Parallel Electron Stress (Fluid)

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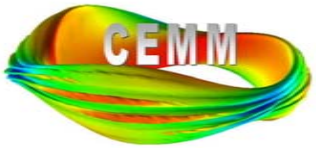
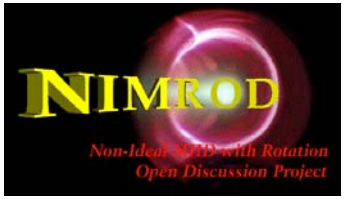
NIMROD Team Meeting

pre-Sherwood

Annapolis Maryland, April 21-22, 2007





Outline

- Introduction
- Numerical advance
- Formulation of electron stress
- Linear implementation
- Tests
- Conclusions

Introduction: Like other forces on the electron fluid, the force from parallel stress appears as an electric-field contribution in Ohm's law.

- In NIMROD, we do not use an expansion for **E**.
- Like other electron forces, the force from electron stress needs to be computed locally for the advance of **B**.

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left[\eta \mathbf{J} - \mathbf{V} \times \mathbf{B} + \frac{1}{ne}\left(\mathbf{J} \times \mathbf{B} - T_e \nabla n - \nabla \cdot \Pi_e \right) \right] + \kappa_{divb} \nabla \nabla \cdot \mathbf{B}$$ Faraday's / Ohm's law

$$\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$$ low-ω Ampere's law

$$\rho \left(\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \Pi_i(\mathbf{V})$$ flow evolution

$$\frac{\partial n}{\partial t} + \nabla \cdot (n \mathbf{V}) = \nabla \cdot D \nabla n$$ particle continuity with artificial diffusivity

$$\frac{n}{\gamma - 1}\left(\frac{\partial T_\alpha}{\partial t} + \mathbf{V}_\alpha \cdot \nabla T_\alpha \right) = -p_\alpha \nabla \cdot \mathbf{V}_\alpha - \nabla \cdot \mathbf{q}_\alpha + Q_\alpha$$ temperature evolution

- The magnetic divergence term and particle diffusion term are used for numerical purposes.

The parallel electron stress will provide some neoclassical effects and (hopefully) smoothing of electron spatial scales.

- An implicit computation of the Braginskii fluid stress will be solved in the magnetic advance.
  - The fluid stress with arbitrary coefficient will allow direct modeling of Pfirsch-Schlüter regime bootstrap current.
  - The viscosity coefficient can be scaled to reproduce neoclassical effects for other plasmas.
  - The implicit operator will be used to stabilize nonlocal kinetic stress computations. [Held, PoP **11**, 2419 (2004)]
- The collisional form of the parallel stress is:

$$\Pi_e = -3\eta_0\left(\hat{\mathbf{b}}\cdot\nabla\mathbf{V}_e\cdot\hat{\mathbf{b}} - \frac{1}{3}\nabla\cdot\mathbf{V}_e\right)\left(\hat{\mathbf{b}}\hat{\mathbf{b}} - \frac{1}{3}\mathbf{I}\right)$$

with $\mathbf{V}_e = \mathbf{V} - \left(\frac{1}{1+Zm_e/m_i}\right)\frac{1}{ne}\mathbf{J} \cong \mathbf{V} - \frac{1}{ne}\mathbf{J}$

Numerical Algorithm (Review): NIMROD uses an implicit leapfrog method to advance the two-fluid equations.

- The number density appearing in the advances of T and **B** is time-averaged, as is the T appearing in the **B** advance.
- A Newton-like computation is used for momentum advection and the Hall term.

$$m_i n^{j+1/2}\left(\frac{\Delta \mathbf{V}}{\Delta t}+\frac{1}{2}\mathbf{V}^j\cdot\nabla\Delta\mathbf{V}+\frac{1}{2}\Delta\mathbf{V}\cdot\nabla\mathbf{V}^j\right)-\Delta t L^{j+1/2}(\Delta\mathbf{V})+\nabla\cdot\Pi_i(\Delta\mathbf{V})=\mathbf{J}^{j+1/2}\times\mathbf{B}^{j+1/2}$$
$$-m_i n^{j+1/2}\mathbf{V}^j\cdot\nabla\mathbf{V}^j-\nabla\left[n^{j+1/2}\left(T_e^{j+1/2}+Z^{-1}T_i^{j+1/2}\right)\right]-\nabla\cdot\Pi_i\left(\mathbf{V}^j\right)$$

