# Update on activities at Utah State

E. Held S. Kruger J.Y. Ji A. Spencer R. Datwyler.

NIMROD at Sherwood, Seattle, WA 2010

# Outline

1. Neoclassical closures benchmark with NEO.
   - Equation and solution method for NIMROD.
   - Results.
2. Fully centered advance of ideal two-fluid equations with displacement current.
   - Background.
   - Results.
3. Update on 2D-FE/Fourier representation for full kinetics.
   - Implementation of field terms.

# Solve Chapman-Enskog-like (CEL) drift kinetic equation.

- Solve CEL-DKE given Grad-Shafronov equilibrium:

$$\frac{\partial F}{\partial t} + \mathbf{v}_{||} \cdot \nabla F + \frac{q}{m} E_{||} \frac{v_{||}}{v} \frac{\partial F}{\partial v} =$$

$$\langle C(F + f_M) \rangle - \frac{mv^2}{T} P_2(v_{||}/v) f_M \left(\mathbf{bb} - \frac{\mathbf{I}}{3}\right) : \nabla \mathbf{u} +$$

$$\frac{2f_m}{3p} L_1^{(3/2)} \left[ \nabla \cdot \mathbf{q} + \mathbf{\Pi} : \nabla \mathbf{V} - Q - S_0^{rf} \right] +$$

$$\mathbf{v}_{||} \cdot \left[ \frac{f_m}{p} \left( \nabla \cdot \mathbf{\Pi} - \mathbf{R} - F_0^{rf} \right) + \frac{f_m}{T} L_1^{(5/2)} \nabla T \right]$$

- Expand $F = \sum_{l=0}^{nl} F_l(\mathbf{x}, t, s) P_l(v_{||}/v)$, where the coefficients $F_l(\mathbf{x}, t, s)$ are determined on a grid of *ns* grid points in the normalized speed, $s = v/v_T$.

# Include drift drives.

- In PSFC/JA-10-5, Ramos has provided following form for the electron drift drives :

$$\frac{2f_m}{3eB}\left\{2P_0L_2^{1/2}\left[\mathbf{b}\times\left(\nabla\ln\mathbf{B}+\kappa\right)\right]\cdot\nabla\mathbf{T}+\right.$$

$$\left.P_2s^2L_1^{3/2}\left[\mathbf{b}\times\left(\nabla\mathbf{lnB}-\mathbf{2}\kappa+\nabla\ln\mathbf{n}\right)\right]\cdot\nabla\mathbf{T}\right\}$$

- NIMROD computes and stores $|B|$ and magnetic curvature $\kappa = \mathbf{b}\cdot\nabla\mathbf{b}$.
- Maxwellian, $f_M = (n/\pi^{3/2}v_T^3)\,e^{-v^2/v_T^2}$, computed and stored at speed grid points.

# Implement flux surface average.

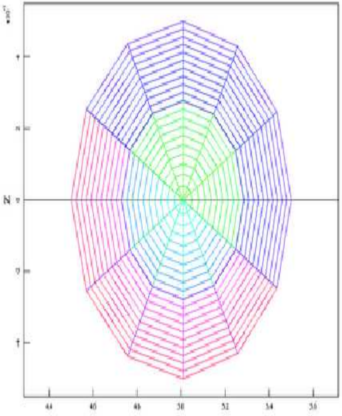
- NIMROD computes $\pi_{||} = m \int d\mathbf{v}(v_{||}^2 - v_{\perp}^2/2)\, F$ for electrons and ions.
- Bootstrap current is given by

