Update on general moment calculation: Moment approach to solving kinetic equation

Jeong-Young Ji and Eric D. Held

Department of Physics, Utah State University

NIMROD Team Meeting

April 30, 2011, Austin, TX

Moment expansion of a distribution function

- Landau-Fokker-Planck kinetic equation

$$\frac{\partial f_a}{\partial t} + \mathbf{v} \cdot \nabla f_a + \frac{q_a}{m_a}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \frac{\partial f_a}{\partial \mathbf{v}} = \sum_b C(f_a, f_b)$$

- Moment expansion: M^{lk}'s, are symmetric traceless fluid moments

$$\begin{aligned} f_a(t, \mathbf{x}, \mathbf{v}) &= f_a^{(0)} \sum_{lk} \mathsf{M}_a^{lk}(t, \mathbf{x}) \cdot \hat{\mathsf{P}}_a^{lk} \\ f_b(t, \mathbf{x}, \mathbf{v}) &= f_b^{(0)} \sum_{nq} \mathsf{M}_b^{nq}(t, \mathbf{x}) \cdot \hat{\mathsf{P}}_b^{nq} \end{aligned}$$

- Moment equations $\int d\mathbf{v} \hat{\mathsf{P}}^{jp} \Rightarrow$

$$\sum_{lk} \hat{D}_a^{jp,lk}(n_a \mathsf{M}_a^{lk}) = \sum_b \sum_{lk,nq} n_a \hat{C}_{ab}^{jp,lk,nq} \overline{\mathsf{M}_a^{lk} \cdot^{\frac{l+n-j}{2}} \mathsf{M}_b^{nq}}$$

General moment equations for uniform plasmas

- Isotropic distribution $j = 0:$ $\mathrm{M}^{0p} = M^p$, $C^{0|pq} = C^{pq}$, $A^{0|pq} = A^{pq}$, $B^{0|pq} = B^{pq}$

$$\frac{\partial}{\partial \hat{t}} \hat{T}_a = -\frac{2z_{ab}}{\sqrt{\hat{T}_a^3}} \frac{m_a}{m_b} \frac{(\hat{T}_a - \hat{T}_b)}{\left(1 + \frac{T_b m_a}{T_a m_b}\right)^{3/2}} + \cdots$$

$$\frac{\partial}{\partial t} \begin{bmatrix} M_a^2 \\ M_a^3 \\ M_b^2 \\ M_b^3 \end{bmatrix} = \begin{bmatrix} C_{aa}^{22} + A_{ab}^{22} & C_{aa}^{23} + A_{ab}^{23} & B_{ab}^{22} & B_{ab}^{23} \\ C_{aa}^{32} + A_{ab}^{32} & C_{aa}^{33} + A_{ab}^{33} & B_{ab}^{32} & B_{ab}^{33} \\ B_{ba}^{22} & B_{ba}^{23} & C_{bb}^{22} + A_{ba}^{22} & C_{bb}^{23} + A_{ba}^{23} \\ B_{ba}^{22} & B_{ba}^{23} & C_{bb}^{32} + A_{ba}^{32} & C_{bb}^{33} + A_{ba}^{33} \end{bmatrix} \begin{bmatrix} M_a^2 \\ M_a^3 \\ M_b^2 \\ M_b^3 \end{bmatrix}$$

