Simulations of Interchange Modes in Spheromaks

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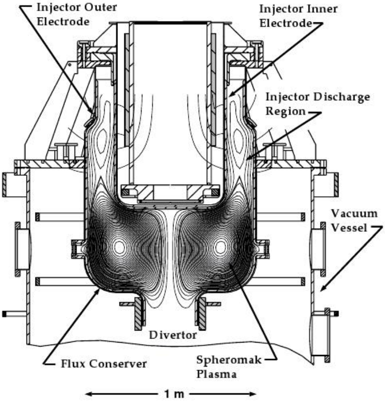
Outline

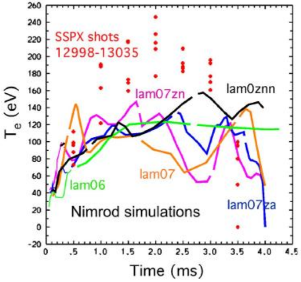
The Sustained Spheromak eXperiment at LLNL explored magnetic field generation and energy confinement.

- Coaxial current injection created and sustained the spheromak.
- Optimal performance was obtained by pulsing the guns to obtain a safety factor profile with $q_{axis} < 2/3$ and $q_{min} > 1/2$. [H.S. McLean et al, POP 13, 2006]
- Formation current amplifies poloidal flux, but prevents formation of flux surfaces.
- Reduced gun current after formation allows flux surface formation. [C.R. Sovinec et al, PRL 94,2005]



Resistive MHD simulations using NIMROD have reproduced many aspects of SSPX discharges but often underestimate peak temperatures.

- Highest temperatures were observed during the controlled decay phase when the guns are turned down.
- $T_e > 400eV$ were routinely observed in high performance SSPX discharges.
- Resistive MHD NIMROD simulations produce electron temperatures $\sim 40\%$ less than high performance discharges.
- MHD activity results in poor confinement and rapid cooling of the spheromak.



NIMROD run no.	Z_{eff}	kin-visc	n_e (10^{19} m^{-3})
lam06	1	1000	5
lam07	1	500	5
lam07za	2.3	500	5
lam07zn	2.3	500	3.5
lam07znn	2.3	1000	3.5

[Hooper et al, POP 15, 2008]

Interchange modes in spheromak equilibria are studied using NIMROD.

$$\rho\left(\partial_t\vec{V}+\vec{V}\cdot\nabla\vec{V}\right)=\vec{J}\times\vec{B}-\nabla P-\nabla\cdot\pi_i$$

$$\pi_i=-\rho\nu_{iso}W+\frac{P_i}{4\Omega_{ci}}\left[\hat{b}\times W\cdot\left(I+3\hat{b}\hat{b}\right)-\left(I+3\hat{b}\hat{b}\right)\cdot W\times\hat{b}\right]$$

$$W=\nabla\vec{V}+\nabla\vec{V}^T-2/3I\nabla\cdot\vec{V}$$

$$\partial_t n+\nabla\cdot(n\vec{V})=\nabla\cdot(D\nabla n-D_h\nabla\nabla^2 n)$$

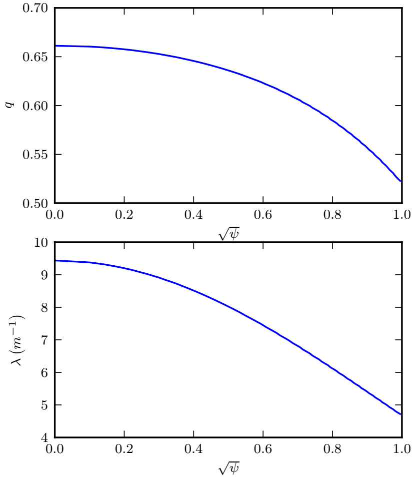
$$\frac{n}{\gamma-1}\left(\partial_t T_s+\vec{V}_s\cdot\nabla T_s\right)=-P_s\nabla\cdot\vec{V}_s-\nabla\cdot\vec{q}_s+Q_s$$

$$\partial_t\vec{B}=-\nabla\times\left[\eta\vec{J}-\vec{V}\times\vec{B}+\frac{1}{ne}\left(\vec{J}\times\vec{B}-T_e\nabla n\right)+\mu_0 d_e^2\partial_t\vec{J}\right]+k_{divb}\nabla\nabla\cdot\vec{B}$$