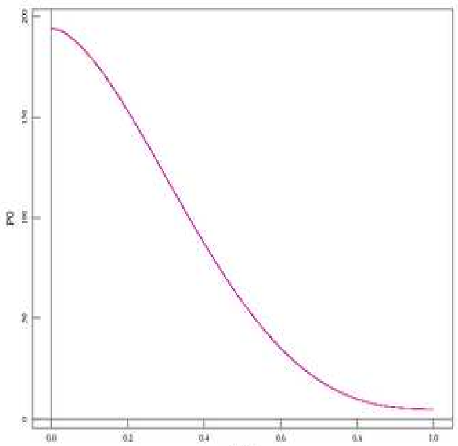
$$J_{||BS} = (\sigma_{||}/ne)\left[\mathbf{b}\cdot\nabla\cdot\pi_{||}\left(\mathbf{bb}-\mathbf{I}/3\right)\right]$$
$$= (\sigma_{||}/ne)\left[\frac{2}{3}\mathbf{b}\cdot\nabla\pi_{||} - \pi_{||}\mathbf{b}\cdot\nabla\ln B\right]$$

Flux surface average implemented in nimplot using $\langle J_{||BS}\rangle = \int_0^{2\pi} d\theta\, J_{||BS}/\mathbf{B}\cdot\nabla\theta / \int_0^{2\pi} d\theta/\mathbf{B}\cdot\nabla\theta.$

# Start with high-aspect ratio, collisional case.

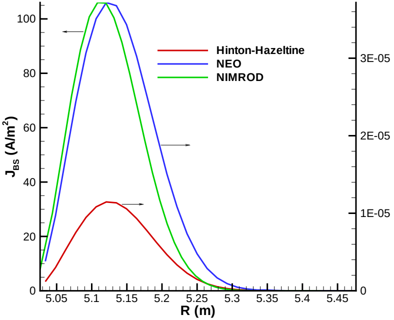
- Circular cross section, $\epsilon = 0.1$, Pfirsch-Schluter regime very low $\beta = 1\%$.

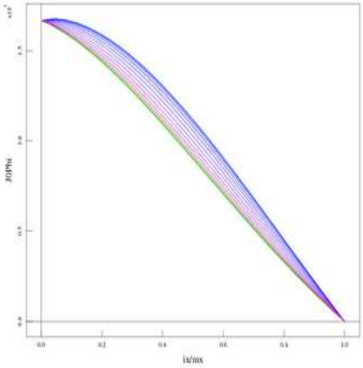




# $J_{||BS}$ has similar spatial structure.

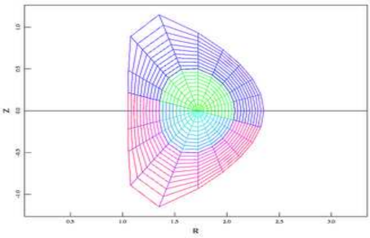
- Theory says $J_{||BS}/J_{||Ohmic} \sim \sqrt{\epsilon}\beta_p \sim .003$. NIMROD has $J_{||BS}/J_{||Ohmic} \sim 100/1.5 \times 10^5 = .006$.

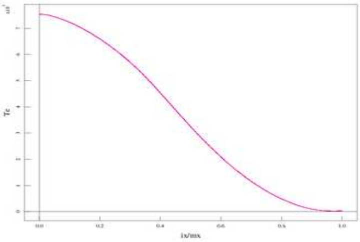


# Proceed to shaped, high-$\beta$ equilibriua.

- Adjust $\nu* = \nu/(v_T/qR)$ by tweaking density and temperature profiles.
- Banana, plateau and PS regimes have $\nu* = 4 \times 10^{-4}$, 0.3, and 10.3, respectively.



# Future work on NEO benchmark.

- Turn on moment terms in linearized collision operator.
- Compare distribution functions between NIMROD and NEO.
- Compute other moments, like radial heat and particle fluxes, for comparison with NEO.