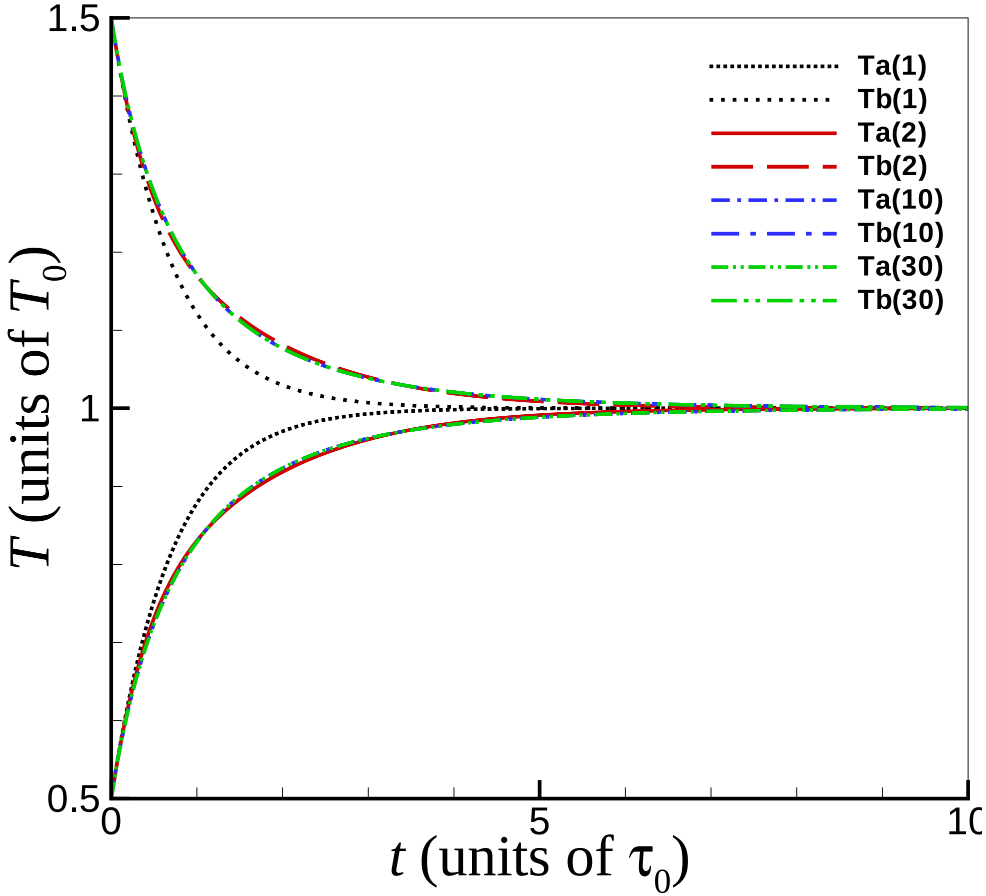
$$- \frac{1}{T_a} \frac{\partial T_a}{\partial t} [\Xi] \begin{bmatrix} M_a^2 \\ M_a^3 \\ M_b^2 \\ M_b^3 \end{bmatrix} + \begin{bmatrix} \sum_{kq} (C_{aa}^{2kq} M_a^k M_a^q + C_{ab}^{2kq} M_a^k M_b^q) \\ \sum_{kq} (C_{aa}^{3kq} M_a^k M_a^q + C_{ab}^{3kq} M_a^k M_b^q) \\ \sum_{kq} (C_{bb}^{2kq} M_b^k M_b^q + C_{ba}^{2kq} M_b^k M_a^q) \\ \sum_{kq} (C_{bb}^{3kq} M_b^k M_b^q + C_{ba}^{3kq} M_b^k M_a^q) \end{bmatrix}$$