$$\frac{\Delta n}{\Delta t}+\frac{1}{2}\nabla\cdot\left(\mathbf{V}^{j+1}\cdot\Delta n-D\nabla\Delta n\right)=-\nabla\cdot\left(\mathbf{V}^{j+1}\cdot n^{j+1/2}-D\nabla n^{j+1/2}\right)$$

$$\frac{3n}{2}\left(\frac{\Delta T_\alpha}{\Delta t}+\frac{1}{2}\mathbf{V}_\alpha^{j+1}\cdot\nabla\Delta T_\alpha\right)+\frac{n}{2}\Delta T_\alpha\nabla\cdot\mathbf{V}_\alpha^{j+1}+\frac{1}{2}\nabla\cdot\mathbf{q}_\alpha(\Delta T_\alpha)$$
$$=-\frac{3n}{2}\mathbf{V}_\alpha^{j+1}\cdot\nabla T_\alpha^{j+1/2}-nT_\alpha^{j+1/2}\nabla\cdot\mathbf{V}_\alpha^{j+1}-\nabla\cdot\mathbf{q}_\alpha\left(T_\alpha^{j+1/2}\right)+Q_\alpha^{j+1/2}$$

Implicit $\nabla\cdot\Pi_e$ here.

$$\frac{\Delta\mathbf{B}}{\Delta t}-\frac{1}{2}\nabla\times\left(\mathbf{V}^{j+1}\times\Delta\mathbf{B}\right)+\frac{1}{2}\nabla\times\frac{1}{ne}\left(\mathbf{J}^{j+1/2}\times\Delta\mathbf{B}+\Delta\mathbf{J}\times\mathbf{B}^{j+1/2}\right)+\frac{1}{2}\nabla\times\eta\Delta\mathbf{J}$$
$$=-\nabla\times\left[\frac{1}{ne}\left(\mathbf{J}^{j+1/2}\times\mathbf{B}^{j+1/2}-T_e\nabla n\right)-\mathbf{V}^{j+1}\times\mathbf{B}^{j+1/2}+\eta\mathbf{J}^{j+1/2}\right]$$

Explicit $\nabla\cdot\Pi_e$ here.

- The temperature advance is repeated for **B**- or T-dependent thermal conduction.

Implementation of parallel electron stress with basis functions having C^0 continuity requires an auxiliary field.

- NIMROD's polynomial spaces are sufficiently general to represent functions with square-integrable first derivatives.

- Since electron velocity depends on the curl of **B**, adding electron stress to Ohm's law implies that $d\mathbf{B}/dt$ depends on fourth-order derivatives of **B**.

- Just like ODE systems, introducing auxiliary fields is a way to have fewer derivatives appearing explicitly.

- With θ-centering in the time advance, we define $\nu \equiv \sqrt{3\Delta t \eta_0 \theta}\,/\,e\sqrt{\mu_0}$ and

$$f - \nu\left[\hat{\mathbf{b}}\cdot\nabla\left(\frac{\nabla\times\Delta\mathbf{B}}{n}\right)\cdot\hat{\mathbf{b}} - \frac{1}{3}\nabla\cdot\left(\frac{\nabla\times\Delta\mathbf{B}}{n}\right)\right] \equiv -\sqrt{\frac{3\eta_0\mu_0\Delta t}{\theta}}\left\{\hat{\mathbf{b}}\cdot\nabla\left(\mathbf{V} - \frac{1}{ne}\mathbf{J}^{\mathrm{n}}\right)\cdot\hat{\mathbf{b}} - \frac{1}{3}\nabla\cdot\left(\mathbf{V} - \frac{1}{ne}\mathbf{J}^{\mathrm{n}}\right)\right\}$$

The auxiliary field f is essentially the parallel component of the electron stress, and the ν-coefficient is defined to make implicit terms symmetric.

$$\Delta\mathbf{B} - \nabla\times\left\{\frac{1}{n}\nabla\cdot\left[\nu\, f\left(\hat{\mathbf{b}}\hat{\mathbf{b}} - \frac{1}{3}\mathbf{I}\right)\right]\right\} = -\Delta t\nabla\times\mathbf{E}_{other}$$

We use the weak form of the system for $\Delta\mathbf{B}$ and f.

