ITG-driven instabilities in extended MHD

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Can ion-temperature-gradient (ITG) instabilities exist in nimrod simulations?

- ITG effects found in recent nimrod simulations of DIII-D experiments [Schnack 10]
- Extended MHD model with two-fluid physics implemented in NIMROD code [Sovinec *et al.* 04].
- To what extent are ITG physics captured in extended MHD nimrod simulations?
  - May help explain unstable modes in MHD-stable configurations.
  - May help distinguish truly numerical instabilities.
- Recent theory and nimrod calculations developed.

Single fluid formulation of extended MHD model is implemented in NIMROD code [Sovinec *et al.*, 2004]

$$\frac{d\rho}{dt} = -\rho\nabla\cdot\mathbf{u} + D\nabla^2\rho \quad (1)$$

$$\rho\frac{d\mathbf{u}}{dt} = -\nabla p + \mathbf{J}\times\mathbf{B} - \nabla\cdot\boldsymbol{\pi} \quad (2)$$

$$\frac{n}{\gamma-1}\frac{dT}{dt} = -\frac{p}{2}\nabla\cdot\mathbf{u} - \boldsymbol{\pi}:\nabla\mathbf{u} - \nabla\cdot\mathbf{q} + Q \quad (3)$$

$$\frac{\partial\mathbf{B}}{\partial t} = -\nabla\times\mathbf{E} \quad (4)$$

$$\mu_0\mathbf{J} = \nabla\times\mathbf{B} \quad (5)$$

$$\mathbf{E} = -\mathbf{u}\times\mathbf{B} + \eta\mathbf{J} + \frac{\lambda}{ne}(\mathbf{J}\times\mathbf{B} - \nabla(1-\tau)p) \quad (6)$$

where

$$\boldsymbol{\pi}_i = \frac{\tau p}{4\Omega}\left\{\left[\mathbf{b}\times(\nabla\mathbf{u}+\nabla\mathbf{u}^{\mathrm{T}})\cdot(\mathbf{I}+3\mathbf{bb})\right] + [\text{previous term}]^{\mathrm{T}}\right\} \quad (7)$$

Two-fluid equilibrium in shear-less slab configuration and fluite-like perturbation are assumed

- Equilibrium

$$\mathbf{u} = 0 \tag{8}$$

$$\mathbf{B} = B\mathbf{e}_z \tag{9}$$

$$\frac{d}{dx}\left(p + \frac{B^2}{2}\right) = \rho\mathbf{g}\cdot\mathbf{e}_x = \rho g \tag{10}$$

$$ne\mathbf{E} = \nabla p_i - \rho\mathbf{g}. \tag{11}$$

- Perturbation

$$\mathbf{u} = (u_x(x), u_y(x), u_z(x)) \exp\left(-i\omega t + ik_y y + ik_z z\right). \tag{12}$$

where $k_z/k_y \sim \epsilon$, $k_y L_A \sim \epsilon^{-1}$, $k_y d_i \sim \lambda \sim \delta$, $u_y \sim \epsilon u_x$, $u_x \sim u_z$, $\epsilon \ll 1$, where $L_A = |d \ln A/dx|^{-1}$ is the spatial scale of field A in x direction, and $d_i = \sqrt{p_i/\rho}/\Omega$ is the ion Larmor radius.

Ideal MHD model yields coupling of shear-Alfvén and slow wave modes

- In absence of **g**

$$(\omega^2 - k_z^2 u_{\mathrm{A}}^2)\left(\omega^2 - \frac{k_z^2 c_s^2}{1+\gamma\beta}\right) = 0 \tag{13}$$

- 3D g mode

$$\omega^4 + \left[\frac{g^2}{u_{\mathrm{A}}^2(1+\gamma\beta)} - \frac{\rho'}{\rho}g - k_z^2 u_{\mathrm{A}}^2 \frac{1+2\gamma\beta}{1+\gamma\beta}\right]\omega^2 + \frac{k_z^2 u_{\mathrm{A}}^2}{1+\gamma\beta}(k_z^2 c_s^2 - \frac{g^2}{u_{\mathrm{A}}^2} + \gamma\beta g\frac{\rho'}{\rho}) = 0 \tag{14}$$