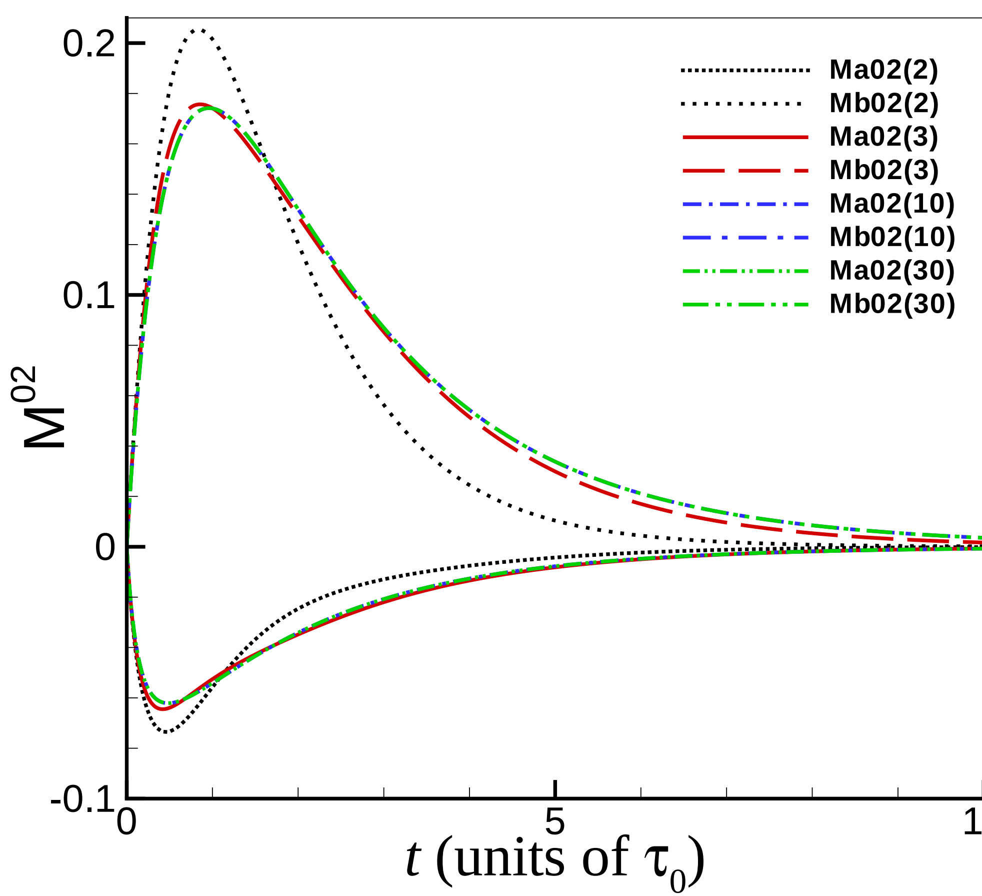
where $\hat{t} = \frac{t}{\tau_{aa}^0}$, $\hat{T}_a = \frac{T_a}{T_0}$, $\tau_{ab} = \frac{6\sqrt{2}\pi^{3/2}\epsilon_0^2 \sqrt{m_a T_a^3}}{n_b q_a^2 q_b^2 \ln \Lambda_{ab}}$

$A_{ab}^{j|pq} \equiv C_{ab}^{jp,jq,00}$, $B_{ab}^{j|pq} \equiv