Progress on the MHD closure with kinetic ions and drift kinetic electrons

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Motivations

- In certain problems, such as Tokamak edge ETG and weak guide field (or no guide field) magnetic reconnection, the gyro-kinetic orderings are not valid. Therefore the current gyro-kinetic model should be extended.
- Also, for GEM turbulence code on small devices like NSTX, the timestep constraint is $\Omega_i \Delta t < 0.2$.
- If electrons and ions are treated as fluid and full-kinetic particles respectively, this simple hybrid model could include the kinetic ion physics and capture MHD physics in a natural way. Meanwhile, realistic electron ion mass ratio could be preserved. It will also serve as a good check for the validity of gyro-kinetic model in the edge.
- We are using the GEM code as a test bed for the model and algorithm. To include kinetic electron effects, drift-kinetic and gyro-kinetic electrons could be added.

Lorentz ion and fluid electron model

- Lorentz force ions:

$$\frac{d\boldsymbol{v}_i}{dt} = \frac{q}{m_i}(\boldsymbol{E} + \boldsymbol{v}_i \times \boldsymbol{B})$$
$$\frac{d\boldsymbol{x}_i}{dt} = \boldsymbol{v}_i$$

- Isothermal fluid electrons as a simple test:

$$\delta P_e = \gamma \delta n_e T_e$$

I am working on drift-kinetic and will add gyrokinetic electrons.

- Ampere's law:

$$\bigtriangledown \times \boldsymbol{B} = \mu_0 e \left(n_i \boldsymbol{u}_i - n_e \boldsymbol{u}_e \right)$$

- Faraday's law

$$\bigtriangledown \times \boldsymbol{E} = -\frac{\partial \ \boldsymbol{B}}{\partial t}$$

Generalized Ohm's law

- Starting from the electron momentum equation:

$$en_e(\boldsymbol{E} + \boldsymbol{u}_e \times \boldsymbol{B}) = en_e\eta\, \boldsymbol{j} - \nabla \cdot \boldsymbol{\Pi}_e - m_e \frac{\partial(n_e \boldsymbol{u}_e)}{\partial t}$$

  where $\boldsymbol{\Pi}_e = \int f_e m_e \boldsymbol{v}\boldsymbol{v} d\boldsymbol{v}$.

- Substitute in Ampere's law $\boldsymbol{j} = e(n_i \boldsymbol{u}_i - n_e \boldsymbol{u}_e) = \frac{1}{\mu_0} \nabla \times \boldsymbol{B}$, we could rewrite the above equation as

$$en_e \boldsymbol{E} = -\boldsymbol{j}_i \times \boldsymbol{B} + \frac{1}{\mu_0} (\nabla \times \boldsymbol{B}) \times \boldsymbol{B} + \frac{en_e}{\mu_0} \eta\, (\nabla \times \boldsymbol{B}) - \nabla \cdot \boldsymbol{\Pi}_e - m_e \frac{\partial(n_e \boldsymbol{u}_e)}{\partial t}$$

  where $\boldsymbol{j}_i = en_i \boldsymbol{u}_i$

- Taking time derivative of Ampere's law

$$\mu_0 e(\frac{\partial n_i \boldsymbol{u}_i}{\partial t} - \frac{\partial n_e \boldsymbol{u}_e}{\partial t}) = \bigtriangledown \times \frac{\partial \boldsymbol{B}}{\partial t} = - \bigtriangledown \times \bigtriangledown \times \boldsymbol{E}$$

The first term on the left hand side comes from ion momentum equation

$$m_i \frac{\partial n_i \boldsymbol{u}_i}{\partial t} = e n_i (\boldsymbol{E} + \boldsymbol{u}_i \times \boldsymbol{B}) - \bigtriangledown \cdot \boldsymbol{\Pi}_i$$

the electron inertial term could be written as

$$m_e \frac{\partial (n_e \boldsymbol{u}_e)}{\partial t} = \frac{m_e}{m_i} (e n_i (\boldsymbol{E} + \boldsymbol{u}_i \times \boldsymbol{B}) - \bigtriangledown \cdot \boldsymbol{\Pi}_i) + \frac{m_e}{\mu_0 e} \bigtriangledown \times (\bigtriangledown \times \boldsymbol{E})$$

- Dropping terms with m_e/m_i, we arrive at the generalized Ohm's law

$$\boldsymbol{E} + \frac{c^2}{w_{pe}^2} \bigtriangledown \times (\bigtriangledown \times \boldsymbol{E}) = -\frac{1}{e n_e} \boldsymbol{j}_i \times \boldsymbol{B} + \frac{1}{\mu_0 e n_e} (\bigtriangledown \times \boldsymbol{B}) \times \boldsymbol{B} + \frac{\eta}{\mu_0} \bigtriangledown \times \boldsymbol{B} - \frac{\bigtriangledown \cdot \boldsymbol{\Pi}_e}{e n_e}$$

Implicit δf algorithm

- Given an ion distribution function $f_i = f_{i0} + \delta f_i$, utilizing the usual δf method, the weight equation is

$$\frac{d}{dt} w_i = -\frac{d \ln f_{i0}}{dt}$$

- For ρ_i scale instabilities $k_\perp \rho_i \sim 1, \beta \sim 0.01$, the compressional wave frequency $\frac{\omega}{\Omega_i} \geq 10$, therefore $\Omega_i \Delta t \ll 0.01$ is needed. To get rid of the fast frequency, we employ an implicit scheme.

- A first-order scheme has been developed. Here we come up with a second-order scheme with an adjustable centering parameter and improved the field solver.

