Solving Ballooning Equations in NIMFL

Ping Zhu

University of Wisconsin-Madison
Collaborators: C. R. Sovinec and C. C. Hegna

NIMROD Team Meeting
Seattle, WA
April 17, 2010

Motivation

- Numerical solution of the infinite-n ballooning eigenmode is required in many applications
  - Two-fluid effects
  - 2D and 3D shaping effects
- `NIMFL` provides useful framework and tools for numerically solving the infinite-n ballooning mode equations for tokamaks.

Infinite-n ballooning eigenmode equations are two coupled ODEs

For general magnetic configuration $\mathbf{B} = \nabla\Psi \times \nabla\alpha$

$$\begin{aligned}
\rho B^2 \Gamma_b^2 \xi^{\parallel} &= B\partial_l \left[\frac{\gamma p}{1+\gamma\beta} \left(B\partial_l \xi^{\parallel} - 2\mathbf{e}_\perp \cdot \boldsymbol{\kappa} \xi^{\Psi} \right) \right] \\
\rho |\mathbf{e}_\perp|^2 \Gamma_b^2 \xi^{\Psi} &= B\partial_l (|\mathbf{e}_\perp|^2 B\partial_l \xi^{\Psi}) + 2\mathbf{e}_\perp \cdot \boldsymbol{\kappa} \mathbf{e}_\perp \cdot \nabla p \xi^{\Psi} \\
&\quad + \frac{2\gamma p \mathbf{e}_\perp \cdot \boldsymbol{\kappa}}{1+\gamma\beta} \left(B\partial_l \xi^{\parallel} - 2\mathbf{e}_\perp \cdot \boldsymbol{\kappa} \xi^{\Psi} \right)
\end{aligned}$$

where

$$\mathbf{e}_\perp \equiv \frac{\nabla\alpha \times \mathbf{B}}{B^2}, \qquad \boldsymbol{\kappa} \equiv \mathbf{b} \cdot \nabla \mathbf{b}.$$

Solve above two coupled ODEs along each flux tube with proper end boundary conditions to find growth rate Γ_b^2.

First step is to solve marginal ballooning eigenmode equation in field line coordinates for tokamaks

Choose field line coordinates (Ψ, θ, ζ), so that

$$\rho|\mathbf{e}_\perp|^2\Gamma_b^2\xi^\Psi = B\partial_\theta(|\mathbf{e}_\perp|^2 B\partial_\theta\xi^\Psi) + 2\mathbf{e}_\perp\cdot\boldsymbol{\kappa}\mathbf{e}_\perp\cdot\nabla p\xi^\Psi$$

where

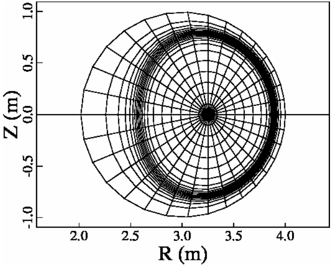
$$|\mathbf{e}_\perp|^2 = |\nabla\alpha|^2, \quad \alpha = q(\Psi)\theta - \zeta$$

For axisymmetric toroidal system, all coefficients of above equation are functions of Ψ and θ. For each flux surface Ψ, the above equation is solved in covering space $-\infty < \theta < \infty$ for Γ_b.

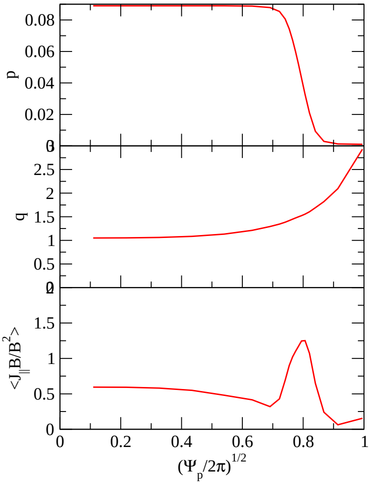
An infinite-n ballooning equation solver is implemented in NIMFL

- First, find all coefficients as functions of θ by tracing along a field line on any given surface using NIMFL.
  - $\Psi(R, Z)$ is obtained from output of `NIMEQ`.
- Second, solve the eigenmode equation using a shooting method.
  - `spline` module is used to interpolate all coefficients.

