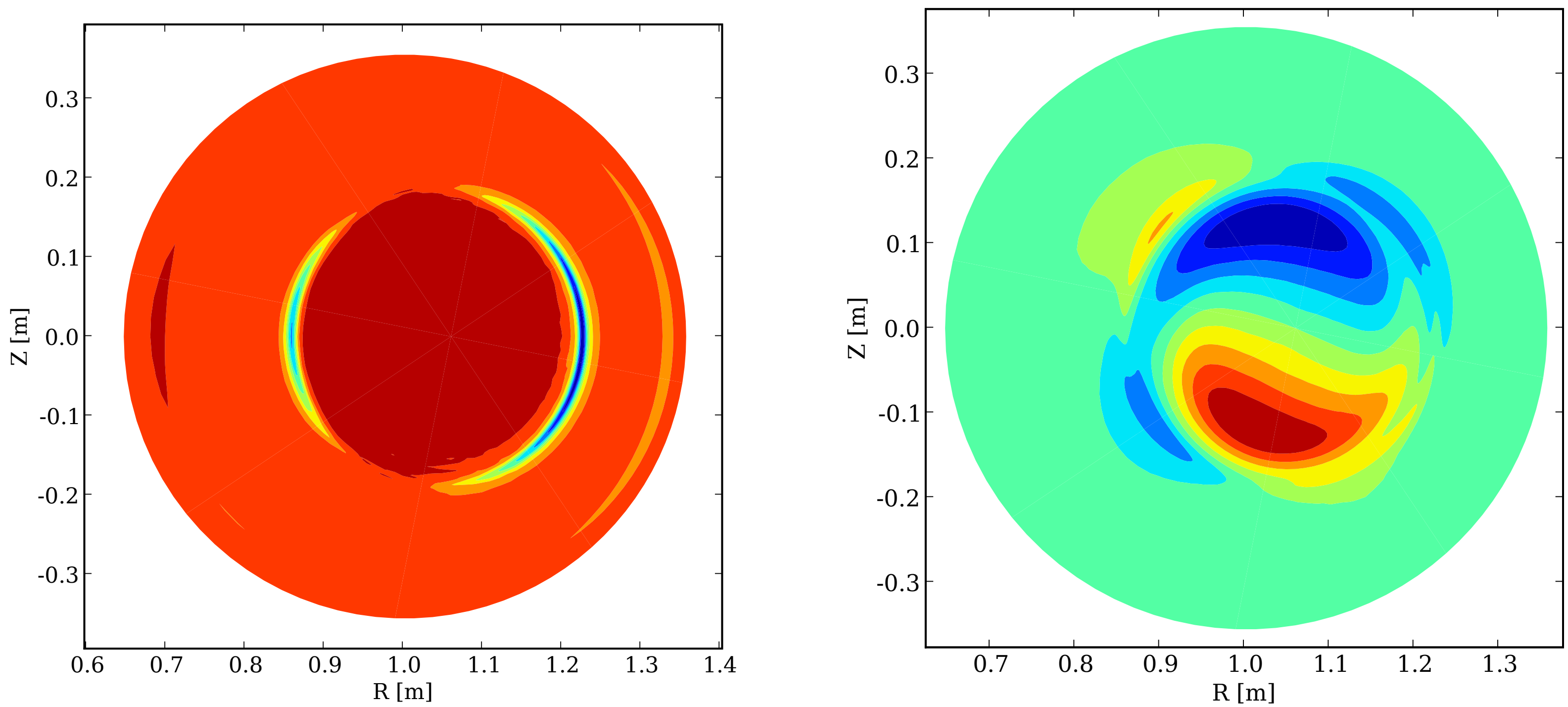


Figure 3: $n = 1 V_\phi$ with passing particles, V_ϕ with trapped particles

- simulations with only passing particles stabilize but do not change V_ϕ mode topology
- trapped particles excite precessional fishbone mode (plot on right)

Impact of increasing velocity cutoff

- benchmark $(1,1)$ performed with $v_{max} = 1 \times 10^6 m/s$
- increasing the velocity cutoff results in stronger stabilization
  - fixed β_h - increase in energy range, decrease in density → **fewer particles are doing more**
  - increase in real frequency
  - examine $\beta_h = .5$

v_{max}	$\gamma\tau_A$	$\omega\tau_A$
1.0	0.0218	0.0257
1.1	0.0180	0.0290
1.2	0.0142	0.0317
1.3	0.0093	0.0345
1.5	small	0.0370

