Numerical investigation of design and operating parameters on CHI spheromak performance

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NIMROD Team Meeting
April 30, 2017

This work is supported by the Defense Advanced Research Projects Agency (DARPA) under Grant Number N66001-14-1-4044.
Introduction & Motivation
The CHI spheromak is formed and sustained through driven magnetic self-organization.

- During CHI, two electrodes connected by vacuum magnetic flux are biased relative to each other.
- Current flows along the magnetic field lines, producing an expanding flux bubble that fills the conducting vacuum vessel.
- A current-driven $n_\phi = 1$ magnetic instability changes the magnetic topology.
- For a simply-connected device, this instability is called the “column mode.”

Sustained Spheromak Physics eXperiment (SSPX) Design

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The column mode is an \( n_\phi = 1 \) kink instability of the current column near the geometric axis.

- The column mode acts as a (semi-) coherent dynamo that converts toroidal flux into poloidal flux, i.e. predominantly poloidal current into toroidal current.\(^2\)

- The column mode is self-stabilizing, as the buildup of poloidal flux effectively reduces the value of \( \lambda = \mu_0 J_\parallel / B \).

- Resistive decay of the core toroidal current increases \( \lambda \), triggering instability and toroidal current drive.

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SSPX achieved encouraging results, despite being limited by the power driving system and wall heat dissipation.

- The Sustained Spheromak Physics eXperiment (SSPX) achieved $T_e \sim 0.5$ keV, $B_{\text{tor}} > 1$ T, $I_p \sim 1$ MA, and peak $\beta_e > 5\%$.
- The power system on SSPX could sustain hundreds of kiloamperes for about 5 ms and was configurable to produce a series of pulses of different amplitudes and durations.
- The wall was a tungsten-coated copper shell, including the injector region, which was subjected to the largest heat loads.
- Despite its promise as a confinement concept, the spheromak has only been studied at the basic plasma science and concept exploration levels.

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The objective of this project is to develop the spheromak concept into a compact, pulsed fusion device for the efficient production of neutrons (and/or electricity).

- We’re exploring two separate approaches (multi-pulse CHI and magnetic flux compression) for sustaining and heating a spheromak plasma to fusion temperatures.
- For both approaches, the initial spheromak plasma is formed by CHI.
- A successful device would achieve high average neutron flux in a relatively compact footprint, e.g. comparable to a few shipping pallets including the power supply.
- This numerical study seeks to explore and optimize the formation of the CHI spheromak and its magnetic compression.
Numerical Model
The computations solve the low-frequency MHD model, starting from vacuum magnetic field and cold fluid.

\[ \frac{\partial n}{\partial t} + \nabla \cdot (nv) = 0 \]

\[ \rho \left( \frac{\partial v}{\partial t} + v \cdot \nabla v \right) = J \times B - \nabla p + \nabla \cdot \Pi(W) \quad \text{where} \quad W = \nabla v + \nabla v^T - (2/3) (\nabla \cdot v) I \]

\[ \frac{2n}{3} \left( \frac{\partial T_e}{\partial t} + v_e \cdot \nabla T_e \right) = -nT_e \nabla \cdot v_e - \nabla \cdot \left[ \kappa_{\parallel e} \hat{b} \hat{b} + \kappa_{\perp e} I \right] \cdot \nabla T_e + n\sigma (T_i - T_e) + \eta J^2 \]

\[ \frac{2n}{3} \left( \frac{\partial T_i}{\partial t} + v_i \cdot \nabla T_i \right) = -nT_i \nabla \cdot v_i - \nabla \cdot \left[ \kappa_{\parallel i} \hat{b} \hat{b} + \kappa_{\perp i} I \right] \cdot \nabla T_i + n\sigma (T_e - T_i) \]

\[ \frac{\partial B}{\partial t} + \nabla \times \left[ \eta J - v \times B \right] = 0 \quad \text{where} \quad J = \mu_0^{-1} \nabla \times B \]

- The computations use realistic, evolving, locally-computed transport coefficients.
- Our model does not include neutral particle effects (e.g. ionization and recombination).
- The NIMROD code\(^5\) (nimrodteam.org) is used to solve these systems.