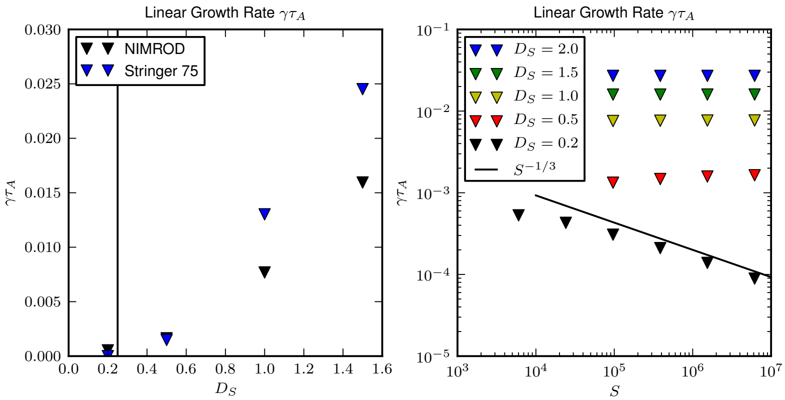
- Artificial particle diffusivity and magnetic divergence diffusions are used to provide numerical stability.
- Gyro-viscosity and two-fluid corrections to Ohm's law are included in two-fluid calculations.

Linear interchange modes are studied in a cylindrical pinch.

- Equilibria adapted from equilibria used in S. Jardin Nuclear Fusion 1982.
- Quadratic safety factor profile: $q = q_0 \left(1 - q_2 \left(\frac{r}{a}\right)^2\right)$.
  - Here $q(0) = 0.66$ and $q(a) = 0.52$.
- Constant D_S: $\mu_0 P' = \frac{\alpha r B_z^2}{8} \left(\frac{q'}{q}\right)^2$
  - $\alpha = 1 \Rightarrow D_s = \frac{1}{4}$.
- Results are for $m = 3$ $n = 5$ mode.

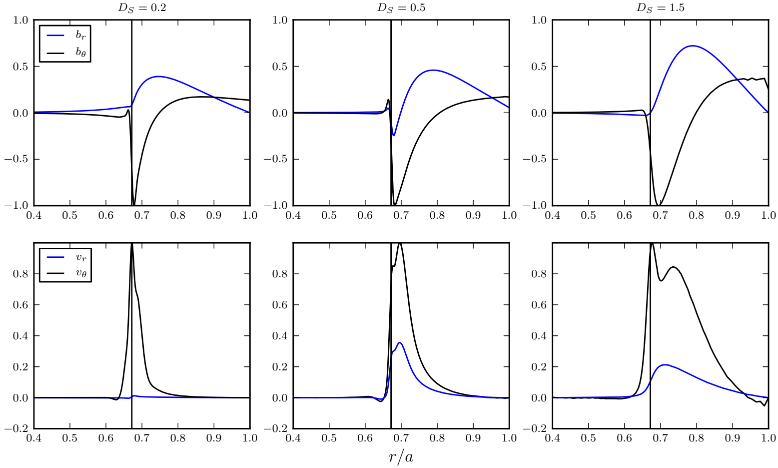


Linear MHD growth rates scale with D_s.

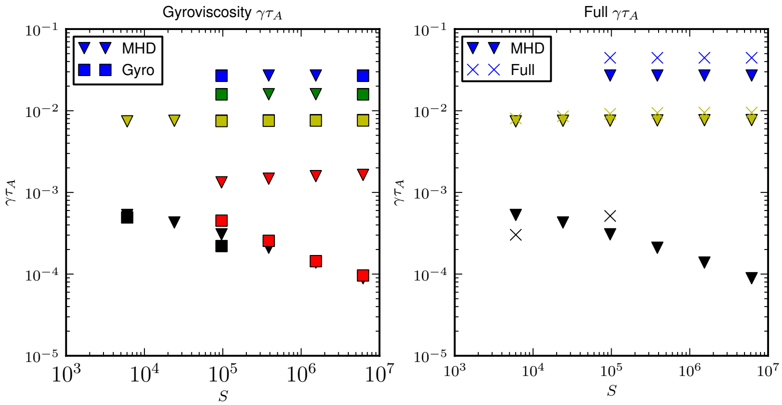


- Growth rates are insensitive to resistivity for $D_s > 0.5$.
- Resistive interchange scaling $S^{-1/3}$ is observed for $D_s = 0.2$
- Stringer 75 growth rates are for local ideal interchange.