Yang Chen, Scott E. Parker, Phys. Plasmas **16**, 052305 (2009)

Second order implicit scheme

- Particle push

$$\frac{\boldsymbol{x}^{n+1}-\boldsymbol{x}^n}{\Delta t} = (1-\theta)\ \boldsymbol{v}^n + \theta\ \boldsymbol{v}^{n+1}$$

$$\frac{\boldsymbol{v}^{n+1}-\boldsymbol{v}^n}{\Delta t} = \frac{q}{m}\left((1-\theta)\ (\boldsymbol{E}^n + \boldsymbol{v}^n \times \boldsymbol{B}^n) + \theta\ (\boldsymbol{E}^{n+1} + \boldsymbol{v}^{n+1} \times \boldsymbol{B}^{n+1})\right)$$

$$\frac{w^{n+1}-w^n}{\Delta t} = \frac{q}{T_{i0}}\left((1-\theta)\ (\boldsymbol{E}^n \cdot \boldsymbol{v}^n) + \theta\ (\boldsymbol{E}^{n+1} \cdot \boldsymbol{v}^{n+1})\right)$$

- Faraday's law

$$\frac{\delta\boldsymbol{B}^{n+1}-\delta\boldsymbol{B}^n}{\Delta t} = -[(1-\theta)\ \bigtriangledown \times \boldsymbol{E}^n + \theta\ \bigtriangledown \times \boldsymbol{E}^{n+1}]$$

- Ohm's law:

$$\begin{aligned}(n_{e0} + \delta n_e)\boldsymbol{E}^{n+1} + \frac{m_e}{\mu_0 e} \bigtriangledown \times (\bigtriangledown \times \boldsymbol{E}^{n+1}) &= -\delta \boldsymbol{j}_i^{n+1} \times \boldsymbol{B}_0 - \delta \boldsymbol{j}_i^{n+1} \times \delta \boldsymbol{B}^{n+1} + \frac{1}{\mu_0}(\bigtriangledown \times \delta\boldsymbol{B}^{n+1}) \times \boldsymbol{B}_0 \\ &+ \frac{1}{\mu_0}(\bigtriangledown \times \boldsymbol{B}_0) \times \delta\boldsymbol{B}^{n+1} + \frac{1}{\mu_0}(\bigtriangledown \times \delta\boldsymbol{B}^{n+1}) \times \delta\boldsymbol{B}^{n+1} \\ &+ \frac{\eta e n_{e0}}{\mu_0} \bigtriangledown \times \delta\boldsymbol{B}^{n+1} + \frac{\eta e \delta n_e}{\mu_0} \bigtriangledown \times \delta\boldsymbol{B}^{n+1} - \gamma T_e \nabla \delta n_i^{n+1}\end{aligned}$$

Ion current

- First half push cycle

$$\begin{aligned}\boldsymbol{v}^\star &= \boldsymbol{v}^n + (1-\theta)\Delta t\ \frac{q}{m}\left(\boldsymbol{E}^n + \boldsymbol{v}^n \times \boldsymbol{B}^n\right)\\ \boldsymbol{x}^\star &= \boldsymbol{x}^n + (1-\theta)\Delta t\ \boldsymbol{v}^n\\ w^\star &= w^n + (1-\theta)\Delta t\ \frac{q}{T_{i0}}\left(\boldsymbol{E}^n \cdot \boldsymbol{v}^n\right)\end{aligned}$$

- Dependence of $\delta \boldsymbol{j}_i^{n+1}$ on $\boldsymbol{E}^{n+1}$

$$\begin{aligned}\delta \boldsymbol{j}_i^{n+1} &= q\ \sum_j w_j^{n+1} \boldsymbol{v}_j^{n+1}\\ &= \delta \boldsymbol{j}_i^\star + \theta\ \Delta t \frac{V}{N} \sum_j \frac{1}{\Delta V} \frac{q}{T_i} \boldsymbol{v}_j \boldsymbol{E}^{n+1}(\boldsymbol{x}_j^{n+1}) \cdot \boldsymbol{v}_j S(\boldsymbol{x} - \boldsymbol{x}_j^{n+1})\\ &\simeq \delta \boldsymbol{j}_i^\star + \theta\ \Delta t\ \frac{q^2}{m}\ \boldsymbol{E}^{n+1} \equiv \boldsymbol{J}_i'\end{aligned}$$

  where this equation follows as the marker distribution is Maxwellian.

- For accuracy issues, we iterate on the differences between $\delta \boldsymbol{j}_i^{n+1}$ and $\boldsymbol{J}_i'$ while solving Ohm's law to obtain $\boldsymbol{E}^{n+1}$.

- Once we have $\boldsymbol{E}^{n+1}$, $\delta\boldsymbol{B}^{n+1}$ is advanced according to the Faraday's law.

$$\frac{\delta\boldsymbol{B}^{n+1} - \delta\boldsymbol{B}^{n}}{\Delta t} = -[(1-\theta)\ \triangledown \times \boldsymbol{E}^{n} + \theta\ \triangledown \times \boldsymbol{E}^{n+1}]$$

- With $\boldsymbol{E}^{n+1}$ and $\delta\boldsymbol{B}^{n+1}$, we could proceed to complete the second half push cycle

$$\begin{aligned}
\boldsymbol{v}^{n+1} &= \boldsymbol{v}^{\star} + \theta\Delta t\ \frac{q}{m}\left(\boldsymbol{E}^{n+1} + \boldsymbol{v}^{n+1} \times \boldsymbol{B}^{n+1}\right) \\
\boldsymbol{x}^{n+1} &= \boldsymbol{x}^{\star} + \theta\Delta t\ \boldsymbol{v}^{n+1} \\
w^{n+1} &= w^{\star} + \theta\Delta t\ \frac{q}{T_{i0}}\left(\boldsymbol{E}^{n+1} \cdot \boldsymbol{v}^{n+1}\right)
\end{aligned}$$

Field solver

- Zero-order B field

$$\boldsymbol{B}_0 = \boldsymbol{e}_y \boldsymbol{B}_{0y} + \boldsymbol{e}_z \boldsymbol{B}_{0z}.$$

.

