Validation of MHD Models for MST Reversed Field Pinch Plasmas

Craig M. Jacobson, Karsten J. McCollam, and the MST Team
Outline

• Introduction to Validation

• MST Validation Activities

• Previous MST MHD Validation Work

• Current MST MHD Validation Work

• Conclusions and Future Work
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Validation is vital to fusion energy programs across the world

- Computational models are improving, but how do we know if their predictions are correct?

- **Validation**: quantitative assessment of the degree to which a computational model is an accurate representation of the real world
  - Asks if simulation agrees with experiment under a rigorous comparison
  - This is *not* simply plotting simulation results against experimental results
  - Quantitative, inclusion of uncertainty is a key consideration
  - Allows *objective* comparisons of conceptual models

- Validation facilitates assessment and improvement of fusion plasma *predictive* capabilities, and is necessary if predictive models are to be trusted to inform the design of future reactors

- Validation is a significant challenge
  - The effort requires substantial resources (physicists, dedicated experiments, computational resources)
  - Validation is still in the exploratory phase in plasma physics
  - The community must evaluate what it means to Validate
  - Validation can aid understanding of physics
Validation is part of a larger process comparing experiments, conceptual models, and computational models

- **Qualification**: theoretical specification of the expected domain of applicability for a model

- **Verification**: assessment of the degree to which a computational model correctly implements a conceptual model

- **Validation**: assessment of the degree to which a computational model is an accurate representation of the real world

- In plasma physics, Qualification and Verification are done routinely, but Validation is not
  - Validation is more routinely performed in computational fluid dynamics and aerospace industry
Validation metrics allow quantitative comparisons between simulation and experiment with consideration of uncertainty

- Validation metrics can take on various forms

\[ M_j = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{y_i - Y_i}{Y_i} \right| \rightarrow \begin{cases} 0 & \text{agreement} \\ \infty & \text{disagreement} \end{cases} \]

\[ M_j = 1 - \frac{1}{n} \sum_{i=1}^{n} \tanh \left( \left| \frac{y_i - Y_i}{Y_i} \right| + \left| \frac{\sigma Y_i}{Y_i} \right| + \left| \frac{\sigma y_i}{y_i} \right| \right) \rightarrow \begin{cases} 0 & \text{disagreement} \\ 1 & \text{agreement} \end{cases} \]

\[ M_j = \frac{1}{N_{\text{degrees}}} \sum_{i=1}^{n} \left( \frac{y_i - Y_i}{\sigma y_i + \sigma Y_i} \right)^2 \rightarrow \begin{cases} 0 & \text{perfect agreement} \\ 1 & \text{expected agreement} \\ \infty & \text{disagreement} \end{cases} \]

- There is no “correct” metric
  - Metric must identify salient elements of the model being tested
  - Metric must confront disagreement between simulation and experiment
  - Metric should incorporate uncertainties in both experiment and simulation

\[ \begin{array}{c|c|c}
\text{Value} & Y_i & \bar{Y}_i \\
\text{Uncertainty} & \sigma y_i & \sigma Y_i
\end{array} \]
Comparison of many quantities is essential to meaningfully compare simulation and experiment

- Primacy hierarchy tracks how quantities combine to produce other quantities

![Primacy levels for simulation](image)

- Composite metrics characterize overall agreement of entire simulations

\[
V = \sum_{j}^{n} M_j P_j S_j W_j
\]

\(S\): sensitivity rating

\(W\): repetition weight

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MST is well-suited for Validation

- MST plasmas are well-diagnosed providing many measurements for Validation
  - RFP provides complimentary parameter space to tokamaks
    - For example, $q<1$ everywhere
    - Multiple nonlinearly interacting tearing modes
    - Successful Validation of a code in both RFP and tokamak configurations increases confidence in its ability to be used to predictively model a reactor

