Energetic Particle Effects on Linear Tearing Mode Stability

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Motivation

- Trapped particle precession in toroidal configurations can be resonant with and have significant effects on slow growing resistive MHD instabilities.

- This resonance with the tearing mode is similar to the resonance commonly studied with ideal modes, in that the interaction is with the global eigenfunction, but is affected by the resistive layer which can have its own real frequency contribution.

- With the advent of full and extended MHD coupled to δf codes, we can study this effect both computationally and analytically.

- The underlying physics of the mode-particle interaction is the goal of our study. Computational studies have shown both damping and driving of the resistive mode in the presence of particle resonance. We are focused on AT-like equilibria with an m/n=2/1 tearing mode, where only damping has been observed.
Analytic Model: Energetic Particle Corrections to Resistive Wall Mode Growth Rates

- The eigenfrequencies are numerically approximated using a dispersion relation described by Hu & Betti.

\[
(\langle q_{Va}^{-2} \rangle - q_a^{-2}) \sum m \tilde{\psi}_m / h_m - h_m a \tilde{\psi}_m / m + \frac{3}{2} \frac{\beta}{\epsilon} \left( \frac{m+1}{h_{m+1}} \tilde{\psi}_{m+1} + \frac{m-1}{h_{m-1}} \tilde{\psi}_{m-1} \right) + \delta_{m,k} (\gamma \tau_w) \tilde{\psi}_k
\]

- Where \( K.T. \) consists of a series of rigorously calculated energetic particle pressure contributions that take into account orbital mechanics, poloidal harmonics, and phase space integrals for the ion population.
Analytic Model: Extension of Hu & Betti's Work

- Hu & Betti considered particle effects on the RWM, resulting in an energy eigenvalue correction dependent on scalar trapped particle pressure, the displacement eigenfunction, and field line curvature.

\[
\delta W_K \equiv \frac{1}{2} \sum_{j=i,e} \int d\xi_\perp \xi_\perp^* \cdot \kappa \tilde{p}_j^K
\]

- The resulting analytics produced a stability map for increasing \( \beta \) that gave the criterion for full suppression of the RWM from the kinetic contribution to \( \delta W \)

- We are taking an analogous approach in an effort to give the stability criterion necessary for damping and stabilizing to occur in the presence of energetic particles for the 2/1 tearing mode.
Analytic Model: Tearing Modes in a Step Function Equilibrium

- Using an analysis similar to Brennan & Finn '14, we obtain the dispersion relation for a simple step function equilibrium profile with a tearing layer located between the jumps of pressure and toroidal current density.

- This yields a $\Delta'$ dependent on the amplitude of the equilibrium pressure gradient, and can be modified by the presence of a scalar hot ion pressure.
The equilibrium pressure used in this study depends exponentially on the value of magnetic flux, giving it a direct map to the energetic particle spatial load.
Computational Model: Method of Simulations and Calculations

- Using NIMROD, we simulate the time advancing of energetic particles in the presence of linear tearing instabilities.
- The equilibria used were calculated using Corsica/TEQ.
- Each equilibrium is used as a basis for the simulations with and without energetic particle effects added.
- To the left is a depiction of the initially isotropic equilibrium distribution of energetic particles, with slight shift due to helicity.
Computational Model: Hybrid Equations Used in $\delta f$ Simulations

\[
P(\psi) = P_0 \exp(-4\psi) \\
E = -V \times B + \eta J \\
J_0 \times B_0 = \nabla P + \nabla p_{h0} \\
\frac{\partial T}{\partial t} + n V \cdot \nabla T + (\Gamma - 1) n T \nabla \cdot V = - (\Gamma - 1) \nabla \cdot q + (\Gamma - 1) Q \\
f_0 = \frac{C_0 \exp \left( \frac{P_\xi}{\mu_\rho n} \right)}{(\epsilon^{3/2} + \epsilon_c^{3/2})} \\
p_h = p_{h0} + m_h \int (v - V_h)(v - V_h) \delta f_0 d^3 v \\
p_\perp = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \\
\delta p_h = \begin{pmatrix} 0 & p_\perp & 0 \\ 0 & 0 & p_\parallel \end{pmatrix} \\
p_\parallel = \int v_\parallel^2 \delta f d^3 v \\
p_\perp = \int \mu B \delta f d^3 v \\
\delta f = -\delta v \cdot \nabla f_0 - e v_0 \cdot \delta E \partial_\epsilon f_0 \\
v_0 = v_\parallel \hat{b} + \frac{m}{e B^3} \left( v_\parallel^2 + \frac{v_\perp^2}{2} \right) (B \times \nabla B) + \frac{\mu_0 m v_\parallel^2}{e B^2} J_\perp \\
\delta v = \frac{\delta E \times B}{B^2} + v_\parallel \frac{\delta B}{B} \\
\]
Computational Model: Assumptions for Energetic Particle Model

- Energetic particles are modeled using a slowing down distribution.
- The trapped particle density is assumed to be much lower than that of the bulk plasma. However, the $\beta$ values are of the same order.
- In this model, the initial energetic particle equilibrium pressure tensor is assumed to be a scalar quantity to satisfy a scalar pressure force balance, allowing for only equilibrium particle distribution functions that are isotropic in velocity space.
- The evolution equation is obtained using a drift kinetic model reduced from 6D to 5D in the high $B$ limit, FLR effects are not included.
Previous work developed a stability map of the linear 2/1 mode in $\beta_N/4l_i$ - $S$ space.

In these equilibria, instability persists to low $\beta$ as a current driven instability, future cases will have a stability boundary in $\beta$. 
Observed Frequencies of Linear Resistive Modes

- Changes in growth rates show that the onset of resistive modes is sensitive to the contribution from energetic particle pressure as measured by $\beta_{\text{frac}}$. 
Observed Motion of $B_r$ Eigenfunction
Future Goals

- Previous work has shown that the kinetic contribution to $\delta W$ depends on the anisotropic trapped particle pressure tensor. The reduced calculation of this energy eigenvalue correction for tearing instabilities could provide insight into the nature of mode-particle interactions, and give an energetic particle effect on $\Delta'$ analogous to its effects $\delta W_K$ in the RWM model of Hu & Betti.

- Find an equilibrium with an observed stability boundary and find the sensitivity to changes in the trapped particle pressure contribution.

- Run cases in the lower S regime to obtain a larger resistive unstable region.

- Continue running cases with an observed tearing mode for much longer times to obtain the low-magnitude real frequency.


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