Extended MHD model leads to linear dispersion relation with ITG effects [Zhu *et al.* 2011]

To dominant order in ϵ, the dispersion relation takes the quintic form

$$c_5\omega^5 + c_4\omega^4 + c_3\omega^3 + c_2\omega^2 + c_1\omega + c_0 = 0 \tag{15}$$

where

$$c_5 = 1 + \gamma\beta + \frac{k_y^2\delta^2\tau^2}{4\Omega^2}\frac{p}{\rho}\beta \tag{16}$$

$$c_4 = -\frac{k_y^3\tau^3}{4\Omega^3}\frac{pp'}{\rho^2}\beta(1+\beta) + \frac{k_y\lambda}{\Omega}\left[(1+\tau+\gamma\beta)\frac{p'}{\rho} - \tau c_s^2\frac{\rho'}{\rho}\right] + \dots \tag{17}$$

$$c_3 = -k_z^2u_A^2(1+2\gamma\beta) - \frac{k_y^2\lambda^2}{\Omega^2}\left[k_z^2u_A^2c_s^2 + \tau\frac{p'}{\rho}(-\frac{p'}{\rho} + c_s^2\frac{\rho'}{\rho})\right] + \dots \tag{18}$$

$$c_2 = -\frac{k_y\lambda}{\Omega}k_z^2u_A^2\left[(1+\tau+\gamma\beta)\frac{p'}{\rho} - \tau c_s^2\frac{\rho'}{\rho}\right] + \frac{k_y\delta\tau}{\Omega}k_z^2u_A^2(1+\beta)\frac{p'}{\rho} + \dots \tag{19}$$

$$c_1 = k_z^4u_A^2c_s^2 + \frac{k_y^2\lambda^2}{\Omega^2}k_z^2u_A^2\tau\frac{p'}{\rho}\left(-\frac{p'}{\rho} + c_s^2\frac{\rho'}{\rho}\right) + \dots \tag{20}$$

$$c_0 = \frac{k_y^3\lambda^2\delta\tau}{\Omega^3}k_z^2u_A^2(1+\beta)\left(\frac{p'}{\rho} - c_s^2\frac{\rho'}{\rho}\right)\left(\frac{p'}{\rho}\right)^2 \tag{21}$$

Further reduction can be made in low frequency and small Larmor radius regime

- To first order in $(\omega_{*i}, \omega_{*p}) \ll \omega_{\mathrm{A}}$ and $k_y \rho_{\mathrm{L}} \ll 1$, the dispersion can be further reduced to

$$(1+\gamma\beta)\left(\frac{\omega}{\omega_{\mathrm{A}}}\right)^4 - \lambda\left[\frac{\omega_{*i}}{\omega_{\mathrm{A}}}\left(\eta-\frac{2}{3}\right)+\frac{\omega_{*p}}{\omega_{\mathrm{A}}}(1+\gamma\beta)\right]\left(\frac{\omega}{\omega_{\mathrm{A}}}\right)^3 - (1+2\gamma\beta)\left(\frac{\omega}{\omega_{\mathrm{A}}}\right)^2 + \left\{\lambda\frac{\omega_{*i}}{\omega_{\mathrm{A}}}\left(\eta-\frac{2}{3}\right)+\frac{\omega_{*p}}{\omega_{\mathrm{A}}}[\lambda(1+\gamma\beta)-\delta\tau(1+\beta)]\right\}\left(\frac{\omega}{\omega_{\mathrm{A}}}\right)+\gamma\beta = 0 \qquad (22)$$

where $\eta = d\ln T/d\ln\rho$, $T = p/\rho$, $\gamma = 5/3$, and

$$\omega_{*i} = -\frac{k_y}{\Omega}\frac{\tau p}{\rho}\frac{\rho'}{\rho}, \quad \omega_{*p} = -\frac{k_y}{\Omega}\frac{p}{\rho}\frac{p'}{p} \qquad (23)$$