- MST Validation efforts:
  - MHD relaxation: tearing modes and relaxation processes (in progress)
  - Fast ions: energetic ion beam-driven processes (emerging)
  - Electrostatic turbulence: high-$k$ fluctuations, micro-instabilities (exploratory)

- The remainder of this presentation focuses on MHD Validation efforts
Dominant thermal energy transport mechanism is based on interaction of MHD tearing fluctuations

\[ \chi_{RR} \sim \tilde{b}_{m,n}^2 \]

Lundquist number must increase for a reactor, so understanding how \( \tilde{b} \) scales with \( S \) is key

\[ S = \frac{\tau_R}{\tau_A} = \frac{\mu_0 a^2}{\eta (Z_{eff}, n_e, T_e)} \cdot \frac{B}{a \sqrt{\mu_0 \rho}} \sim T_e^{3/2} I_P \]

Previous study found \( \tilde{b}_a \sim S^{-0.18} \) or \( \tilde{b}_a \sim S^{-0.07} \)
- However, \( Z_{eff} \) measurements were wrong
- MST diagnostics and plasma control systems have significantly improved since then

Integrated data analysis (IDA) determines \( Z_{eff} \approx 2 \) in core

M.E. Galante, EPR invited talk (2014)

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Two MHD codes are used to simulate MST RFP plasmas

- **DEBS**
  - Nonlinear, single-fluid, visco-resistive MHD code, can be operated with finite-\(\beta\)
  - Dynamic viscosity is necessary for operation at higher \(S\)
  - Used in RFP and tokamak configurations, but restricted to cylindrical geometry

\[
\frac{\partial \mathbf{A}}{\partial t} = SV \times \mathbf{B} - \eta \mathbf{J}
\]
\[
\rho \frac{\partial \mathbf{V}}{\partial t} = -S \rho \mathbf{V} \cdot \nabla \mathbf{V} + S \mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{V}
\]

- **NIMROD**
  - Nonlinear, two-fluid, extended MHD code, can be operated with finite-\(\beta\)
  - Includes gyroviscous stress terms
  - Flexible geometry
  - Used in RFP, tokamak, spheromak, reconnection simulations, and other configurations

\[
\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \frac{\mathbf{J} \times \mathbf{B}}{en} - \frac{\nabla p_e}{en} + \eta \mathbf{J} + \frac{m_e}{e^2 n} \frac{\partial \mathbf{J}}{\partial t}
\]
\[
m_i n \frac{d\mathbf{v}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \mathbf{\Pi}_{gyro} - \nabla \cdot \nu m_i n \mathbf{W}
\]

- Linear growth rates of tearing modes as function of \(S\) and \(Pm = \mu_0 v / \eta\) benchmarked
  - Typical discrepancy < 1%, but maximum discrepancy is 4.5%
DEBS simulations found good agreement with sawtooth period and allow Validation metric comparisons

- $S = 4 \times 10^6$ (corresponding to 400 kA), zero-$\beta$, single-fluid, $R_0/a \approx 3$
- $Pm = \mu_0 \nu / \eta = 100$ with dynamic $\nu$
- $\eta_{Spitzer}$ and $\eta_{neo}$ profiles based on experimental measurements used as inputs
- Sawtooth period Validation metric shows neoclassical case performing better
- Composite metric shows neoclassical case performs better than Spitzer case, but not by much
  - $\tilde{b}_a$, $q$, and sawtooth period included
- In both cases, edge mode amplitudes are ~2 times higher than in experiment

\[\begin{array}{|c|c|c|}
\hline
& \text{Spitzer} & \text{Neoclassical} \\
\hline
\text{Relative Error} & 32.8\% \pm 8.4\% & 0.16\% \pm 6.8\% \\
\text{\chi}^2 & 15.2 & 0.0005 \\
\text{Tanh} & 0.622 & 0.9249 \\
\hline
\end{array}\]

\[\begin{array}{|c|c|c|}
\hline
& \text{Spitzer} & \text{Neoclassical} \\
\hline
V_{NE} (0 = \text{agree}) & 0.676 \pm 0.172 & 0.581 \pm 0.114 \\
V_{x^2} (0 = \text{agree}) & 25.04 & 24.87 \\
V_{\text{tanh}} (1 = \text{agree}) & 0.55 & 0.62 \\
\hline
\end{array}\]