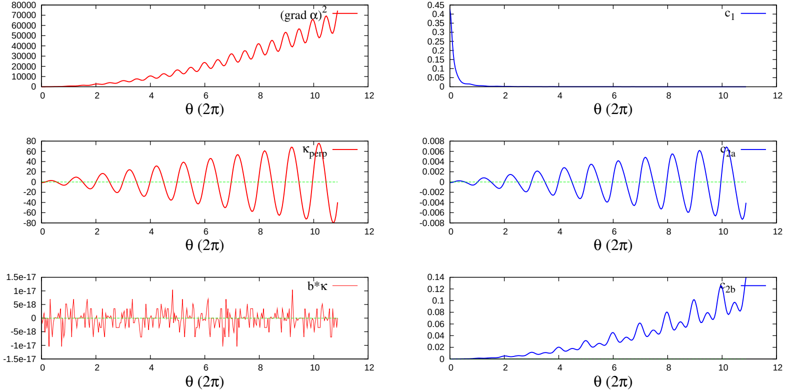
Example: Circular-shaped limiter tokamak



- Equilibrium from ESC [Zakharov and Pletzer,1999]
- Use finite element representation in dump file as input to ballooning solver.

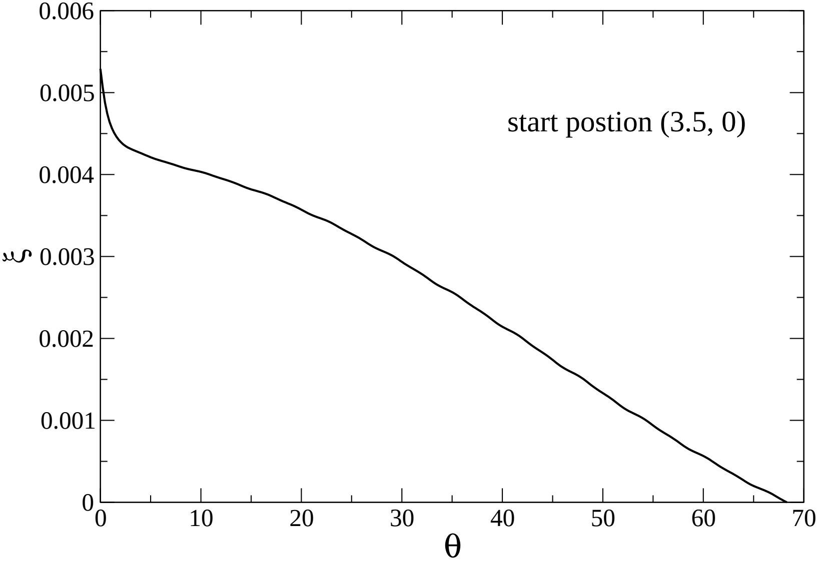


Ballooning coefficients along poloidal angle θ: core region

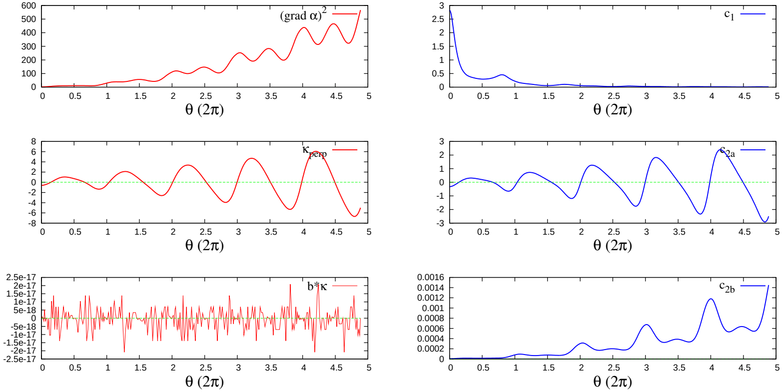


$$C_1 = (B^\theta |\mathbf{e}_\perp|^2)^{-1}, \quad C_{2a} = -2\mathbf{e}_\perp \cdot \boldsymbol{\kappa} p' (B^\theta)^{-1}, \quad C_{2b} = \rho |\mathbf{e}_\perp|^2 (B^\theta)^{-1}$$