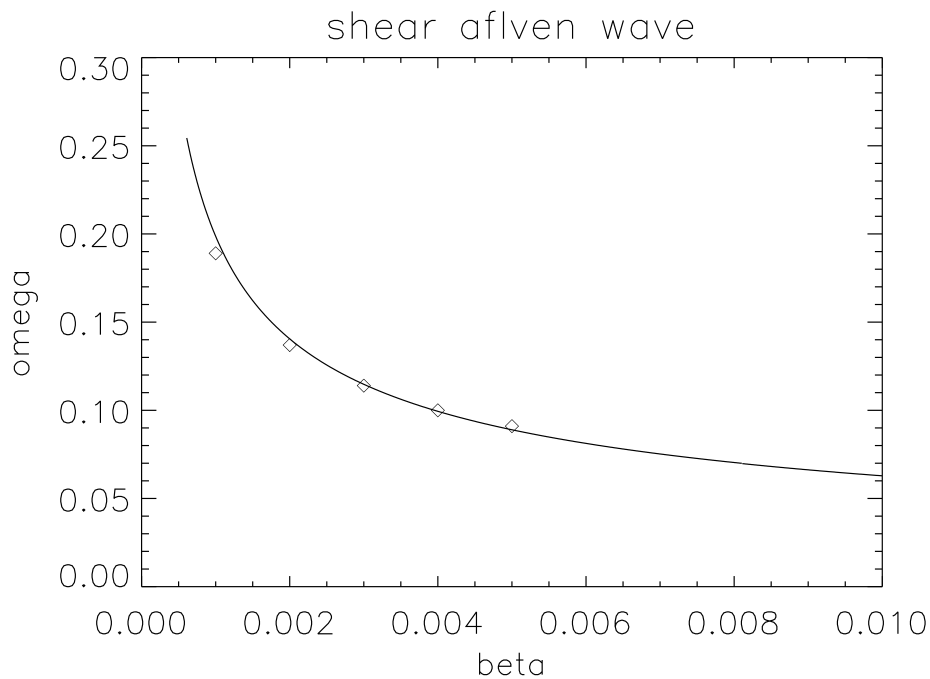
- In the Ohm's law,

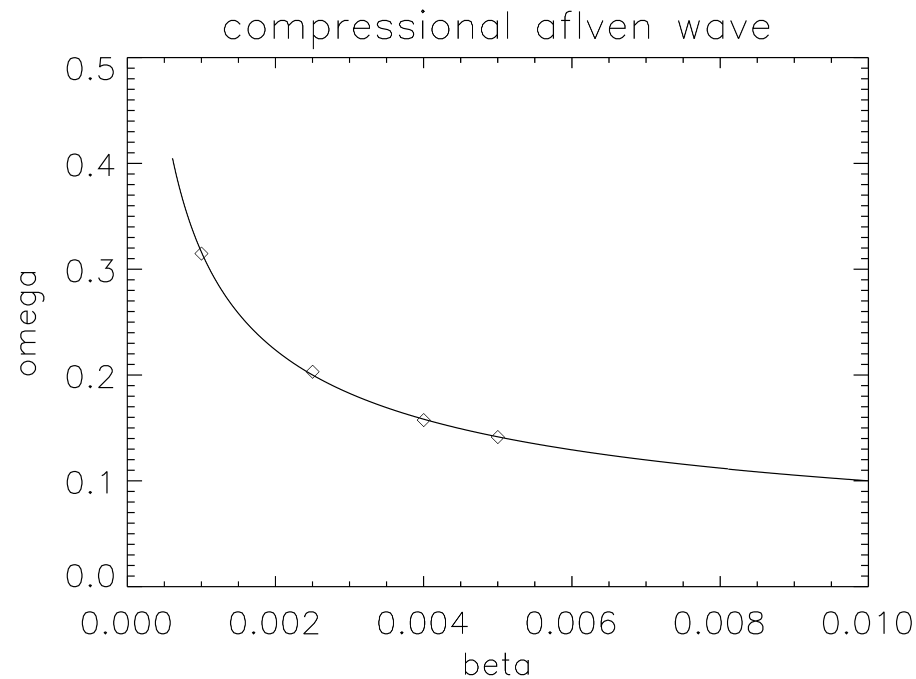
$$n_e\, \boldsymbol{E}^{n+1} + \frac{n_e}{\beta_e}(\frac{m_e}{m_i} + \theta \Delta t \eta) \bigtriangledown \times (\bigtriangledown \times \boldsymbol{E}^{n+1}) + \theta\, \frac{\Delta t}{\beta_e} \left(\bigtriangledown \times \bigtriangledown \times \boldsymbol{E}^{n+1}\right) \times \boldsymbol{B}_0 + \cdots$$
$$= -\delta \boldsymbol{j}_i^{n+1} \times \boldsymbol{B}_0 + \cdots$$

the third term on the left hand side of involves the cross product of $\boldsymbol{E}^{n+1}$ and $\boldsymbol{B}_0$,

  - If $\boldsymbol{B}_0$ is space-dependent, we could not obtain a clean single mode equation through Fourier transformation. As in the Harris sheet equilibrium, $\boldsymbol{B}_0$ only depends on x, we could Fourier transform $\boldsymbol{E}^{n+1}(x, y, z)$ to $\boldsymbol{E}^{n+1}(x, k_y, k_z)$ and solve the latter by direct matrix inversion for every k_y, k_z mode.