Examination of δf in phase space

- more energetic particles drive higher frequency
- plots of velocity space density of $n = 1$ projection of δf shows dominance of trapped particles as cut-off is increased
- this is reflected in decreasing relative amplitude of passing particle "wing"
- note the asymmetry in the profile
  - trapped cone region biased toward counter propagating (negative $v_{\|}$)
  - passing region biased towards co-propagating

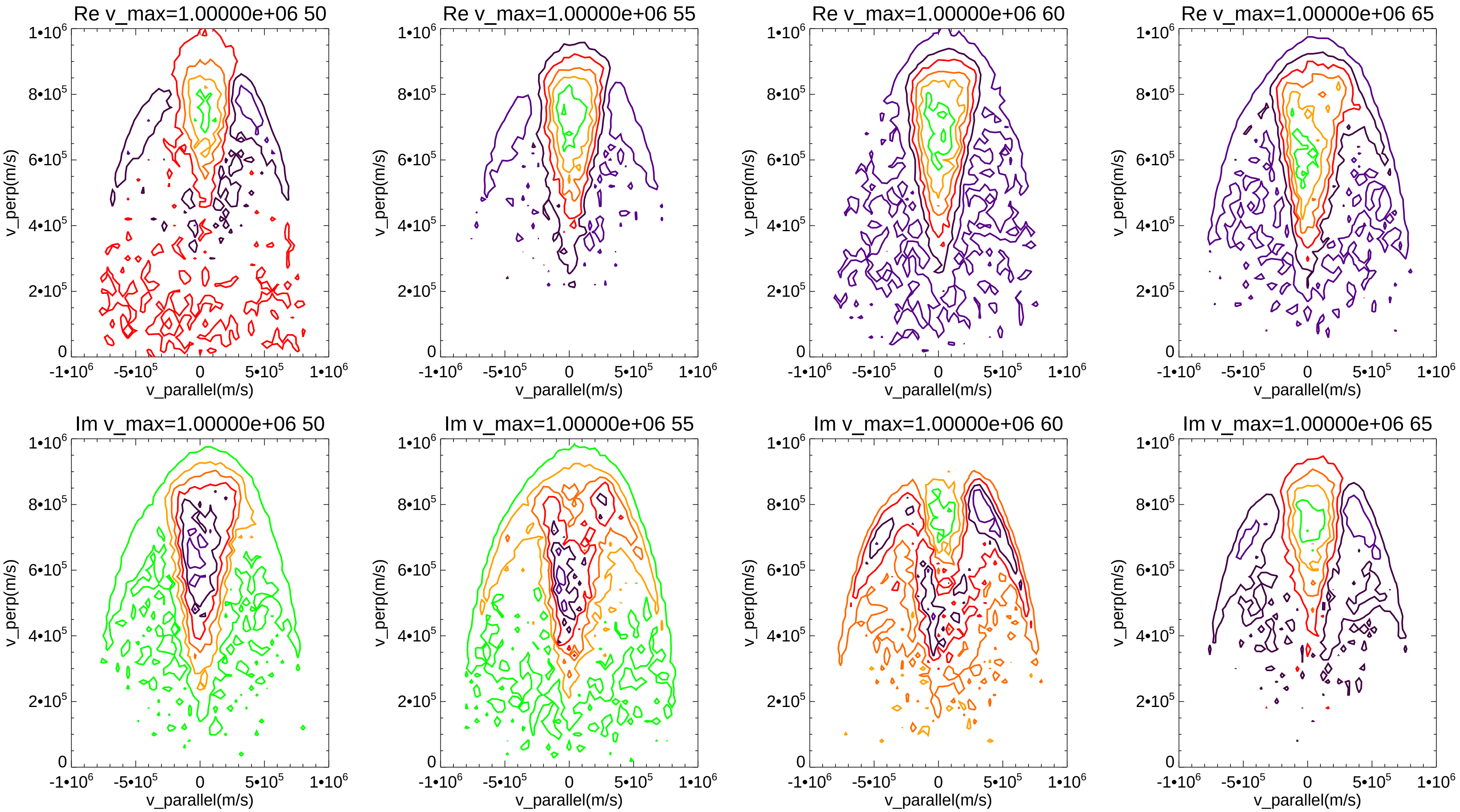


Figure 4: $n = 1$ projection of δf at $v_{max} = 1.0 \times 10^6 \mathrm{m/s}$

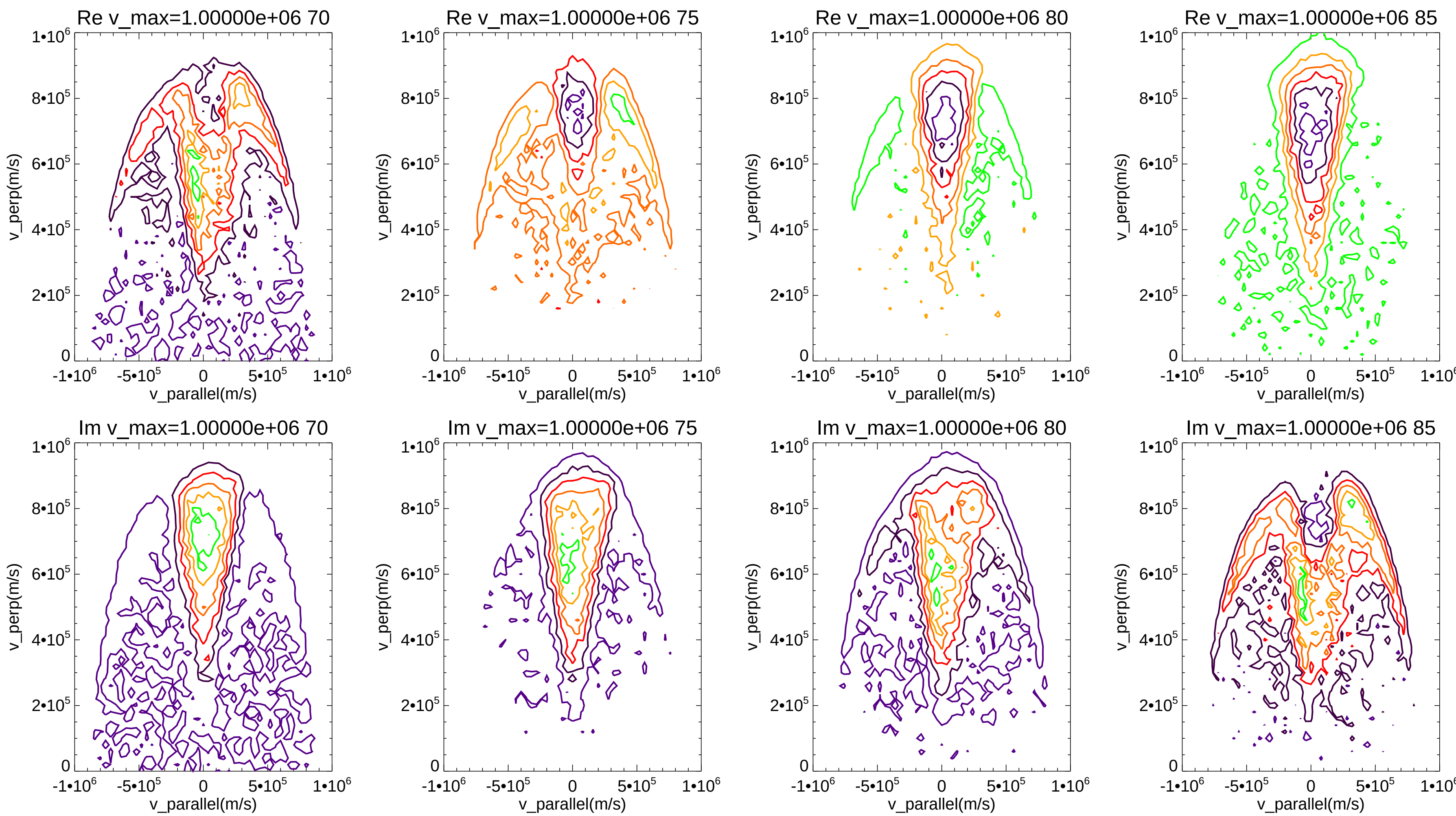