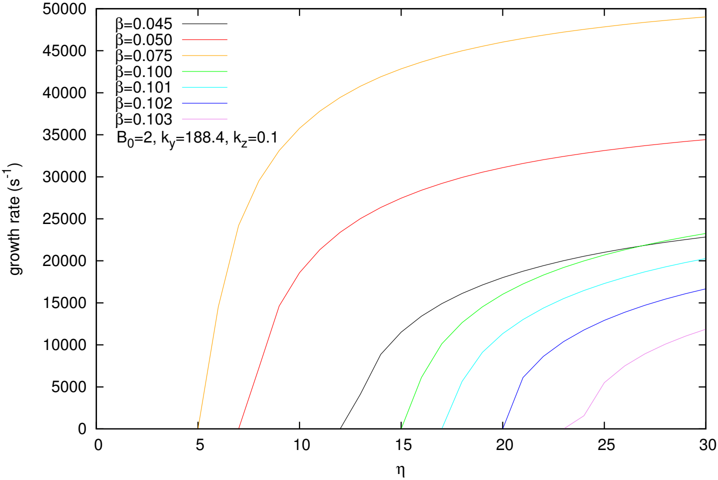
- Formally, the zero β limit gives the cubic form of dispersion relation

$$\left(\frac{\omega}{\omega_{\mathrm{A}}}\right)^3 - \lambda\frac{\omega_{*i}}{\omega_{\mathrm{A}}}\left(\eta-\frac{2}{3}+\frac{\eta+1}{\tau}\right)\left(\frac{\omega}{\omega_{\mathrm{A}}}\right)^2 - \frac{\omega}{\omega_{\mathrm{A}}} + \lambda\frac{\omega_{*i}}{\omega_{\mathrm{A}}}\left(\eta-\frac{2}{3}\right)+\frac{\omega_{*p}}{\omega_{\mathrm{A}}}(\lambda-\delta\tau) = 0 \quad (24)$$

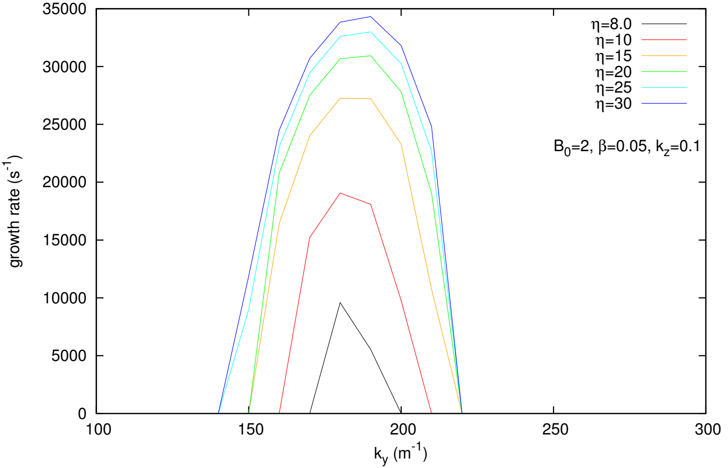
Two cases are considered for the solution of dispersion relation

- Case 1 [Schnack *et al.* 11]: $B_0 = 2$, $\beta = 0.05$, $\tau = 0.2$
- Case 2 [Zhu *et al.* 07]: $B_0 = 6$, $\beta = 0.1$, $\tau = 0.5$
- Parameters common to both cases:
  $\rho = 2 \times 10^{20} m_i$, density (pressure) gradient scale length L_ρ (L_p).

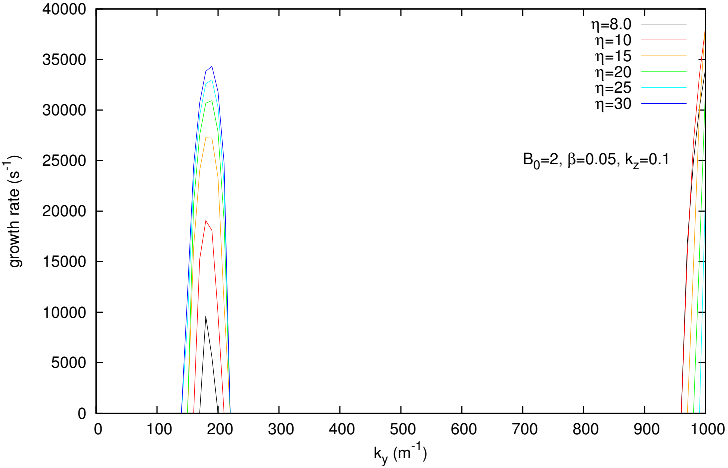
η thresholds for ITG-driven instability onset are sensitive to β