Ballooning eigenmode solution along poloidal angle θ: core region



Ballooning coefficients along poloidal angle θ: pedestal top



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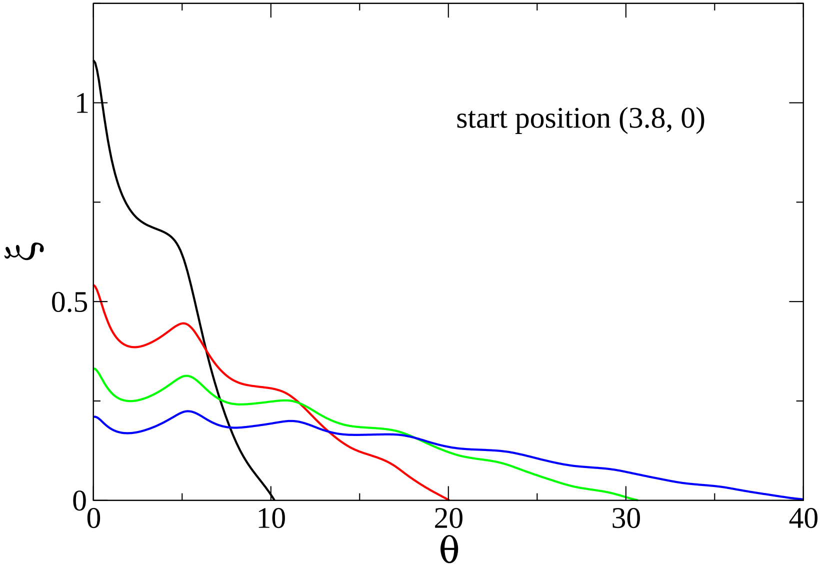
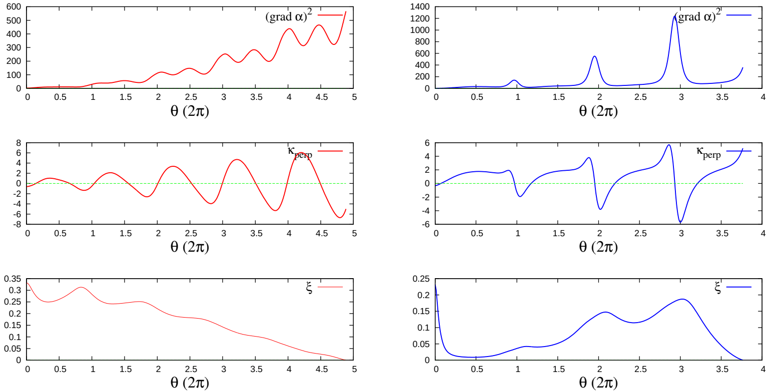
Ballooning eigenmode solution along poloidal angle θ: pedestal top



Ballooning coefficients and mode structure appear different in pedestal top and foot regions



Left: top, start from (3.8, 0); right: foot, start from (3.93, 0)

Summary

- An infinite-n ballooning mode solver is being developed using the framework and tools in NIMFL.
- Numeric solution of the marginal ballooning mode is obtained using the solver for a circular shaped limiter tokamak.
  - Shooting scheme is sensitive to the initial guess.
  - Covergence over $\theta_{\max}$ appears slow.
- Future work
  - To improve shooting scheme.
  - To use large θ asymptotic solution as B.C.
  - To include the parallel component equation (compressional).
  - To compare with linear NIMROD simulation of high-n ballooning modes.