Linear mode is concentrated outside of the rational surface.

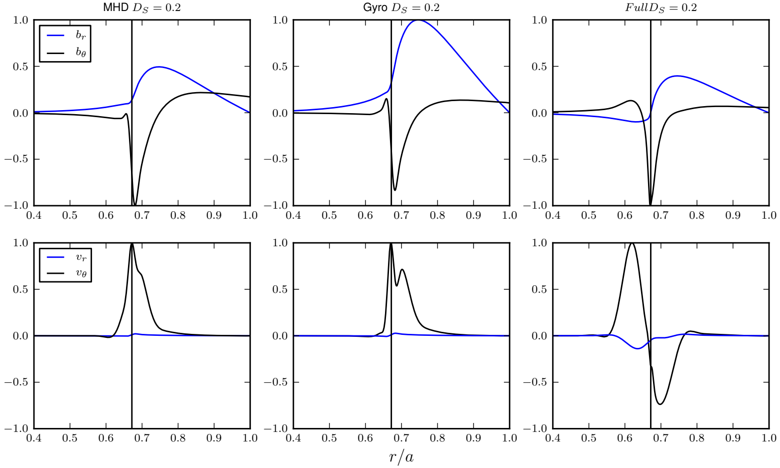


Two fluid effect increase the linear growth rates at large D_S.



- Gyroviscosity has a significant stabilizing effect for $D_S \leq 0.5$.
- Full model uses both gyroviscosity and the two fluid Ohm's Law.

Two fluid effects alter the parity of the mode



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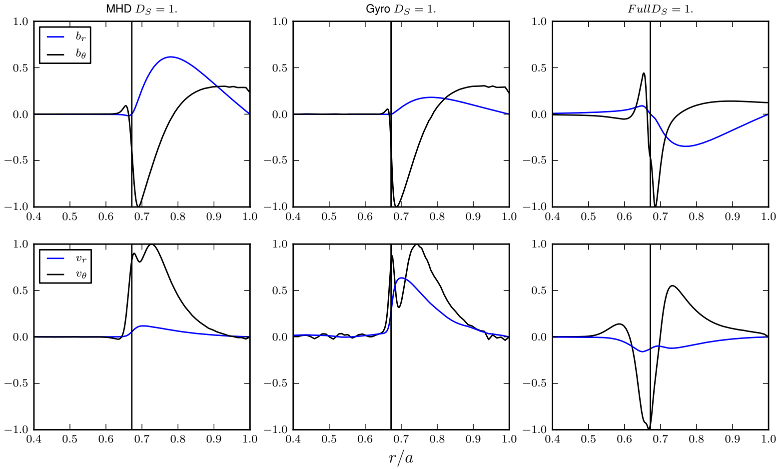
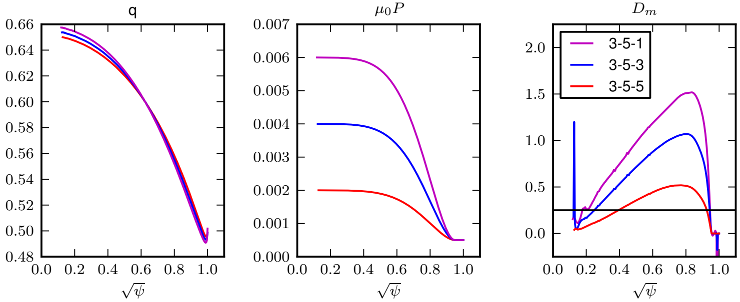


Grad-Shafranov equilibria for the decay phase are constructed using experimentally relevant parameters.