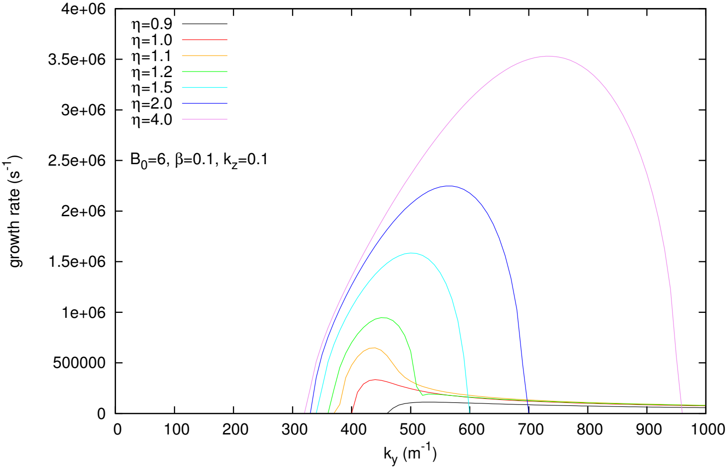
Solutions from analytic dispersion relation show instabilities when $\eta > 8$ in $k_y \rho_L \sim 0.1$ regime



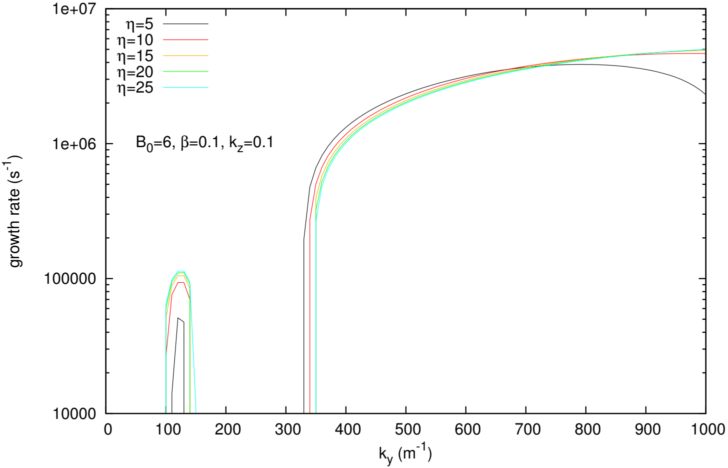
In $k_y \rho_L \sim 0.1 - 1$ regime there are two branches of ITG-driven instabilities



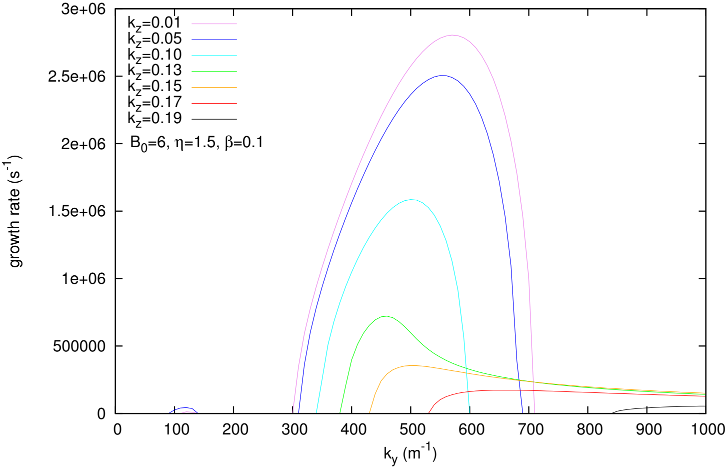
High ITG branch ($k_y \rho_L \sim 1$) more dominant in higher β regime ($\eta_{\rm crit} \sim 0.8$)