3-D Shearless Slab Alfven waves





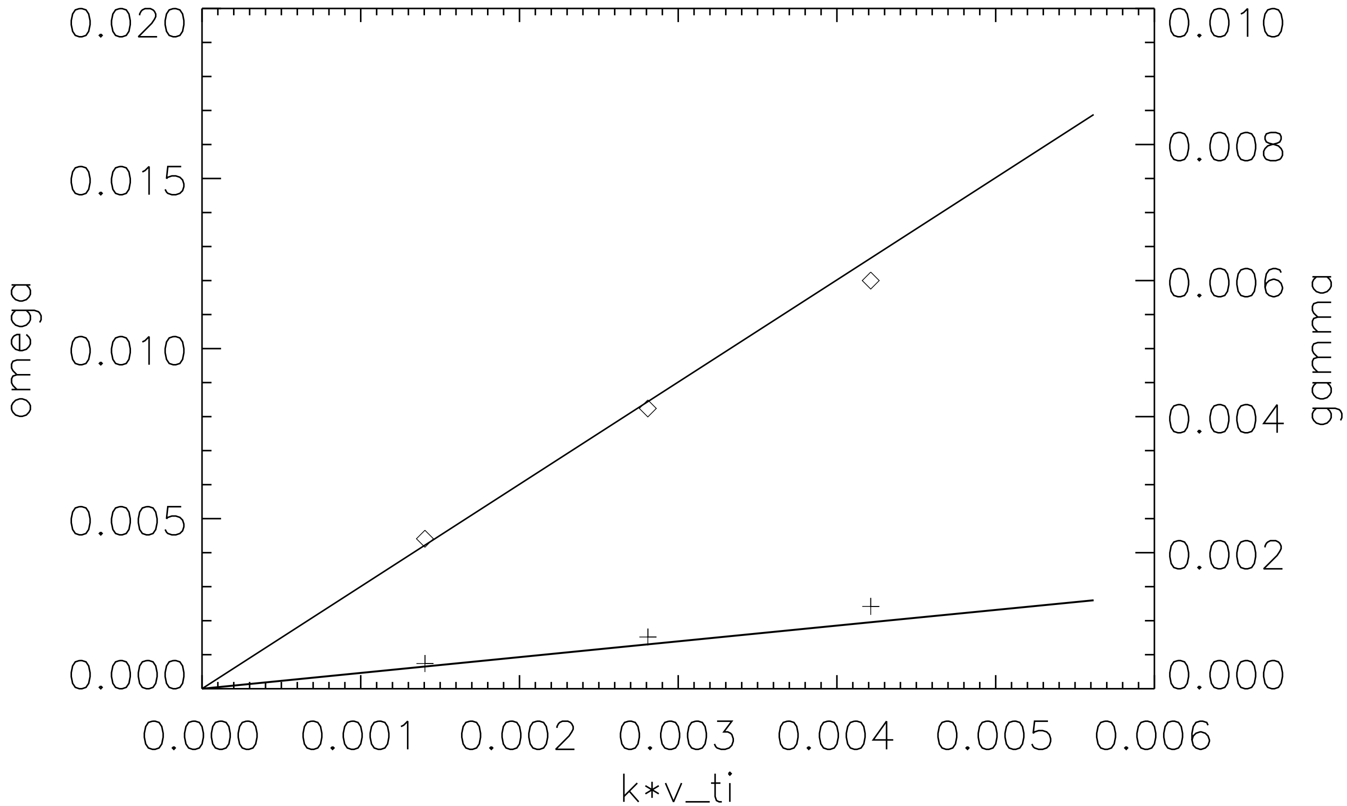
$2 \times 32 \times 32$ grids, 131072 particles.

For shear Alfven wave, $k_\perp = 0$, $k_\parallel \rho_i = 0.00628$, initialize with $\delta \boldsymbol{B}_\perp$.

For compressional Alfven wave, $k_\parallel = 0$, $k_\perp \rho_i = 0.01$, initialize with $\delta \boldsymbol{B}_\parallel$.

These simulations are done in a tilted B_0 field.

Ion acoustic wave



$2 \times 32 \times 32$ grids, 131072 particles. $k_\perp = 0$.

Whistler wave

- By neglecting ion current and electron inertia, the Ohm's law yields

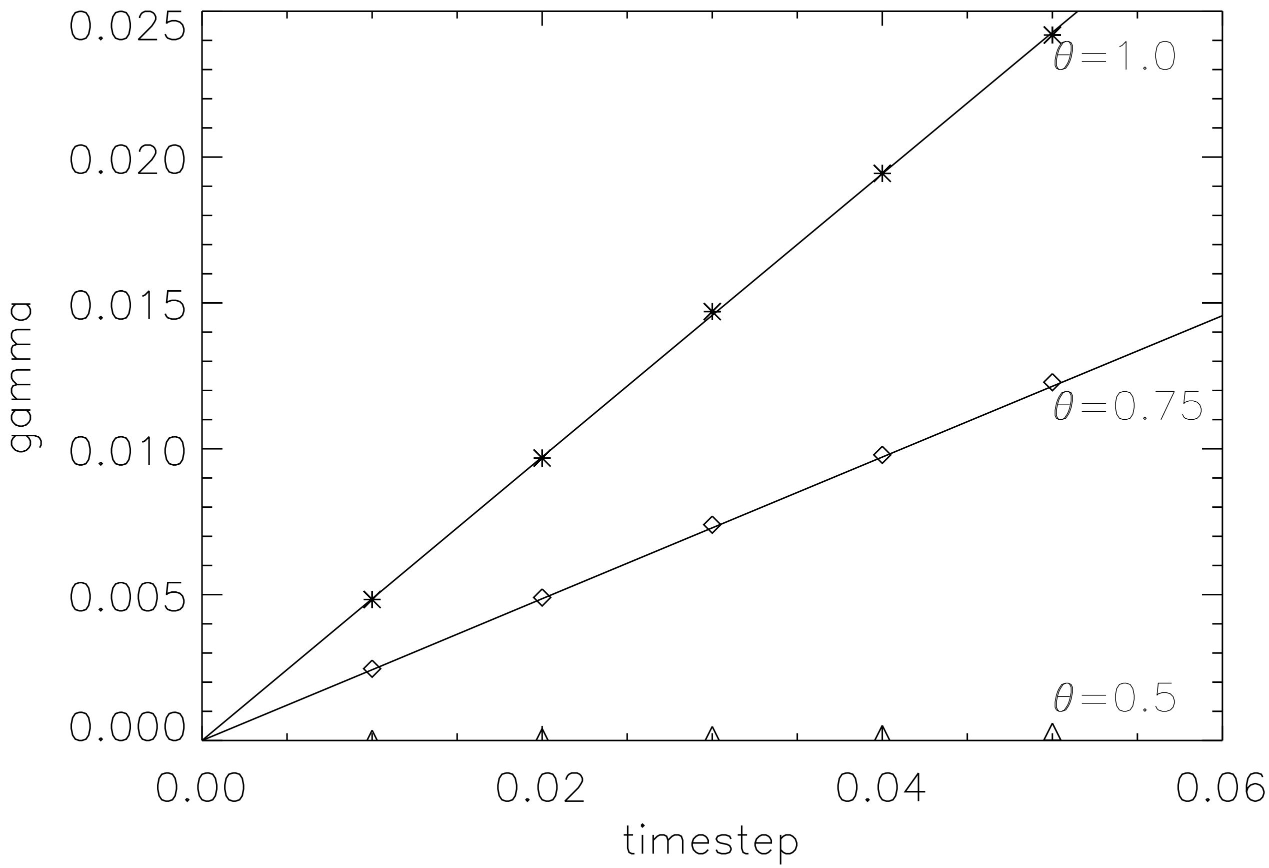
$$\boldsymbol{E} = \frac{1}{\beta_e}(\nabla \times \delta\boldsymbol{B}) \times \boldsymbol{B}_0.$$