	SSPX shot 14590	Numerical Equilibrium
Ψ_{edge}	25.72 mWb	25.71 mWb
Ψ_{axis}	115.50 mWb	113.38 mWb
λ_{gun}	8.86 m^{-1}	8.86 m^{-1}
λ_{axis}	9.69 m^{-1}	10.19 m^{-1}
$T_e(edge)$	25 eV	25eV
$T_e(axis)$	325 eV	50-300 eV
n	$5 - 10x10^{19} m^{-3}$	$5x10^{19} m^{-3}$

- Numerical calculations use a rectangular mesh to approximate the SSPX flux conserver.
- Equilibria are generated using:
  - $F'(\psi) = a(1 + b\hat{\psi}^2)$
  - $P(\psi) = P_0 + P_1(1 - 4(\frac{\hat{\psi}}{0.9})^3 + 3(\frac{\hat{\psi}}{0.9})^4)$
  - F' is constant on the open field ($\hat{\psi} > 1$).
  - P is constant for ($\hat{\psi} > 0.9$).

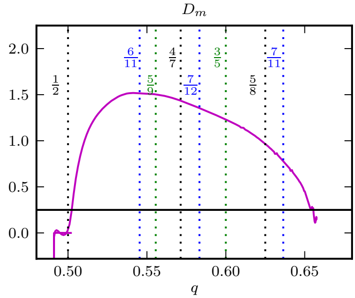
Equilibria are ideal Mercier unstable even at low pressures.

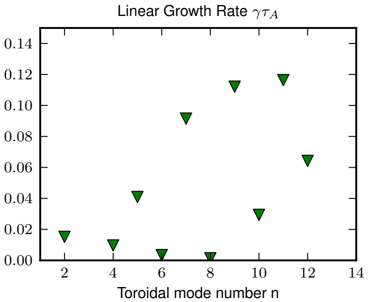


- $D_m > \frac{1}{4}$ is ideal interchange unstable.
- Noise in D_m near the magnetic axis is due to small numerical errors in calculating flux surface averaged quantities.

Equilibrium	T_e on axis
3 – 5 – 1	**300eV**
3 – 5 – 2	$250eV$
3 – 5 – 3	**200eV**
3 – 5 – 4	$150eV$
3 – 5 – 5	**100eV**
3 – 5 – 6	$50eV$

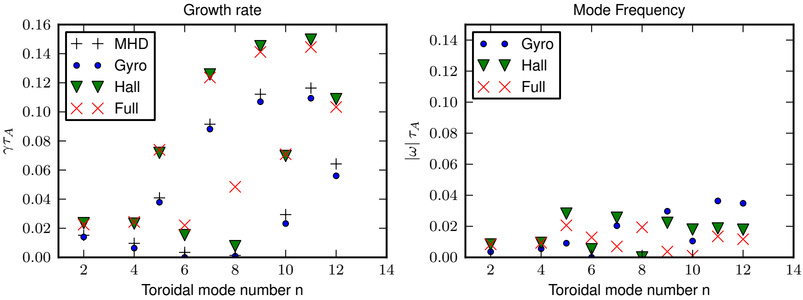
Large growth rates are calculated for the equilibria at experimentally relevant temperatures (300eV).





- The fastest growing modes $(6/11, 5/9, 4/7)$ have the largest D_m.
- Fastest growing modes are located near the cold edge (where q is near $1/2$).
- Numerically calculated growth rates are for $S = 8x10^5$.

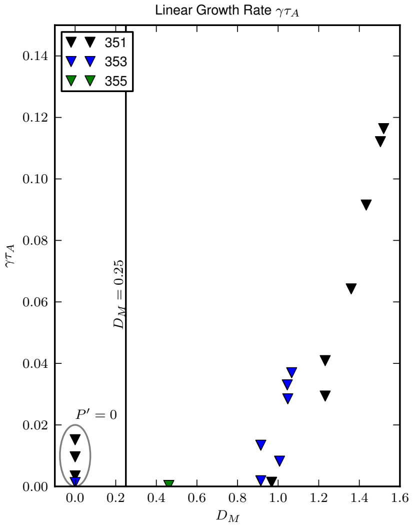
Two fluid effects increase the linear growth rate.



- The Hall term is destabilizing and increases the growth rate of the most unstable modes by 33%.
- Gyro-viscosity is weakly stabilizing and reduces the MHD growth rate.