Figure 5: $n = 1$ projection of δf at $v_{max} = 1.0 \times 10^6$m/s

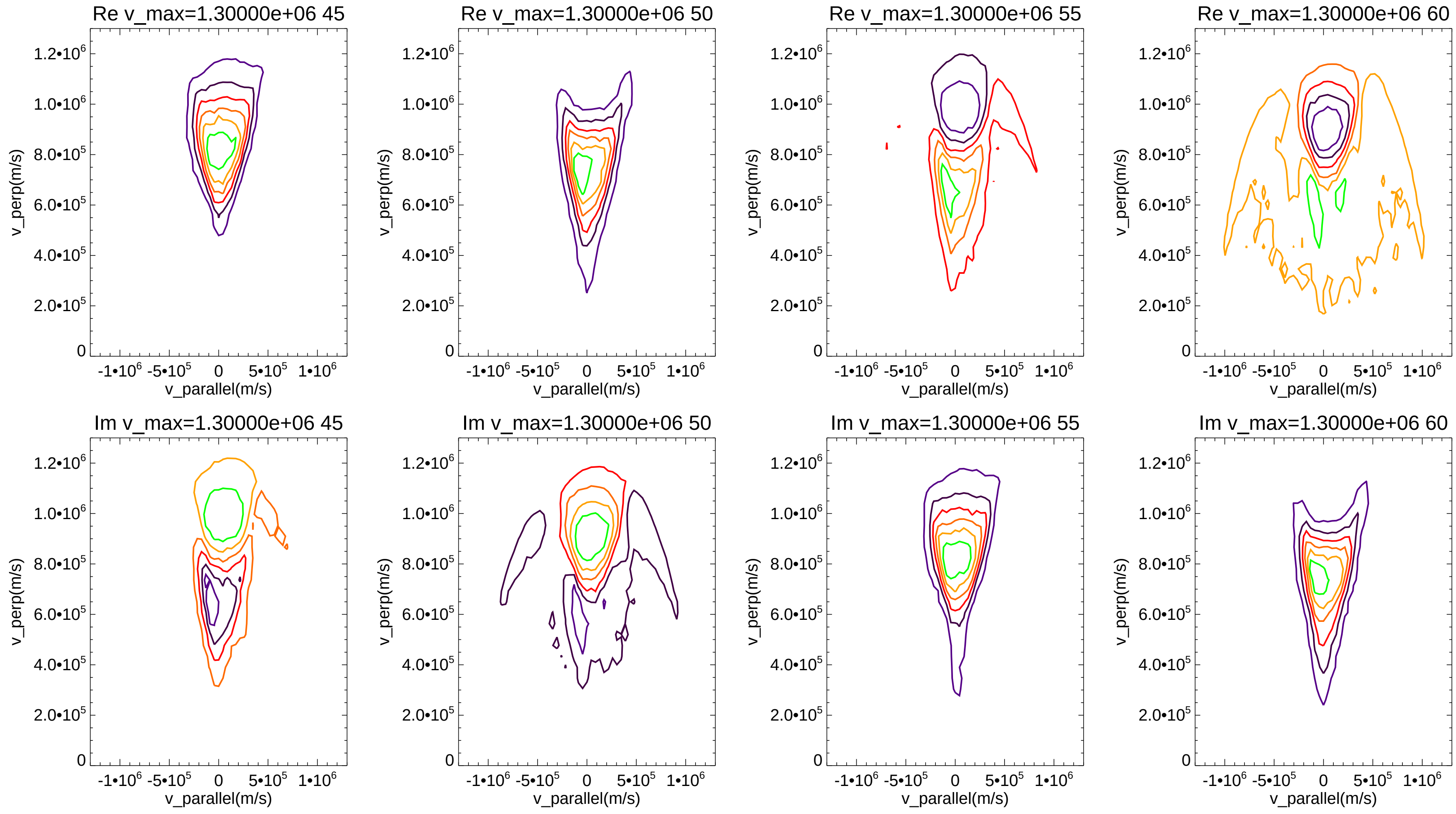


Figure 6: $n = 1$ projection of δf at $v_{max} = 1.3 \times 10^6 \mathrm{m/s}$

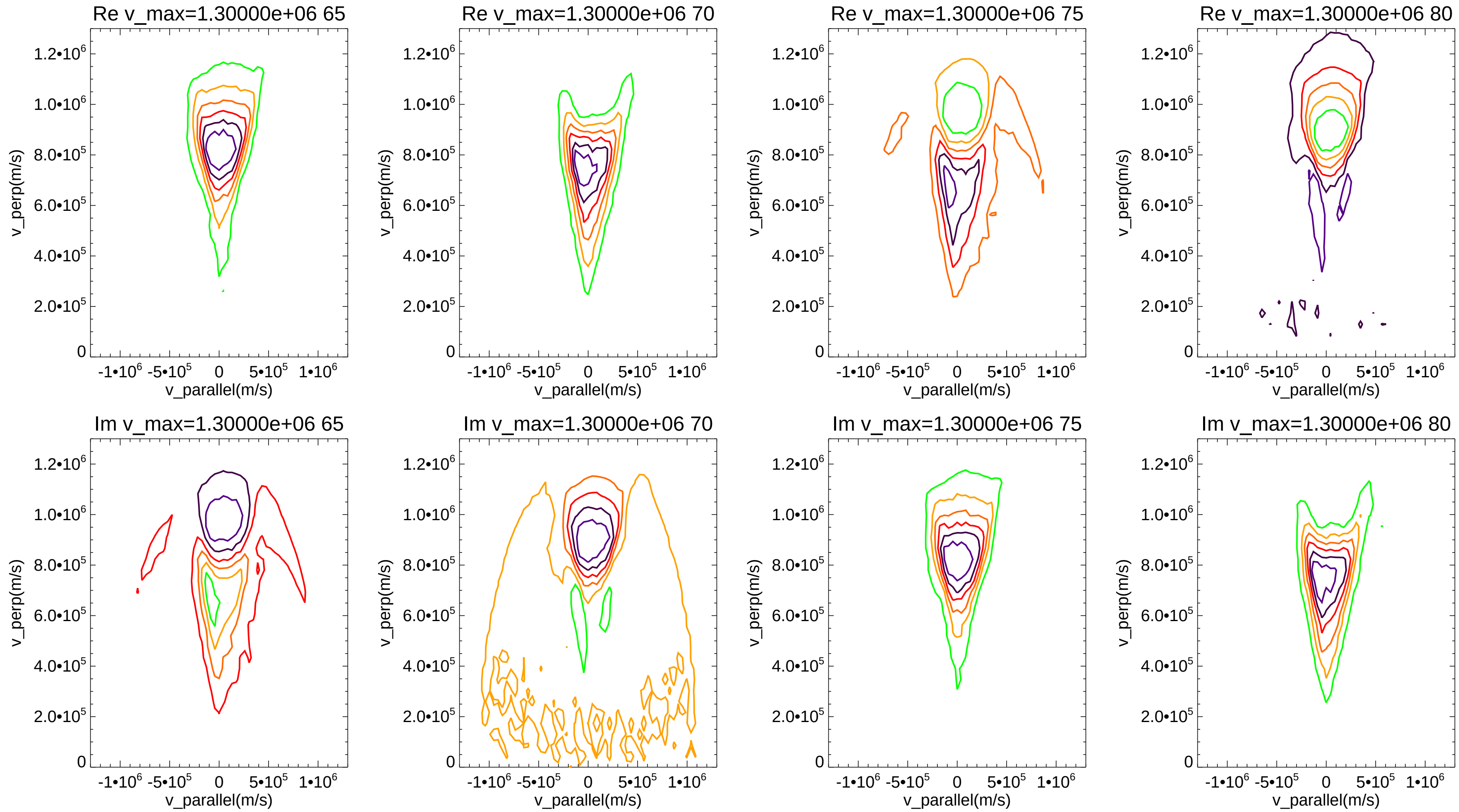