- The numerical dispersion relation from a Von Neumann stability analysis

$$\tan(\omega_r \Delta t) = \frac{\frac{k^2}{\beta}\Delta t}{1 - (\frac{k^2}{\beta}\Delta t)^2\ \theta(1-\theta)}$$

$$\omega_i \Delta t = -\frac{1}{2}\ln\left(\frac{\left(1 - (\frac{k^2}{\beta}\Delta t)^2\ \theta(1-\theta)\right)^2 + (\frac{k^2}{\beta}\Delta t)^2}{\left(1 + (\frac{k^2}{\beta}\Delta t)^2\ (1-\theta)^2\right)^2}\right)$$

Numerical dispersion relation



$16 \times 16 \times 32$ grids, 131072 particles, $k_\perp = 0, k_\parallel \rho_i = 0.0628, \beta = 0.004.$

Harris sheet equilibrium

- Zero-order $\boldsymbol{B}$

$$\boldsymbol{B}_0(\boldsymbol{x}) = B_{y0} \tanh(\frac{x}{L})\ \hat{\boldsymbol{y}} + B_G\ \hat{\boldsymbol{z}}$$

- The equilibrium distribution function is

$$f_{0s} = n_h \ \mathrm{sech}^{\,2}(\frac{x}{L}) \left(\frac{2\pi T_s}{m_s}\right)^{-\frac{3}{2}} \exp\left[-\frac{m(v_x^2 + v_y^2 + (v_z - v_{ds})^2)}{2T_s}\right]$$

$$+n_b \ \left(\frac{2\pi T_s}{m_s}\right)^{-\frac{3}{2}} \exp\left(-\frac{mv^2}{2T_s}\right)$$

- Load particles as Maxwellian

$$g_s = n_0 \left(\frac{2\pi T_s}{m_s}\right)^{-\frac{3}{2}} \ \exp\left(-\frac{m_s \boldsymbol{v}^2}{2T_s}\right)$$

- Weight equation

$$\frac{dw_i}{dt} = \frac{q_s}{T_s} \ \left(\boldsymbol{E} \cdot \boldsymbol{v}(\frac{f_h}{g_s} + \frac{n_b}{n_0}) - \boldsymbol{v}_d \cdot (\boldsymbol{E} + \boldsymbol{v} \times \delta\boldsymbol{B}) \ \frac{f_h}{g_s}\right)$$

$$\frac{f_h}{g_s} = \frac{n_h}{n_0} \ \mathrm{sech}\,(\frac{x}{L})^2 \ \exp\left(\frac{m_s}{2T_s} \ (2\boldsymbol{v}_d \cdot \boldsymbol{v} - \boldsymbol{v}_d^2)\right).$$

Boundary conditions

- Perfect conducting wall boundary condition is employed in x while periodic boundary conditions in y and z direction.

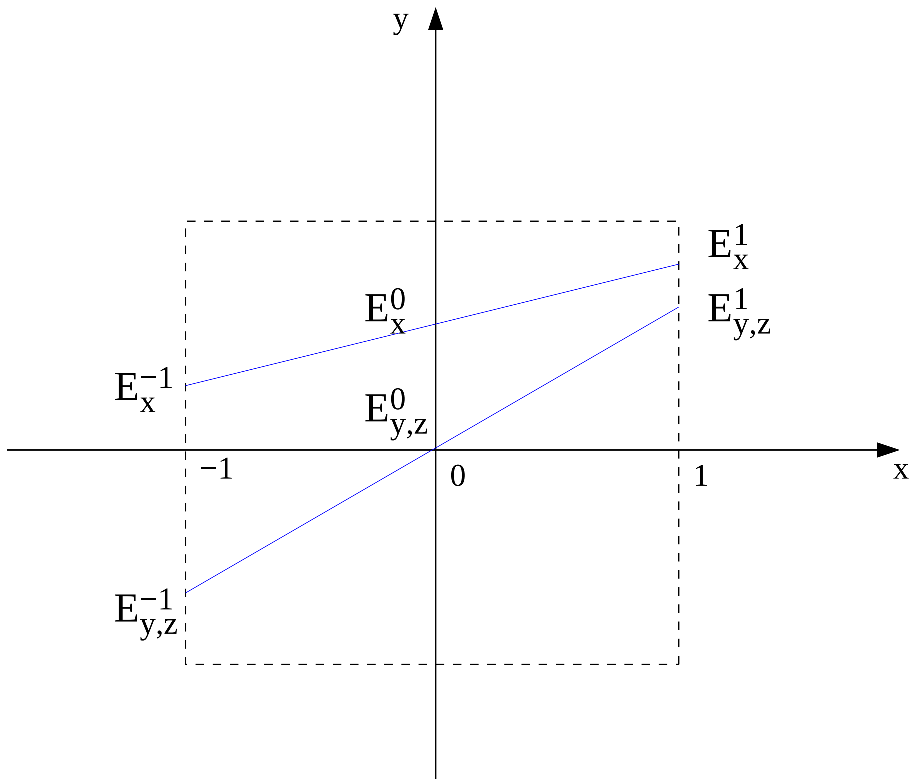
$$\boldsymbol{E}_{y,z}\big|_{x=\pm l_x/2} = 0$$
$$\delta\boldsymbol{B}_x\big|_{x=\pm l_x/2} = 0$$