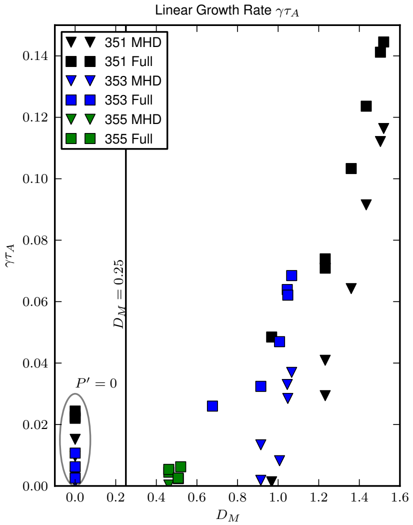
Linear MHD growth rates scale with D_M

- Linear growth rates are small ($\gamma\tau_A < 1\%$) for $D_M < 0.8$
  - Here growth rates strongly depend on resistivity.
- Theory predicts exponentially small ideal growth rates near marginal stability (Kulsrud 63, Stringer 75, Gupta 02).
- The modes with $D_M = 0$ are resonant on the $q = 0.5$ surface.
  - Here $P' = 0$ and $q' \approx 0$.



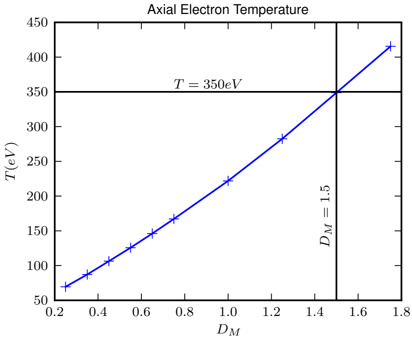
Two fluid effects are destabilizing for $D_M > 0.5$.

- The destabilization is due to the Hall term.
- Gyro-viscosity is weakly stabilizing.
- Theory predicts significant two fluid stabilization for $D_M < 0.5$ or large n.



Large D_M is required to reach experimentally relevant temperatures regardless of the specified pressure profile.

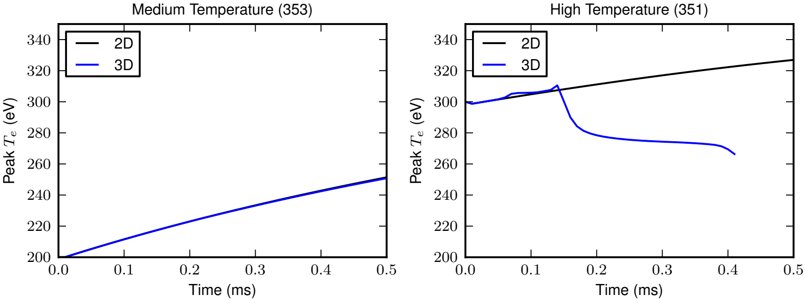
- Equilibria generated with an uniform D_M require $D_M \approx 1.5$ to reach $T_e = 350eV$.
  - The pressure profile is adjusted iteratively to produce a uniform D_M.
- $D_M = 1.5$ is ideal MHD unstable with $\gamma\tau_A > 10\%$
- Two fluid effects are destabilizing at large D_M.



Nonlinear simulations are performed to assess the impact of interchange modes on the evolution of decaying spheromaks

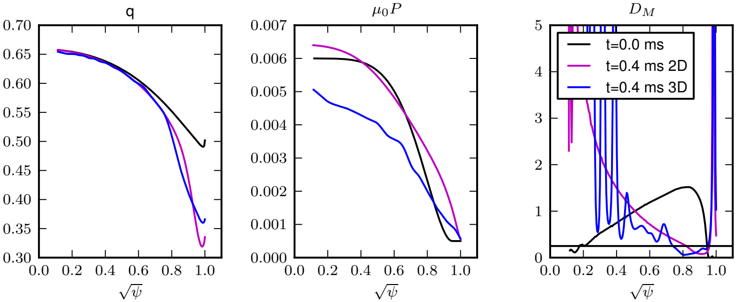
- Spheromaks are allowed to freely evolve and no sustainment current is applied.
- The temperature evolution is modeled self consistently, using a collisional transport model with temperature dependent resistivity, Ohmic heating, and anisotropic temperature dependent thermal conduction.
  - 3D anisotropic transport coefficients are calculated using Braginskii assuming a hydrogen plasma with $Z_{eff} = 1$.
- The high temperature (3-5-1) and moderate temperature (3-5-3) equilibria as initial conditions for the simulation.