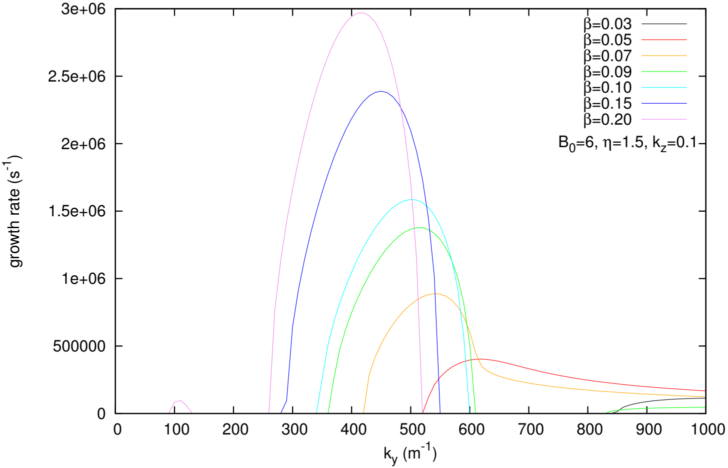
Low ITG branch ($k_y \rho_L \sim 0.1$) becomes less dominant in higher β regime



Instabilities more unstable at smaller k_z and have a cut-off $k_z \sim 0.2$ in high branch ($k_y \rho_L \sim 1$)



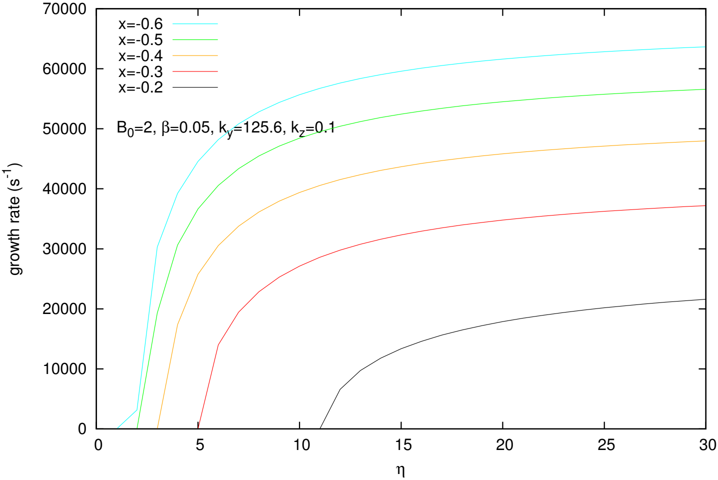
High branch ($k_y \rho_L \sim 1$) growth increases with β ($\beta_{\rm crit} \sim 0.025$) and shifts towards smaller $k_y \rho_L$



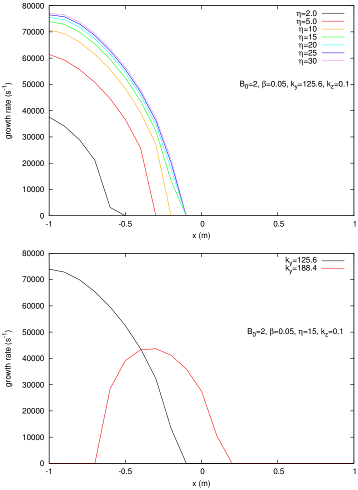
Exponential profiles for both density and pressure are considered in nimrod calculation

- Density and pressure: $\rho(x) = \rho_0 e^{-\frac{x}{L_\rho}}$, $p(x) = p_0 e^{-\frac{x}{L_p}}$, $\eta = \frac{L_\rho}{L_p} - 1$
- Magnetic field $\mathbf{B} = B(x)\hat{\mathbf{Z}}$ is in $R - Z$ plane.
- k_y of perturbation is in periodic direction.
- While η is constant, other key parameters such as β can have large range of variation over x.