C_{ab}^{jp,00,jq}$, $C_{aa}^{j|pq} \equiv A_{aa}^{j|pq} + B_{aa}^{j|pq}$, $C_{aa}^{pkq} \equiv C_{aa}^{0p,0q,0k}$

Convergence with increasing moments

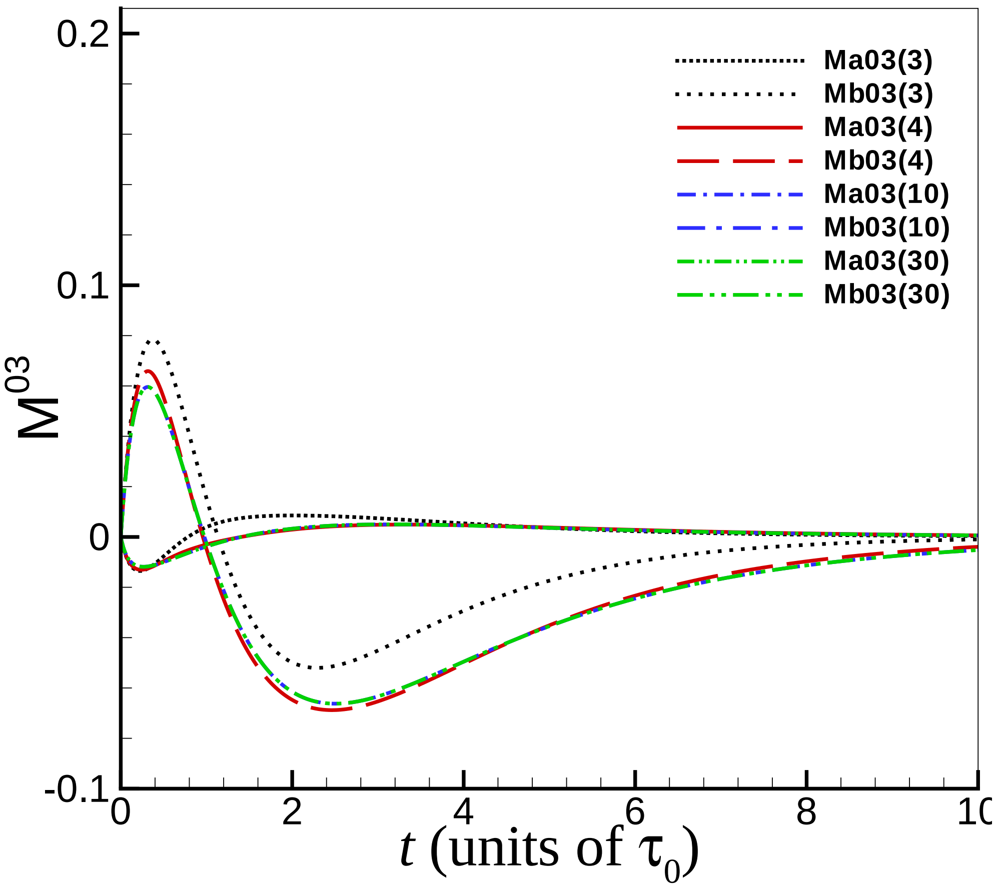
Starting with two Maxwellian distributions $(m_a = m_b)$ with $T_a = 0.5T_0$ and $T_b = 1.5T_0$