- Numerically, the boundary condition for $\boldsymbol{E}$ can be treated as

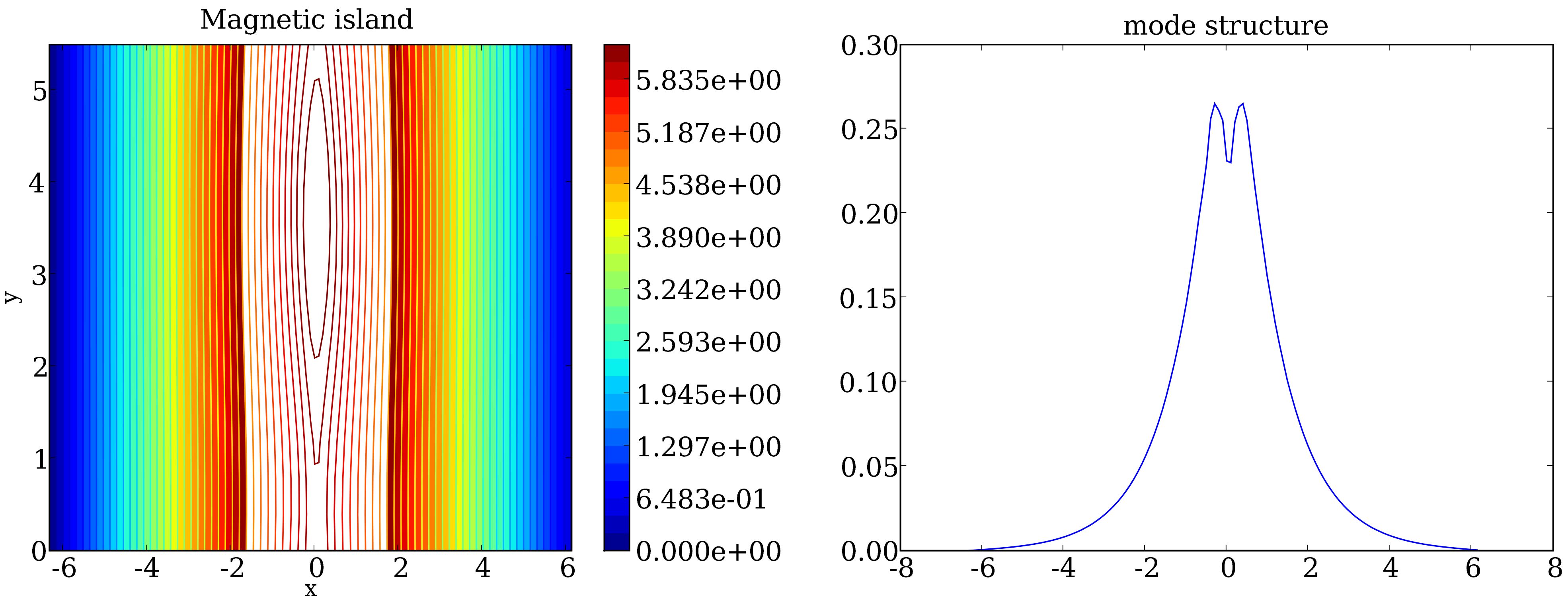
$$\frac{\boldsymbol{E}_{y,z}^{-1} + \boldsymbol{E}_{y,z}^{1}}{2} = 0$$
$$\frac{\boldsymbol{E}_{x}^{-1} + \boldsymbol{E}_{x}^{1}}{2} = \boldsymbol{E}_x^0$$

at $x = \pm l_x/2$.

- Boundary condition for $\delta\boldsymbol{B}$ is assumed in Faraday's equation.



Resistive Tearing mode



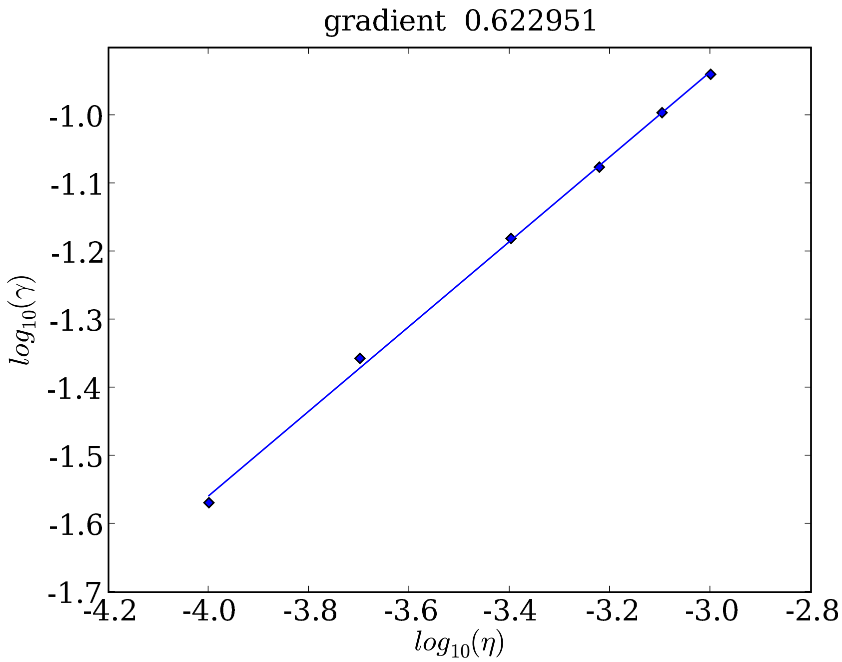
$128 \times 16 \times 4$ grids, 131072 particles. $k_y\rho_i = 1, \frac{L}{\rho_i} = 0.25, \beta_e = \frac{\mu_0 n_0 T_e}{B_0^2} = 0.2,$