# Study stability of equilibria characterized by large charge separation.

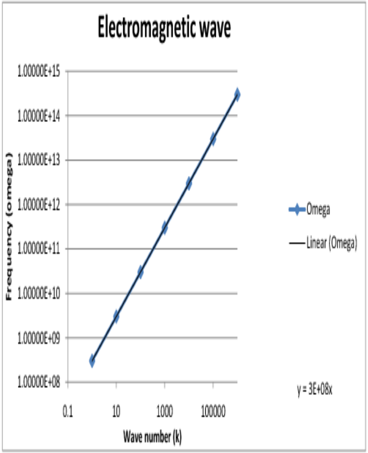
- Equilibria described in Edwards and Held, *PRL* ***93***, 255001 (2004) have large electric fields due to charge separation.
- Implement full two-fluid equations in NIMROD including (Hakim, et al., **JCP** (2005)):

$$\frac{\partial \mathbf{E}}{\partial t} = c^2 \nabla \times \mathbf{B} - \frac{\mathbf{J}}{\epsilon_0} - \chi c^2 \nabla \Phi$$

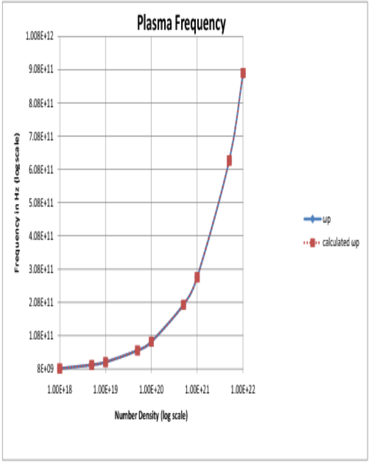
$$\frac{\partial \Phi}{\partial t} = -\chi \nabla \cdot \mathbf{E} + \chi \sum n_s q_s / \epsilon_0$$

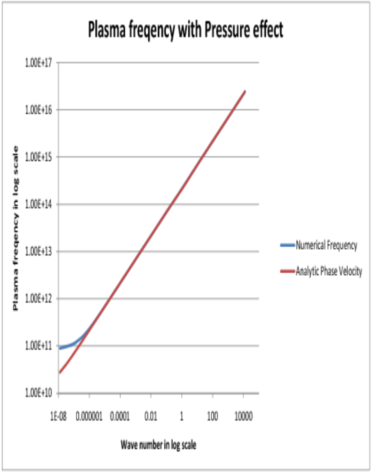
- Solution vector has 18 unknowns: $(n_e, n_i, \mathbf{V}_e, \mathbf{V}_e, \mathbf{B}, \phi, \mathbf{E}, \Phi, T_e, T_i)$.

# Circularly polarized and plane EM waves.



# Electron plasma and Langmuir oscillations.



# Field terms require double integral over velocity space.

Field terms of Landau operator:

$$C\left(f_a, f_b\right) = -\frac{m_a \Gamma_{ab}}{2}\frac{\partial}{\partial \vec{v}} \cdot \int d\vec{v}' \mathrm{U} \cdot \left(\frac{1}{m_b} f_a \frac{\partial f_b}{\partial \vec{v}'} - \frac{1}{m_a}\frac{\partial f_a}{\partial \vec{v}} f_b\right)$$

where $\mathrm{U} = \frac{u^2 \mathbf{I} - \mathbf{uu}}{u^3}$ and $\mathbf{u} = \mathbf{v} - \mathbf{v}'$ becomes (upon conversion to the weak form):

$$\frac{1}{\pi^{3/2}}\frac{q_b^2 \lambda_{ab}}{q_a^2 \lambda_{aa}} \int d\vec{c}_a \alpha \underbrace{\mathrm{e}^{-\mathbf{z}_a^2 - i(n'-n)\gamma}}_{\gamma \text{ integration} \rightarrow 2\pi\delta_{n,n'}}$$

$$\int\int\int dc'_{a\perp} dc'_{a\|} d\Delta\gamma \left(c'_{a\perp}\right)$$

$$\left(\frac{\mathbf{u}_a}{u_a^3} + \mathbf{z}'_a \cdot \mathbf{U}_a\right) \cdot \left[\frac{m_a}{m_b}\left(\frac{\partial f_{b,n}}{\partial c'_{a\|}}\mathbf{b} + \frac{\partial f_{b,n}}{\partial c'_{a\perp}}\hat{\mathbf{r}} + \frac{inf_{b,n}}{c'_{a\perp}}\hat{\gamma}\right) + 2\mathbf{z}'_a f_{b,n}\right] \mathrm{e}^{in\Delta\gamma}$$