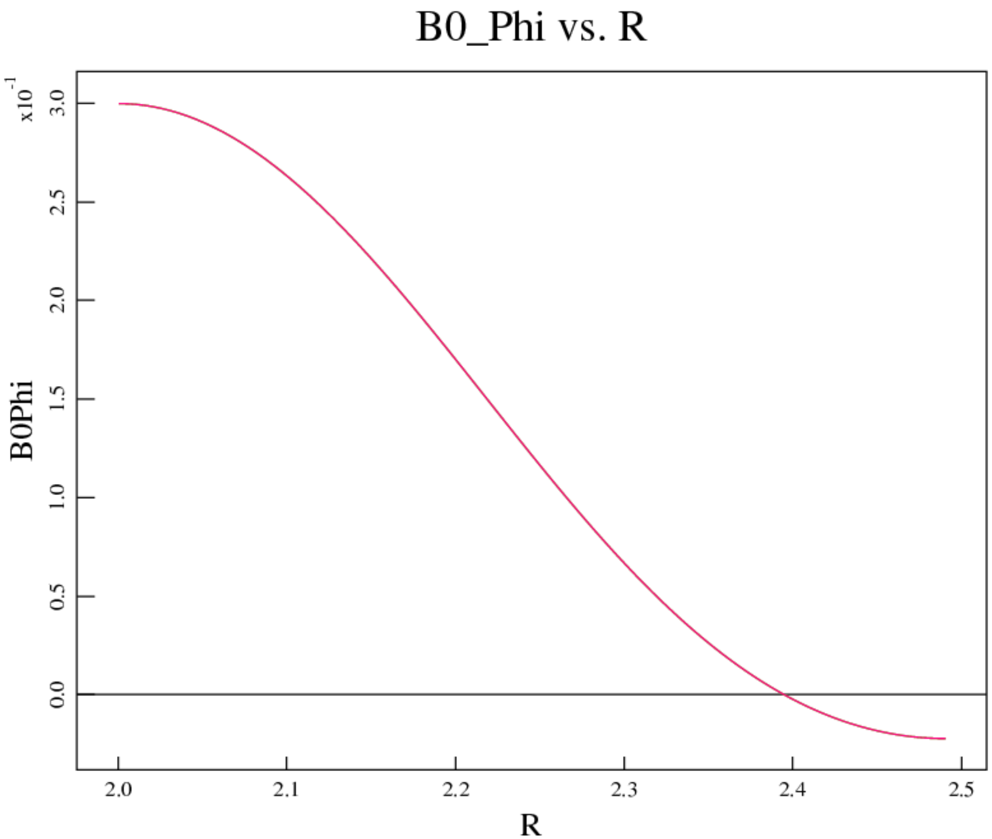
Figure 7: $n = 1$ projection of δf at $v_{max} = 1.3 \times 10^6$m/s

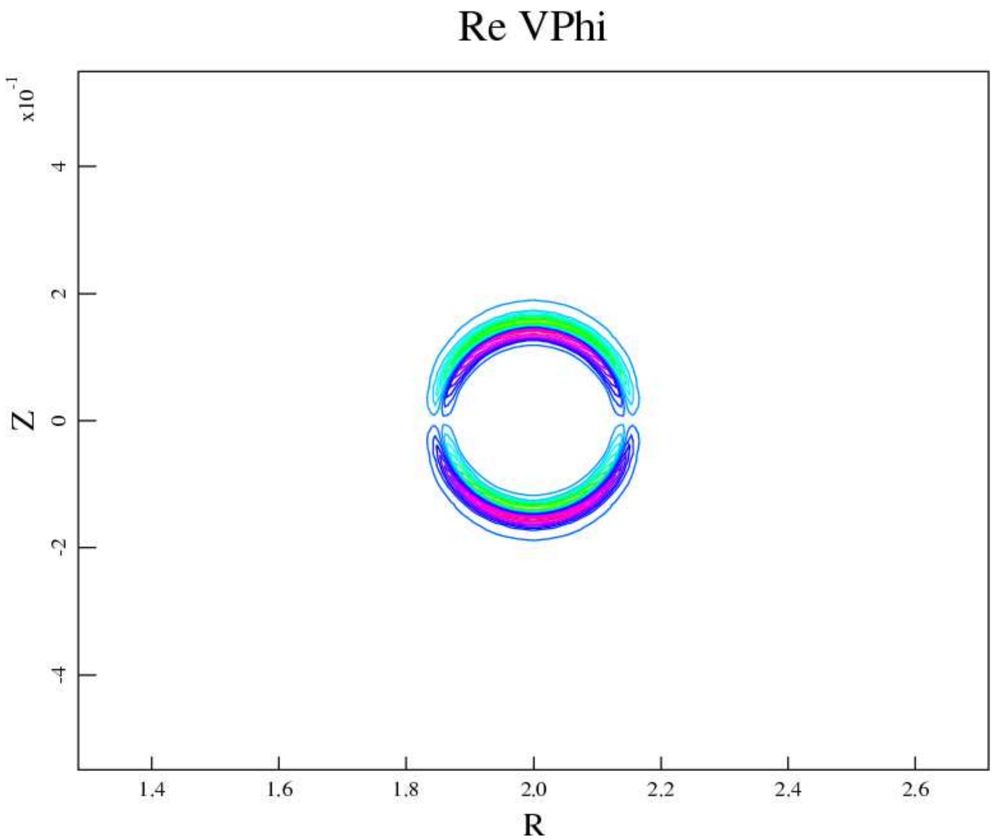
Linear Simulations of Tearing Modes in a RFP