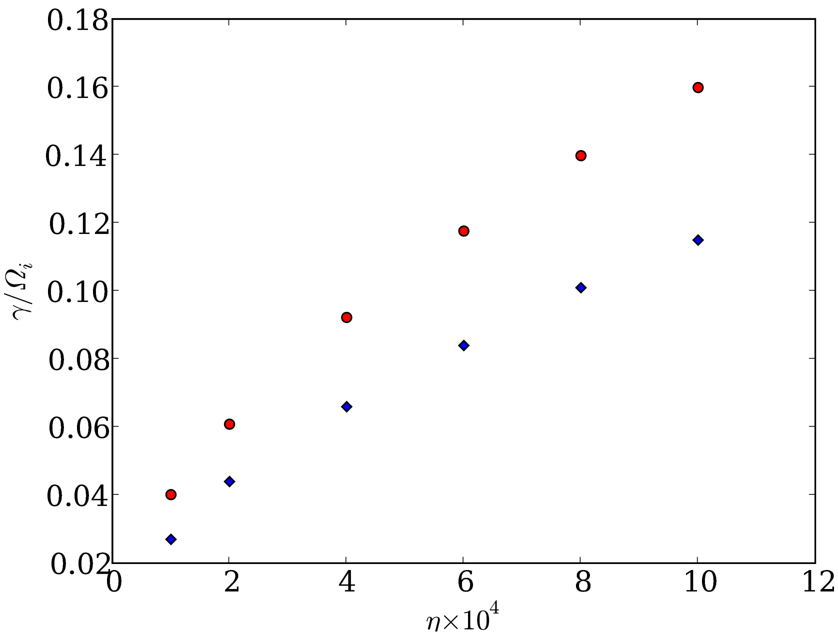
$\eta\frac{en_0}{B_0} = 0.0008, \frac{\boldsymbol{B}_G}{B_0} = 2, \frac{T_i}{T_e} = 1, \frac{l_x}{\rho_i} = 12.56, \frac{l_y}{\rho_i} = 6.28$

Tearing mode growth rate

- Linear Tearing mode theory shows that the growth rate is (scaled)

$$\gamma = 0.55 \; (\frac{\Delta'}{\beta})^{4/5} \; \eta^{3/5} \; (k \; B'_{y0})^{2/5}.$$





$256 \times 16 \times 4$ grids, 131072 particles. $k_y\rho_i = 0.5, \frac{L}{\rho_i} = 0.4, \beta_e = \frac{\mu_0 n_0 T_e}{B_0^2} = 0.2,$
$\frac{\boldsymbol{B}_G}{B_0} = 1, \frac{T_i}{T_e} = 1, \frac{l_x}{\rho_i} = 12.8, \frac{l_y}{\rho_i} = 12.56$

Tearing mode at nonlinear stage

- From an MHD perspective, the nonlinear tearing mode will eventually step into the Rutherford regime where the growth rate is algebraic in time.
- The simulation results doesn't show Rutherford stage. The instability keeps growing exponentially before code crashes.
- One difference of the simulation from MHD is that the ion is not cold enough. The δf method in accordance with the Harris sheet equilibrium distribution places a limit on the ion temperature in the code.

The Lorentz ion/Drift kinetic electron model

Lorentz ions:

$$\frac{d\mathbf{v}_i}{dt} = \frac{q}{m_i}(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}), \qquad \frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i$$

Drift kinetic electrons: $\varepsilon = \frac{1}{2}m_e v^2$

$$\frac{d\mathbf{x}}{dt} = \mathbf{v}_G \equiv v_\parallel \left(\mathbf{b} + \frac{\delta\mathbf{B}_\perp}{B_0}\right) + \mathbf{v}_D + \mathbf{v}_E$$

$$\frac{d\varepsilon}{dt} = -e\mathbf{v}_G \cdot \mathbf{E} + \mu\frac{\partial B}{\partial t}, \qquad \frac{d\mu}{dt} = 0$$

Ampere's equation

$$\nabla \times \mathbf{B} = \mu_0(\mathbf{J}_i - en_e(\mathbf{V}_{\mathrm{e}\perp} + u_{\parallel\mathrm{e}}\mathbf{b}))$$

$$\mathbf{V}_{\mathrm{e}\perp} = \frac{1}{B}\mathbf{E} \times \mathbf{b} - \frac{1}{enB}\mathbf{b} \times \nabla P_{\perp\mathrm{e}}$$

$$\mathbf{J}_i = \int f_i \mathbf{v}\, d\mathbf{v}, \quad u_{\parallel\mathrm{e}} = \int f_e\, v_\parallel\, d\mathbf{v}, \quad P_{\perp\mathrm{e}} = \int f_e \frac{1}{2} m_e v^2\, d\mathbf{v}$$

Faraday's equation,

$$\frac{\partial\mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

- The Harris sheet equilibrium distribution function is a drifted Maxwellian with a non-uniform density profile in x, which doesn't meet the drift-kinetic assumptions. If a strong guide field in z exists, the nonuniformity could be treated as a perturbation, then the drift-kinetic equations is appliable in this case.