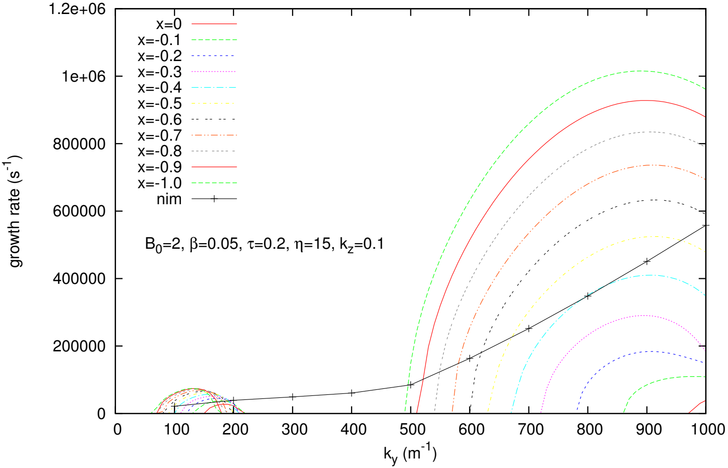
For a fixed k_y ITG growth at each location in x has its own η dependence



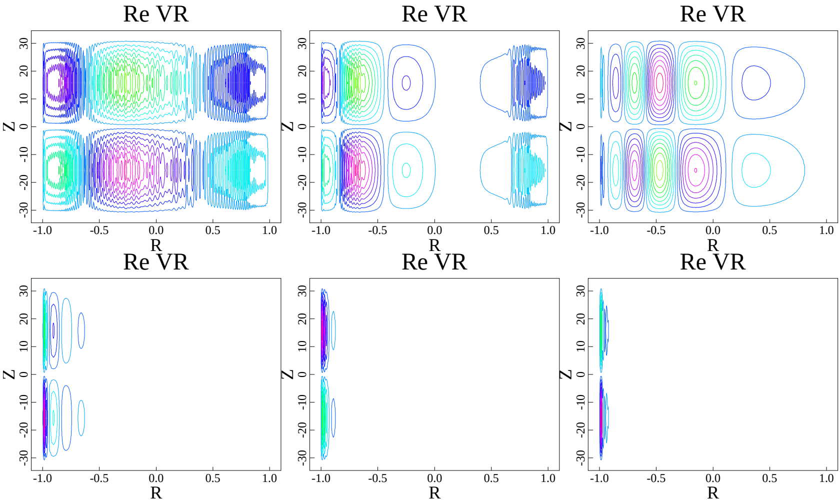
For a fixed η or k_y each location in x is associated with an ITG growth from local dispersion relation



Comparison between nimrod calculation and theory show qualitative consistency

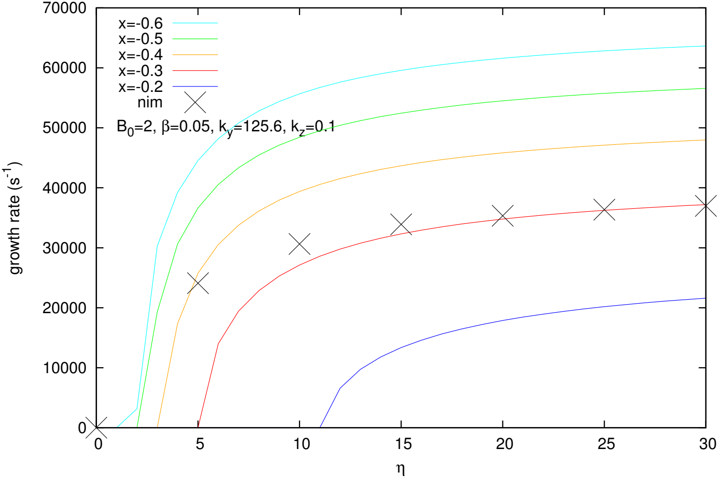


As k_y increases mode structure becomes more localized on the most unstable flux surface in x



Upper row: $k_y = 100, 200, 400$; Lower row: $k_y = 600, 800, 1000$

Comparison between nimrod calculation and theory show qualitative agreement when spatial profile effects taken into account



Summary

- Recent theory and nimrod calculations have identified ITG-driven instabilities in an extended MHD model.
- Extended MHD theory based on single-fluid formulation is able to explain observations from nimrod calculations
  - Two branches of ITG-driven growth in k_y space
  - Asymmetric spatial structure of ITG-mode in exponentially shaped equilibrium
- Discrepancy may due to spatial profile effects.
- Future work
  - Comparison with two-fluid formulation.
  - NIMROD calculation in other equilibrium profiles better for comparison with local dispersion relation.