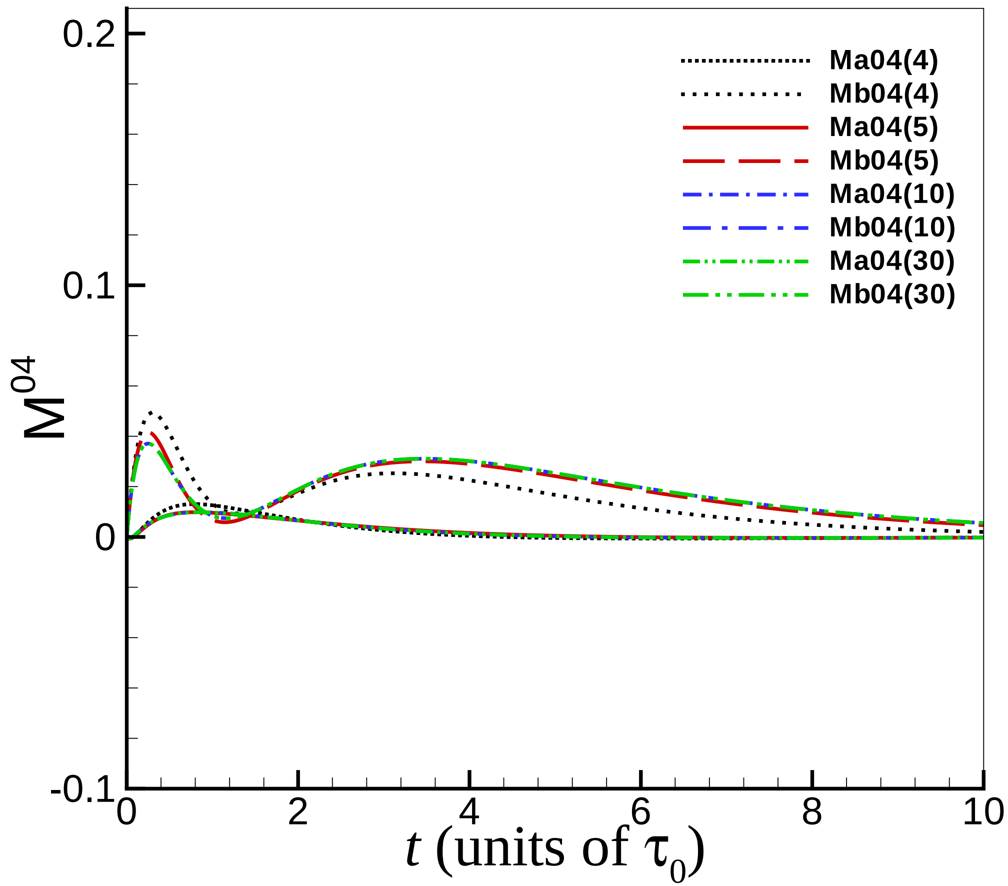




Convergence with increasing moments

Starting with two Maxwellian distributions $(m_a = m_b)$
with $T_a = 0.5T_0$ and $T_b = 1.5T_0$