- After dotting the $\Delta\mathbf{B}$ equation with the vector test functions $\mathbf{A}$ (three of them per node) and multiplying the f equation by the scalar test function g, we use vector identities and integrate over the volume to arrive at:

$$\int_R d\mathbf{x}\left\{\mathbf{A}\cdot\Delta\mathbf{B}-\frac{\nabla\times\mathbf{A}}{n}\cdot\left[\hat{\mathbf{b}}\hat{\mathbf{b}}\cdot\nabla(\nu f)+\nu f\left(\hat{\mathbf{b}}\cdot\nabla\hat{\mathbf{b}}+\hat{\mathbf{b}}\nabla\cdot\hat{\mathbf{b}}\right)-\frac{1}{3}\nabla(\nu f)\right]\right\}$$

$$+\int_{\partial R} d\mathbf{S}\cdot\left\{\mathbf{A}\times\left[\frac{1}{n}\nabla\cdot\nu f\left(\hat{\mathbf{b}}\hat{\mathbf{b}}-\frac{1}{3}\mathbf{I}\right)\right]\right\}=-\Delta t\int_R d\mathbf{x}\,\mathbf{E}_{other}\cdot\nabla\times\mathbf{A}+\Delta t\int_{\partial R} d\mathbf{S}\cdot\mathbf{A}\times\mathbf{E}_{other}$$

$$\int_R d\mathbf{x}\left\{gf+\frac{\nabla\times\Delta\mathbf{B}}{n}\cdot\left[\hat{\mathbf{b}}\hat{\mathbf{b}}\cdot\nabla(g\nu)+g\nu\left(\hat{\mathbf{b}}\cdot\nabla\hat{\mathbf{b}}+\hat{\mathbf{b}}\nabla\cdot\hat{\mathbf{b}}\right)-\frac{1}{3}\nabla(g\nu)\right]\right\}$$

$$-\int_{\partial R} d\mathbf{S}\cdot\left\{g\nu\left(\frac{\nabla\times\Delta\mathbf{B}}{n}\right)\cdot\hat{\mathbf{b}}\hat{\mathbf{b}}-\frac{g\nu}{3}\left(\frac{\nabla\times\Delta\mathbf{B}}{n}\right)\right\}$$

$$=\int_R d\mathbf{x}\left\{\frac{\mu_0 e}{\theta}\left(\mathbf{V}-\frac{1}{ne}\mathbf{J}^{\mathrm{n}}\right)\cdot\left[\hat{\mathbf{b}}\hat{\mathbf{b}}\cdot\nabla(g\nu)+g\nu\left(\hat{\mathbf{b}}\cdot\nabla\hat{\mathbf{b}}+\hat{\mathbf{b}}\nabla\cdot\hat{\mathbf{b}}\right)-\frac{1}{3}\nabla(g\nu)\right]\right\}$$

$$-\int_{\partial R} d\mathbf{S}\cdot\left\{\frac{\mu_0 eg\nu}{\theta}\left(\mathbf{V}-\frac{1}{ne}\mathbf{J}^{\mathrm{n}}\right)\cdot\hat{\mathbf{b}}\hat{\mathbf{b}}-\frac{\mu_0 eg\nu}{3\theta}\left(\mathbf{V}-\frac{1}{ne}\mathbf{J}^{\mathrm{n}}\right)\right\}$$

- For the curvature terms, use $\hat{\mathbf{b}}\cdot\nabla\hat{\mathbf{b}}+\hat{\mathbf{b}}\nabla\cdot\hat{\mathbf{b}}=\frac{1}{B^2}\mathbf{B}\cdot\nabla\mathbf{B}-\frac{2}{B^4}\mathbf{B}[\mathbf{B}\cdot(\nabla\mathbf{B})\cdot\mathbf{B}]$
- Surface terms in the scalar equation are not automatically zero.

The linear parts of the electron stress for spatially uniform η_0 have been implemented in a development branch.