Two-fluid NIMROD simulations have magnetic fluctuation amplitudes half as large as single-fluid NIMROD simulations

- $S = 8 \times 10^4$, $Pm = 1$, cylindrical geometry, finite-\(\beta\) with flat pressure profile
- MST-like $q$ profile, aspect ratio $R_0/a \approx 3$
- Two-fluid simulations have magnetic fluctuation amplitudes half as large as single-fluid simulations
  - Trends toward better agreement with experiment

Motivates further study with NIMROD code for Validation

These simulations are computationally expensive
  - This makes estimation of uncertainty very difficult

King, Phys. Plasmas 19, 055905 (2012)
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Details of nonlinear NIMROD simulations

- Single-fluid, visco-resistive MHD (isotropic viscosity)
- Cylindrical geometry
- $Pm = 1$
- $R_0/a = 3$
- Runs performed on UW-Madison’s High Performance Computer Cluster
  - 20 cores/node
  - 1 job/node
  - Typically 5-10 of my jobs running at the same time for parameter scans

$$\eta(r) = \eta_0 \left(1 + \left(\sqrt{20} - 1\right) \left(\frac{r}{a}\right)^2\right)^2$$

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<thead>
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<th>Setting</th>
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Paramagnetic pinch is used as a starting point for these simulations

Cylindrical equilibrium with applied axial electric and magnetic field

Uniquely specified by \( \lambda_0 \) for a given \( A = R/a \) and resistivity profile

May be unstable to tearing modes

Preliminary nonlinear single-fluid NIMROD runs give $\tilde{b}_a$ scalings

- Single-fluid, cylindrical geometry, $Pm = 1$, $R_0/a = 3$

- Preliminary best fits for $\tilde{b}_a$ scalings for several modes:

  $\tilde{b}_{a,1,5} \sim S^{-0.26\pm0.04}$, \quad $\tilde{b}_{a,1,7} \sim S^{-0.27\pm0.02}$
  
  $\tilde{b}_{a,1,6} \sim S^{-0.21\pm0.03}$, \quad $\tilde{b}_{a,1,8} \sim S^{-0.22\pm0.03}$

- At $S = 10^4$, $S$ and $Pm$ varied by $\pm10\%$ to examine sensitivity
The Lundquist number of MST plasmas spans five orders of magnitude and can now match the capabilities of simulations

- New configuration of programmable power supply (PPS) to control $I_p$
- Very low current (~60 kA) reversed plasmas achieved (standard plasmas limited to ~180 kA)
- Long flat tops (up to 65 ms)
- MST is Ohmically heated, so increase in $I_p$ results in increased $T_e$, and $S \sim T_e^{3/2} I_p$ increases
- MST experiments and MHD RFP simulations can now match $S$, enables more meaningful Validation
Dedicated experiments measure fluctuation amplitudes at the edge and Lundquist numbers

- MST RFP plasmas in a range of parameters
- Thomson scattering used to determine $T_e$, interferometer used to determine $n_e$, $Z_{eff} = 2$ assumed
- $S = \frac{\tau_R}{\tau_A} = \frac{\mu_0 a^2}{\eta(Z_{eff}, n_e, T_e)} \cdot \frac{B}{a \sqrt{\mu_0 \rho}} \sim T_e^{3/2} I_P$
- Edge magnetic probes used to determine $\tilde{b}_a$ for each $n$
- Data analyzed with respect to time of sawtooth crash to form large ensembles
Experimental $T_e$ and $S$ span large range