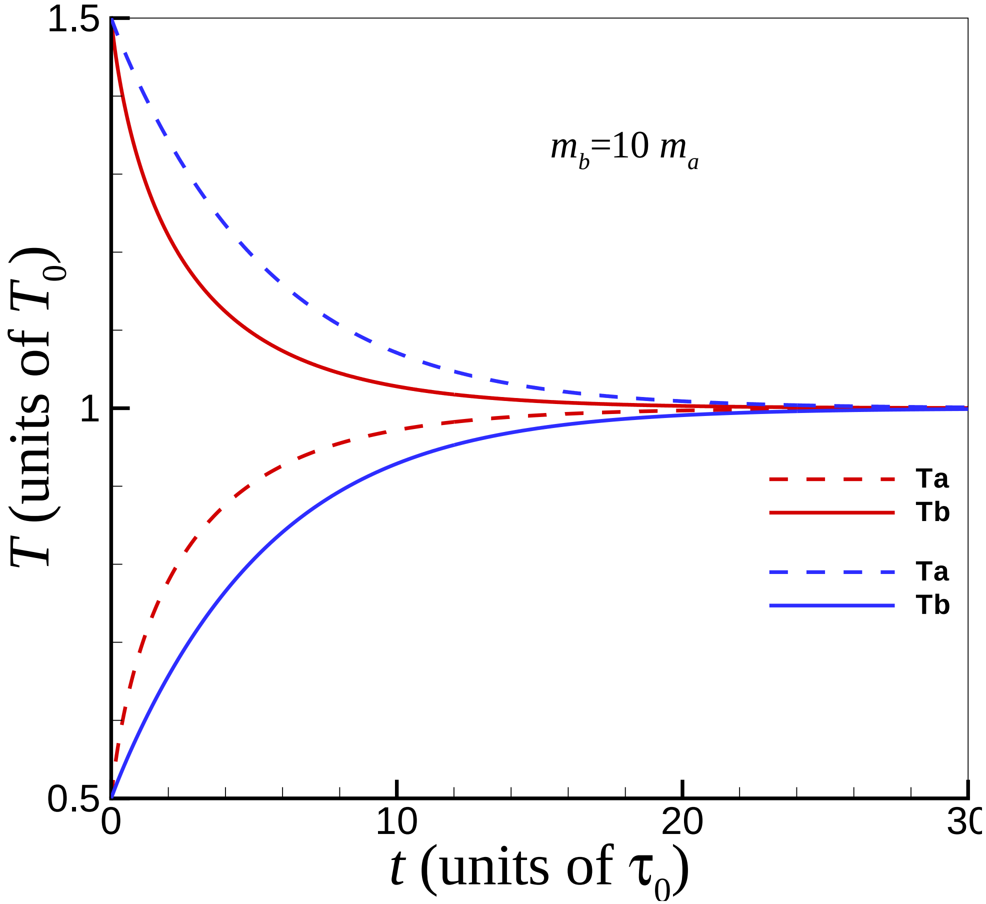


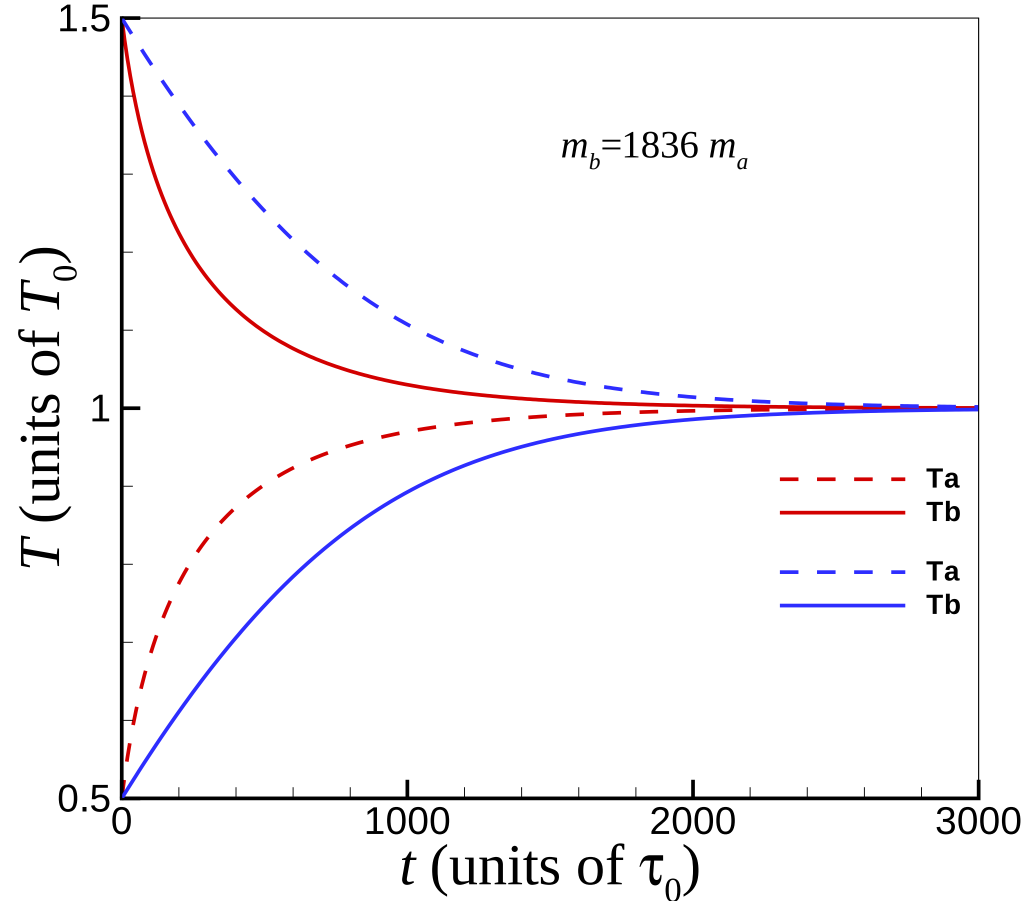


Mass ratio effect

Starting with two Maxwellian distributions $(m_a < m_b)$