- With drift-kinetic electrons, we could study the roles of kinetic electrons in reconnection problem by making direct comparison with our fluid electron model.

- Currently I am adding the drift-kinetic electrons into the code.

Including gyrokinetic electrons

- Gyrokinetic equations are usually derived in terms of $\mathbf{A}$ and ϕ, to make explicit the ordering

$$\frac{\partial \mathbf{A}}{\partial t} \sim \epsilon_\delta \nabla_\perp \phi$$

- The Frieman-Chen gyrokinetic equation, assuming isotropy ($\partial F_0/\partial\mu = 0$),

$$\hat{L}_g \delta H_0 \equiv \left(\frac{\partial}{\partial t} + v_\parallel \mathbf{b} \cdot \nabla + \mathbf{v}_D \cdot \nabla \right) \delta H_0 = -\frac{q}{m}(S_L + \langle R_{\mathrm{NL}} \rangle),$$

where δH_0 is related to the perturbed distribution δF through $\delta F = \frac{q}{m}\phi\frac{\partial F_0}{\partial \epsilon} + \delta H_0$

$$S_L = \frac{\partial}{\partial t} \langle \phi - \mathbf{v} \cdot \mathbf{A} \rangle \frac{\partial F_0}{\partial \epsilon} - \nabla \langle \phi - \mathbf{v} \cdot \mathbf{A} \rangle \times \frac{\mathbf{b}}{\Omega} \cdot \nabla F_0,$$

$$\langle R_{\mathrm{NL}} \rangle = -\nabla \langle \phi - \mathbf{v} \cdot \mathbf{A} \rangle \times \frac{\mathbf{b}}{\Omega} \cdot \nabla \delta H_0.$$

- Define $\delta f = \frac{q}{m} \langle \phi \rangle \frac{\partial F_0}{\partial \epsilon} + \delta H_0$. The gyrokinetic equation for δf is, written in terms of $\mathbf{E}_1$ and $\mathbf{B}_1$

$$\frac{D}{Dt}\delta f = -\left(\frac{1}{B_0} \langle \mathbf{E}_1 \rangle \times \mathbf{b} + v_\parallel \frac{\langle \mathbf{B}_{1\perp} \rangle}{B_0} \right) \cdot \nabla F_0 + \frac{1}{m}\dot{\epsilon}\frac{\partial F_0}{\partial \epsilon}$$

$$\frac{D}{Dt} = \hat{L}_g + \left(\frac{1}{B_0} \langle \mathbf{E}_1 \rangle \times \mathbf{b} + v_\parallel \frac{\langle \mathbf{B}_{1\perp} \rangle}{B_0} \right) \cdot \nabla, \qquad \dot{\epsilon} = q \left(v_\parallel \mathbf{b} + \mathbf{v}_D + v_\parallel \frac{\langle \mathbf{B}_{1\perp} \rangle}{B_0} \right) \cdot \langle \mathbf{E}_1 \rangle + q \langle \mathbf{v}_\perp \cdot \mathbf{E}_{1\perp} \rangle$$

- The perturbed electron diamagnetic flow comes from δf,

$$n_0 \mathbf{V}_D(\mathbf{x}) = \int (v_\parallel \mathbf{b} + \mathbf{v}_\perp(\mathbf{R}', \epsilon, \mu, \alpha)) \delta f(\mathbf{R}', \epsilon, \mu) \delta(\mathbf{x} - \mathbf{R}' - \boldsymbol{\rho}) \, J d\mathbf{R}' \, d\epsilon \, d\mu \, d\gamma$$

  $n_0 \mathbf{V}_D$ is computed by depositing the particle current along the gyro-ring. In the drift-kinetic limit $\mathbf{V}_D$ reduces to the electron diamagnetic flow.

- The electron $\mathbf{E} \times \mathbf{B}$ flow comes from the first term in δF,

$$n_0 \mathbf{V}_E(\mathbf{x}) = \frac{\mathbf{q}}{\mathbf{m}} \int \mathbf{v} \left(\phi(\mathbf{x}) - \langle \phi \rangle (\mathbf{x} - \boldsymbol{\rho}, \epsilon, \mu)\right) \frac{\partial \mathbf{F_0}}{\partial \epsilon} \, \mathbf{J} \, \mathbf{d}\epsilon \, \mathbf{d}\mu \, \mathbf{d}\gamma$$

  in eikonal form,

$$n_0 \mathbf{V}_E = n_0 \frac{h}{B_0} \delta \mathbf{E}_k \times \mathbf{b}$$

  with $b = k_\perp^2 v_T^2 / \Omega^2$ and

$$h(b) = -\frac{1}{b^2} \int_0^\infty e^{-x^2/2b} J_0(b) J_0'(b) x^2 \, dx$$

  In the limit of small $k\rho \ll 1$ the factor $h(b)$ become unity, so that $n_0 \mathbf{V}_E$ become the total guiding center $\mathbf{E} \times \mathbf{b}$ flow.

Summary

1. We implemented an implicit algorithm with Lorentz force ions and isothermal fluid electrons which is
   - Quasi-neutral and fully electromagnetic.
   - Suitable for MHD scale plasmas.

2. Second order implicit scheme allows bigger time step, $\Omega_i \Delta t \gtrsim 0.1$.
   - Compared the first order and second order scheme with Whistler waves

3. Demonstrated 3-D slab simulation for compressional and shear Alfven waves, Whistler wave, and the ion acoustic wave.

4. With conducting wall boundary condition on x and periodic boundary condition on y and z, the resistive tearing mode instability is investigated with Harris sheet equilibrium.

5. Working on nonlinear tearing mode saturation and drift-kinetic electrons.