Nonlinear interchange reduces the temperature in the highest temperature simulation.



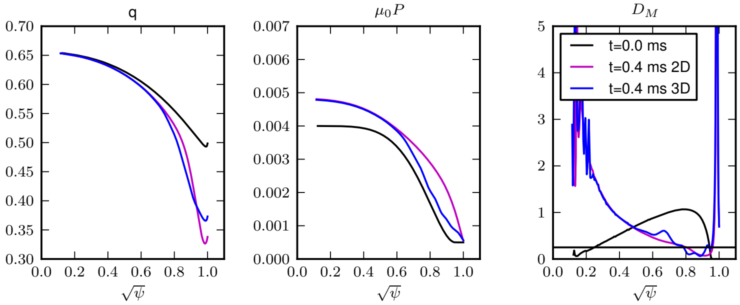
- 2D temperature evolution is due to the collisional transport model (no instabilities).
- Instabilities are 'weak' and do not affect the temperature evolution in 3D simulations started from the moderate temperature equilibrium.
- The 3D high temperature case undegoes an instability, which reduces the temperature.

Interchange dynamics relax the temperature for the high temperature spheromak (3-5-1).



- Poloidal currents on the cold resistive open fields diffuse inwards, lowering q and increasing the shear near the edge.
- Interchange dynamics relax the pressure gradient reducing D_m.
- Ohmic heating drives finite temperature gradients near the magnetic axis, increasing D_m.

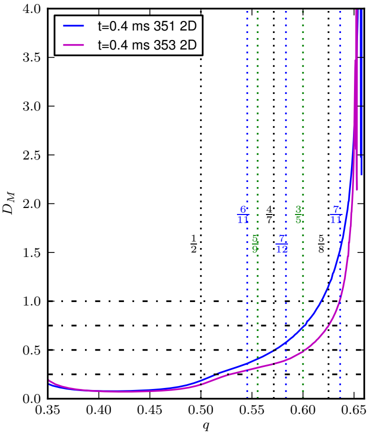
Interchange dynamics have a minimal effect on the evolution of the moderate temperature spheromak.



- Collisional heating and transport dominates the evolution of the temperature profile.
- 3D fluctuations relax the pressure gradient near the edge, but the peak pressure is unaffected.

Largest D_M values are limited to a few low order rational surfaces near the magnetic axis.

- The Mercier criteria is small ($D_M < 0.5$) for $q < 0.6$ for the moderate temperature case.
  - These modes have small linear growth rates.
- For the high temperature case, where interchange dynamics are significant, $D_M > 0.5$ for several low order rational surfaces.
- Low order modes are the most problematic experimentally.



Saturation of nonlinear interchange limits achievable pressure in MHD modeling.

- For the high temperature case, the enhanced transport overpowers Ohmic heating and the spheromak cools.
- For the moderate temperature case, the spheromak continues to heat up throughout the simulation.
- In both cases, the dynamics increase D_m near the magnetic axis, but reduces it for $\sqrt{\psi} > 0.5$.
- The interchange dynamics may have been missed in prior sspx simulations.
  - Unphysically large viscosities have been used in nonlinear simulations to provide numerical stability.
  - These large viscosities significantly dampen interchange modes.
  - In this study we deliberately used small viscosities.

Conclusions

- Linear analysis of SSPX relevant equilibria reveal that they are unstable to pressure driven modes.
  - The ideal Mercier stability criterion is violated even at low pressure.
  - $D_M \approx 1.5$ is needed to reach experimentally relevant T_e.
  - Linear MHD growth rates are $\gamma\tau_A > 10\%$ for $D_M \approx 1.5$.
  - Two fluid effects enhance the linear growth rates at large D_m.
- Nonlinearly, interchange dynamics limit the peak temperature.
  - Interchange dynamics cool simulations started from the highest temperature spheromak equilibrium.
  - Interchange dynamics have a minimal effect on simulations started from the moderate temperature spheromak equilibrium.
- Collisional transport creates a sharply peaked D_M profile.
  - Large D_M are limited to a small range of q.
  - For the moderate temperature simulation, $D_M < 0.5$ for most low order rational surfaces.