with $T_a = 0.5T_0$ and $T_b = 1.5T_0$

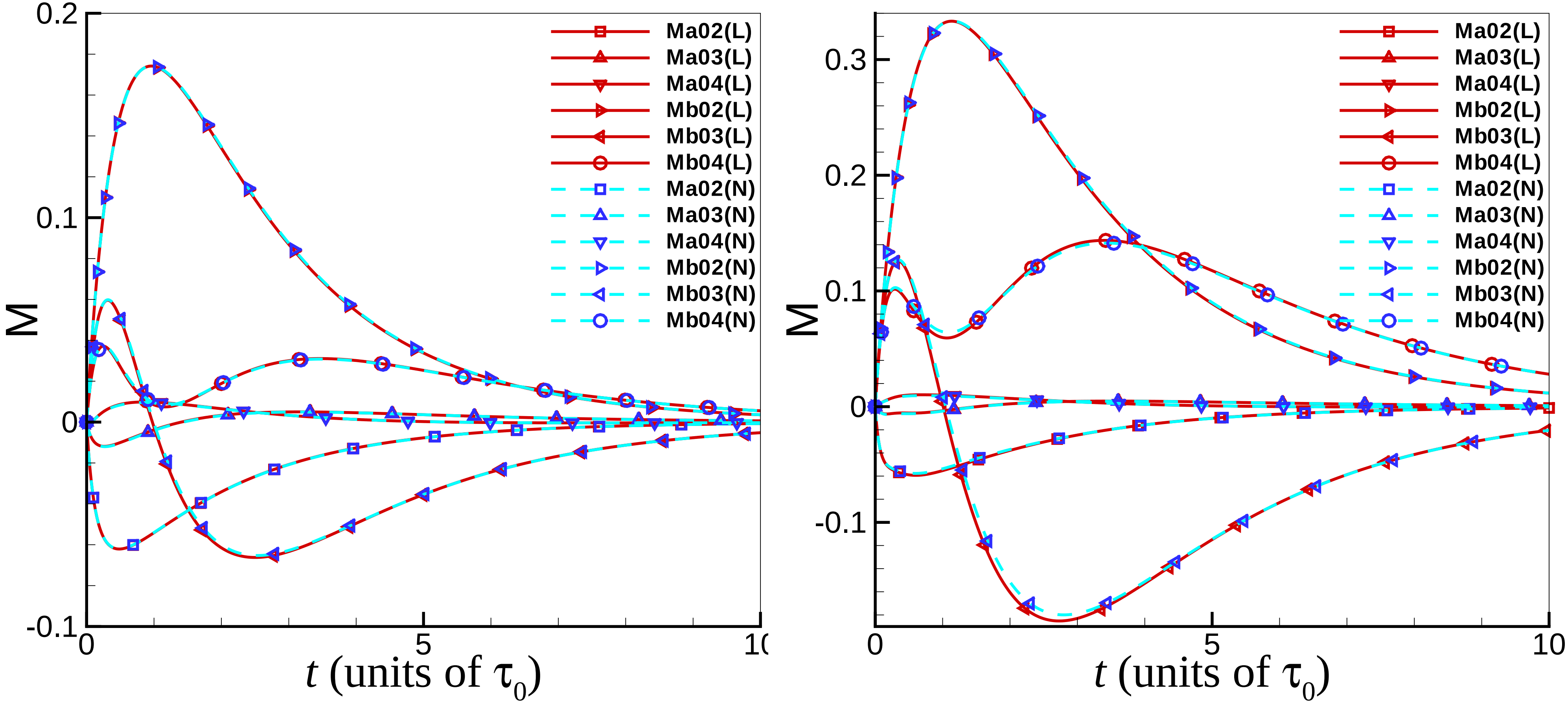
$$\tau_{\rm eq} \sim \frac{m_b}{m_a}\sqrt{\hat{T}_a^3}\,\tau_{aa}^0$$