- Linear contributions can be separated into contributions from perturbed electron flow and from equilibrium electron flow.
  - For linear computations, there are no equilibrium flow terms on the lhs.
  - There are a number of perturbed curvature terms, including terms from the perturbations of B^2 and B^4.
- Boundary condition routines have been modified for 4-vector systems.
- Not all of the 4-vector system solves for the change of a field.
  - The usual step of multiplying the matrix by the existing solution and adding the product to the rhs is not appropriate.

The simplest test of the parallel electron stress is the damping of whistler waves in a doubly periodic box.

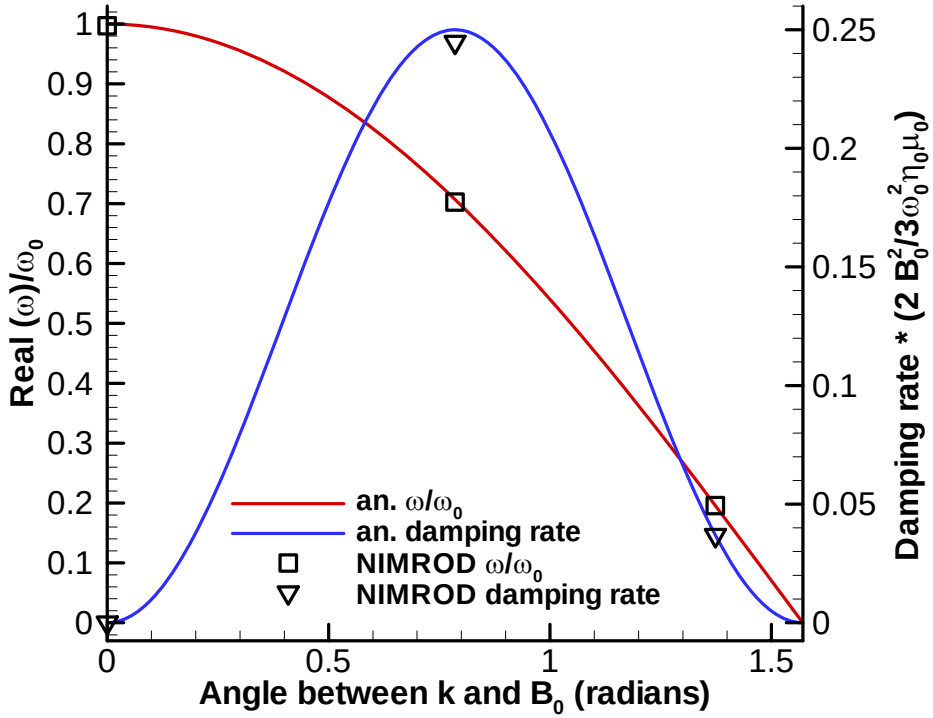
• The NIMROD implementation presently has all of the linear terms for Π_e when η_0 is spatially uniform.

• Whistler tests for NIMROD have **k** and $\mathbf{B}_0$ in various orientations in the finite element plane and in the periodic (Fourier) coordinate.

• Electron stress alone in a uniform background and $\mathbf{B}\sim e^{i\mathbf{k}.\mathbf{x}-i\omega t}$ has

$$-i\omega\mathbf{B} \sim -i\mathbf{k}\times\left\{-i\mathbf{k}\cdot\left[-\hat{\mathbf{b}}\cdot i\mathbf{k}\left(-i\mathbf{k}\times\mathbf{B}\right)\cdot\hat{\mathbf{b}}\right]\right\}$$

$$\sim -\left(\mathbf{k}\cdot\hat{\mathbf{b}}\right)^2\left(\mathbf{k}\times\hat{\mathbf{b}}\right)\left(\mathbf{k}\times\hat{\mathbf{b}}\right)\cdot\mathbf{B}$$



Results obtained with an 8×8 mesh of bicubic elements and $\omega_r\Delta t$~0.1-0.2.

• The whistler dispersion relation is $\omega^2 + \left(\omega_0^2\frac{k_\parallel^2}{k^2}\right)\left(i\frac{3\eta_0\mu_0 k_\perp^2}{B_0^2k^2}\omega - 1\right) = 0\,,\ \omega_0^2 \equiv \Omega_i^2 d_i^4 k^4$

and for weak damping, $\omega_r^2 \cong \omega_0^2\frac{k_\parallel^2}{k^2}$, $\omega_i \cong -\omega_0^2\left(\frac{3\eta_0\mu_0 k_\parallel^2 k_\perp^2}{2B_0^2k^4}\right)$

Our first results on reconnection with parallel electron viscosity show little effect on the growth rate and eigenfunction. This is expected due to the resonance condition together with the anisotropic nature of the stress.