- alpha model equilibrium $\nabla \times \mathbf{B} = \mu \mathbf{B} \quad \mu = 2\Theta \left[1 - \left(\frac{r}{a} \right)^{\alpha_0} \right]$
- parameters for straight cylinder

  $a = .5m,\ B_0 = .3T,\ \Theta = 1.75,\ \alpha_0 = 3,$

  $S = 1.e4,\ ka = 2,\ \gamma\tau_A = 1.3e-3$





δf and the Lorentz Equations

- Lorentz equation of motion

$$\dot{\mathbf{x}} = \mathbf{v}$$
$$\dot{\mathbf{v}} = \frac{q}{m}\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right)$$

- for full kinetic equations use[a]

$$f_0 = f(\mathbf{x}, v^2) + \frac{1}{\omega_c}\left(\mathbf{v} \cdot \mathbf{b} \times \nabla f\right)$$

- weight equation is

$$\dot{\delta f} = -\frac{\delta \mathbf{E} + \mathbf{v} \times \delta \mathbf{B}}{B} \cdot \mathbf{b} \times \nabla f - \frac{2q}{m} \delta \mathbf{E} \cdot \mathbf{v} \frac{\partial f}{\partial v^2}$$

- satisfying in that it is very similar to drift kinetic weight equation

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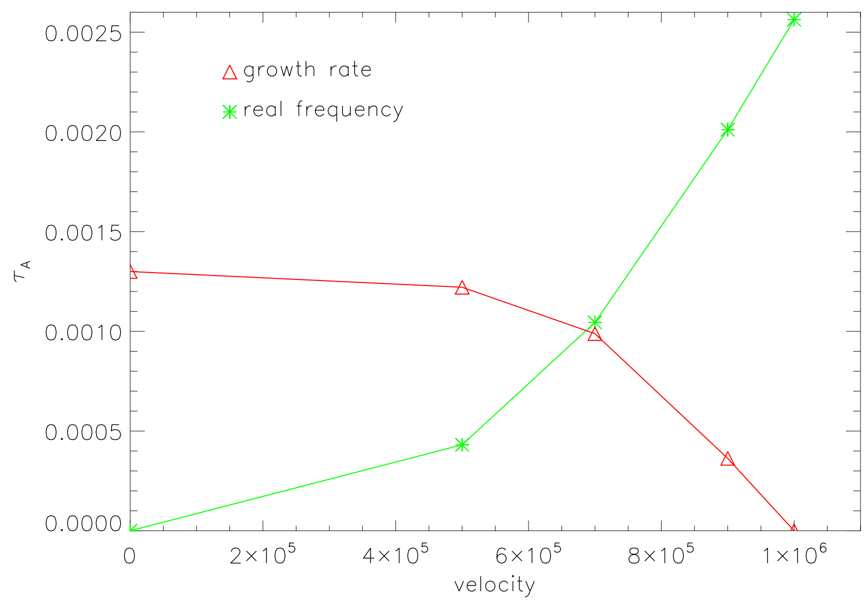
[a] M. N. Rosenbluth and N. Rostoker "Theoretical Structure of Plasma Equations", Physics of Fluids **2** 23 (1959)

FLR Stabilization of RFP Tearing Mode

- initialize with monoenergetic particles, only $\mathbf{v} \times \delta\mathbf{B}$ in weight equation
- use **only** perpendicular pressure for comparison with theory



- stabilization at $\rho_h \simeq 4cm$
- simulation sees real frequency - probably due to finite spread in velocity
- simplify problem to slab tearing mode - work is beginning

Some stuff at PSI Center

- mimetic GS solver for triangles and quads (MatLab)
  - generates equilibria suitable for NIMROD
  - uses NIMROD finite element nodes
  - good equilibria for HIT-SI, FRCs, (LDX soon)
  - see C. Akcay poster 2C-6 Tue 4:30-6:30
- migrating to NIMDevel (would like to set up mirror)
  - particles are running
  - porting experiments in process
- LDX simulations show promise - show movie
- optimization flags - turn off IEEE conformance

Future Work

- add more diagnostics
  - look at spatial dependence of phase space
  - synthetic diagnostics
- make particle domain decomp independent of fluids
- high order particles!
- apply to PSI Center experiments
- continue collaboration with MST
  - tentative extended visit in early June