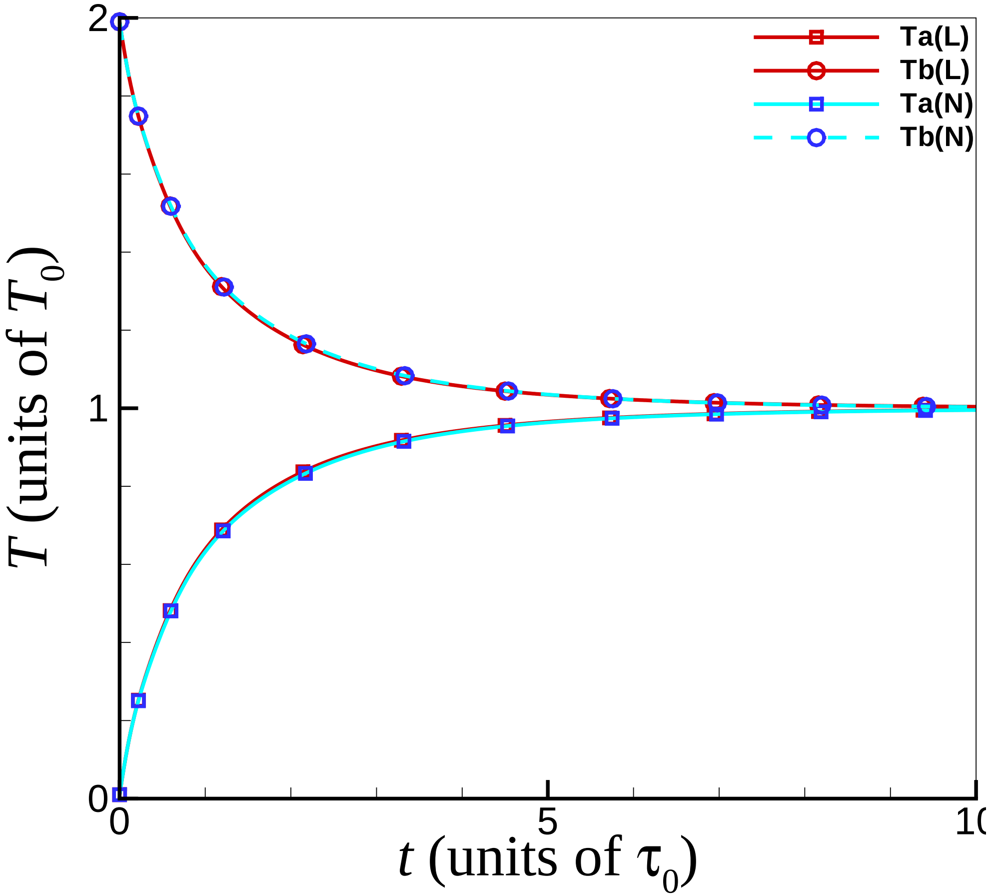
Effect of nonlinear collision terms

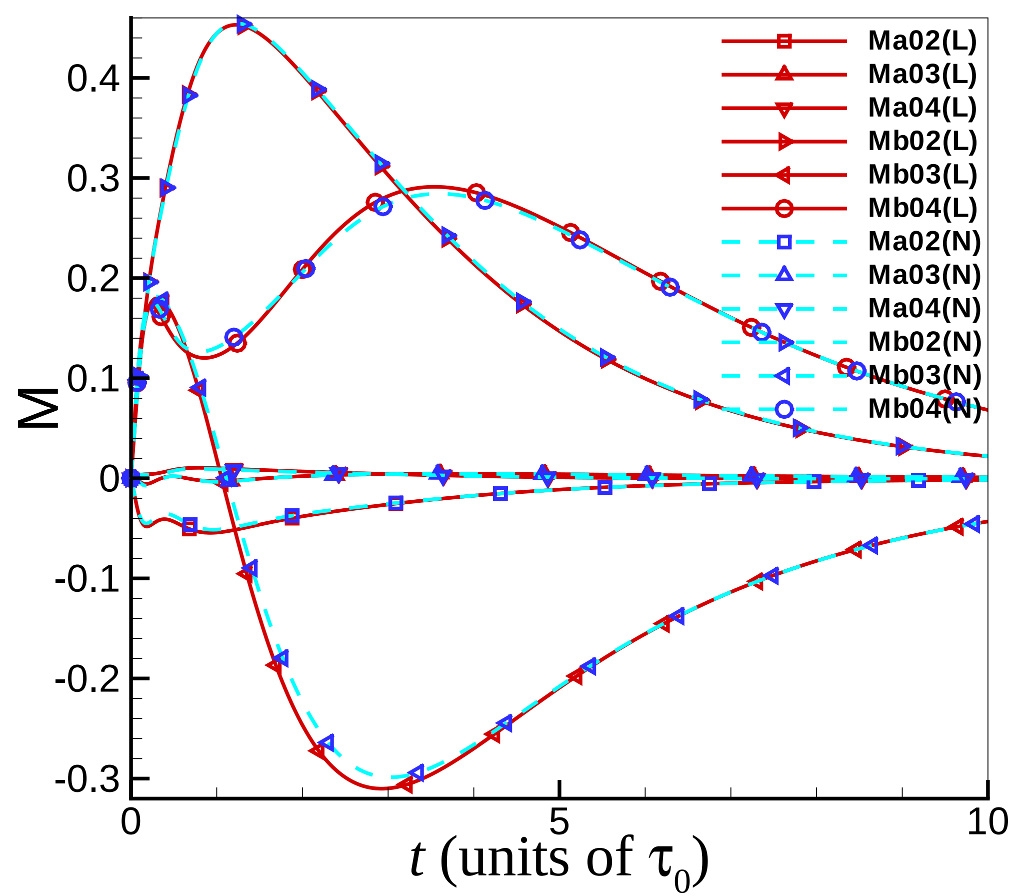
Starting with two Maxwellian distributions $(m_a = m_b)$
with $T_a = 0.5T_0$ and $T_b = 1.5T_0$ vs. $T_a = 0.2T_0$ and $T_b = 1.8T_0$



Effect of nonlinear collision terms

Starting with two Maxwellian distributions $(m_a = m_b)$
with $T_a = 0.01T_0$ and $T_b = 1.99T_0$





Future work

- Effect of nonlinear collision terms
- Vector moment (ion beam heating)
- Temperature gradient effect (time-dependent transport)
- Implementing improved transport coefficients in NIMROD
- Developing 21 or 29 moment equations (dynamic closures) in NIMROD