- First $T_e$ measurements in ultra-low current plasmas
- $T_e$ as low as 40 eV, $S$ as low as $4 \times 10^4$
Edge measured magnetic fluctuations fit to power law scaling

\[ \tilde{b}_{a,1,5} \sim S^{-0.45 \pm 0.14}, \quad \tilde{b}_{a,1,7} \sim S^{-0.33 \pm 0.09} \]

\[ \tilde{b}_{a,1,6} \sim S^{-0.28 \pm 0.08}, \quad \tilde{b}_{a,1,8} \sim S^{-0.36 \pm 0.10} \]
Edge measured magnetic fluctuations fit to power law scaling

Scalings assume fluctuations only scale with $S$, what about other parameters?
There are several parameters to consider other than $S$

- Pinch parameter or normalized current
  \[ \Theta = \frac{B_P(a)}{\langle B_T \rangle} \]

- Magnetic Prandtl number
  \[ Pm = \frac{\mu_0 v}{\eta} \]
  - Braginskii perpendicular ion viscosity gives $Pm = 0.25$
  - Independent MST experiments suggest $Pm = 55 - 90^{(1)}$ and $Pm = 60 - 280^{(2)}$
  - How $Pm$ scales with $I_P$ or $n_e$ in MST is unknown
  - Anisotropy could be important

- Toroidal flux
  - Simulations typically conserve $\Phi$ over sawteeth, but MST does not
  - MST has external power supplies with $L, R$ that could affect sawtooth dynamics

- Reversal parameter
  \[ F = \frac{B_T(a)}{\langle B_T \rangle} \]

\((1)\) A. F. Almagri, Phys. Plasmas 5, 3982 (1998)
\((2)\) R. Fridström, submitted
Reversal parameter $F$ can be tuned by changing initial $a\lambda_0$

- $F$ decreases with $S$ for fixed initial $a\lambda_0$
  - As $F$ changes, the location of the reversal surface changes, which can affect mode coupling, and therefore mode amplitudes

![Graphs showing $F$ vs. $S$ and $a\lambda_0$ for fixed $a\lambda_0 = 3.88$ and $S = 3 \times 10^4$](image)

- Changing initial $a\lambda_0$ changes resulting $F$ (and $\Theta$)
  - Our approach is to target $F = -0.2$ to match experimental conditions
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• MST is an ideal test bed for Validation

• Previous MHD simulations show some agreement with MST plasmas
  – Single-fluid DEBS Validation shows that simulations with neoclassical resistivity produces somewhat better agreement than simulations with Spitzer resistivity, but edge mode amplitudes are high
  – Previous two-fluid NIMROD simulations produce better agreement with MST edge mode amplitudes

• Validation of MHD simulations focus on edge mode amplitude scalings
  – PPS configuration allows access to very low $S$, matching simulations
  – Preliminary $S$ scalings for single-fluid simulations and experiments have been produced, but scalings alone can obscure differences in mode amplitudes
  – Other parameters should be considered
  – This is still a work in progress
Future Work

- Evaluation of Validation metrics comparing experimental plasmas and single-fluid MHD simulations

- Further simulation
  - Two-fluid MHD
  - Inclusion of toroidal circuit model in simulations

- Verification studies comparing NIMROD and DEBS to previous SPECYL and PIXIE3D simulations in collaboration with RFX RFP

- DEBS simulations with applied helical plasma boundary conditions to examine transition to plasmas dominated by a single mode

- Collaboration with RELAX RFP extends parameter space for Validation…
Collaboration with RELAX group expands parameter range for Validation studies

- RELAX has a lower Lundquist number than MST: $10^4 \leq S \leq 10^5$
  - Feasible for NIMROD simulations
- RELAX has a low aspect ratio: $A = R_0/a = 2$
  - Both cylindrical and toroidal geometry will be examined
- Linear growth rates examined
- Preliminary nonlinear runs in progress ($S = 5 \times 10^3$, $Pm = 1$)
- Results will be compared to MIPS MHD analysis (Mizuguchi, IAEA FEC (2012))