- For the parameters of Case B in the table and a parallel damping rate of 8.0 τ_a^{-1} computed from $3\eta_0 k_y^4 B_{0_y}^2 \big/ n^2 e^2 \mu_0 B_0^2$ with η_0 set to 3×10^{-3} (MKS $\rightarrow$ $Jm^{-3}s^{-1}$), the growth rate of 5.12×10^{-3} τ_a^{-1} increases by less than 10% to 5.62×10^{-3} τ_a^{-1}.

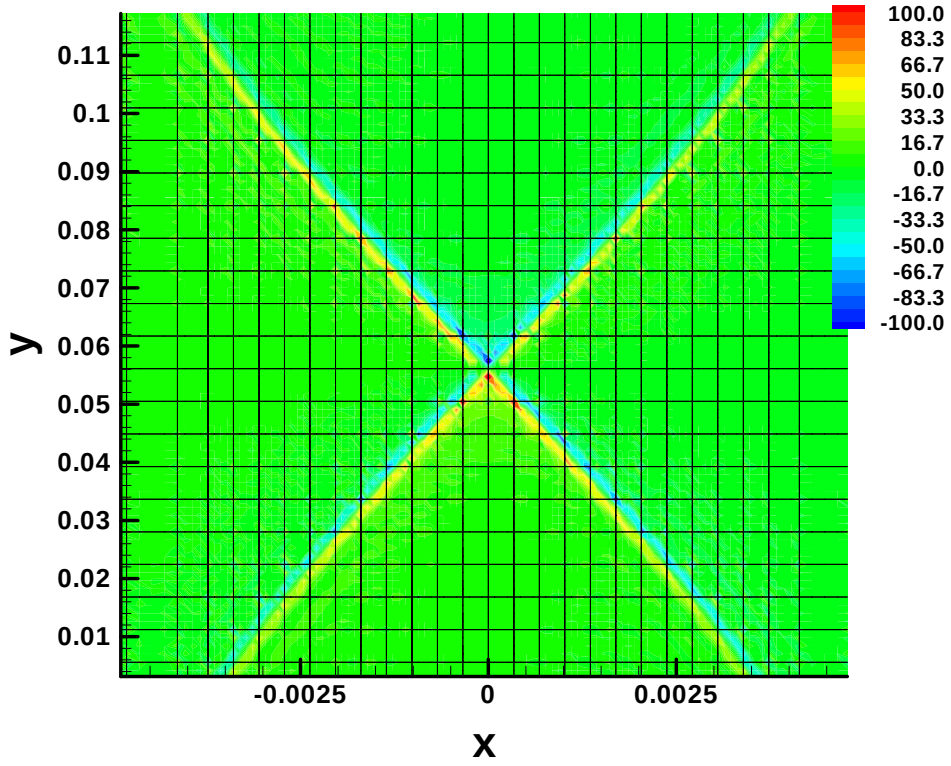
- Near the tearing layer, $k_{||}^2$ is far smaller than at the wall, and the wall conditions have been used in the damping rate estimate of 8.0 τ_a^{-1}.

- There are only very slight changes in the eigenfunction just outside the tearing region.

- Computationally, a small level of thermal conduction has been used with the electron viscosity to avoid slowly growing noise.

- We expect that this term will have a greater effect in nonlinear calculations, where macroscopic changes in $\hat{\mathbf{b}}$ occur.

The nonlinear version is expected to help smooth current generated along separatrices by the Hall effect.



A good candidate for help. This is J_y from a nonlinear two-fluid tearing calculation.

Conclusions

- The proposed formulation appears to be working for linear calculations.
- The cost of the 4-vector system may be an additional 30-50%.
    - Additional local computational work is performed, so the calculations may scale a little better in parallel.
- Development for the nonlinear terms is planned for the summer.
- For more information on the formulation, see www.cptc.wisc.edu/sovinec_research/notes/e_viscosity2.pdf .