Only the injector is prescribed: all dynamics follow self-consistently from the model.

- The injector is simulated by specifying $RB_\phi = \mu_0 I_g / 2\pi$ along the injector boundary.

- To encourage the expansion of the flux bubble into the domain, resistivity is enhanced along the injector boundary:
  \[ \eta \rightarrow \eta + (D_s - 1) \eta_{\text{inj}}. \]

- The density boundary condition along the injector edge is initially no-flux, but transitions to Dirichlet when $n < n_{\text{crit}}$.

- The injector current trace is prescribed, but the injector voltage is produced self-consistently from the plasma model and not an external circuit model.
To consistently model flux compression, the calculations use discrete external field coils.

- The magnetic field from close, discrete compression coils helps stabilize the spheromak to the tilt mode.
- The coils are modeled as a series of single-turn current loops.
- Previous implementations of flux compression in NIMROD have assumed a rectangular (R,Z)-aligned cross-section with uniform $B_Z$ from compression.
- We’ve generalized the implementation to arbitrarily shaped cross-sections.
- During initialization, $B_R$, $B_Z$, and $A_\phi$ are computed along the edge of the domain for each coil at unit current.

\[
B_n (t) = \hat{n} \cdot \sum_{i=1}^{n_{coil}} l_{c,i}(t) \hat{B}_{c,i}
\]

\[
E_\phi (t) = - \sum_{i=1}^{n_{coil}} \frac{\partial l_{c,i}}{\partial t} \hat{A}_{\phi,i}
\]

\[
B_t (t) = \hat{t} \cdot \mathbf{B} (t)
\]

\[
\mathbf{B}^* (t) = (n_r B_n + t_r B_t) \hat{r} + (n_z B_n + t_z B_t) \hat{z}
\]
Multi-pulse CHI in SSPX
We simulated entire shots in the SSPX spheromak and made direct comparisons between experimental data and synthetic diagnostics.

- We determined that a single-temperature resistive MHD model can sufficiently capture relevant physical behavior to qualitatively assess spheromak performance.
- A similar model has been used to study the interaction between thermal transport and magnetic relaxation in previous spheromak studies.  
- With a simplified physics model, we can explore a greater number of candidate operational modes and flux conserver/injector geometries.
- Once we determine parameters that qualitatively optimize spheromak performance, those cases can be explored with a more complete physics model to quantify the performance gains.

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For simulations of multi-pulse shots in SSPX, the injector voltage trace and the onset of the column mode agree with the experiment.

SSPX Shot #19719 – 40 mWb bias flux
The expansion of the injector flux bubble is consistent with experimental discharges in SSPX and the results of prior computational studies.

(a) $t = 80 \mu s$, (b) $t = 97 \mu s$, (c) $t = 116 \mu s$
As the current decreases at the end of an injector pulse, the spheromak plasma recedes into the throat of the injector.

(a) $t = 270 \, \mu s$, (b) $t = 349 \, \mu s$, (c) $t = 500 \, \mu s$

- The contraction of the plasma and high current density present at the separatrix indicate the occurrence of pull-type reconnection in the injector region.
During a refluxing pulse, a second expanding flux bubble forms in the injector region.

When the flux bubble comes into contact with the pre-existing spheromak plasma, reconnection opens and connects the outer flux surfaces of the spheromak to the injector.

- During this process, the spheromak plasma is compressed into the flux conserver.

- (a) $t = 571 \mu s$, (b) $t = 619 \mu s$, (c) $t = 684 \mu s$
Despite producing qualitatively different temperature profiles, the 1T & 2T models yield essentially the same global magnetic evolution.
Shots with similar $\lambda_{inj}$ traces produce qualitatively different behavior for flux amplification and spheromak lifetime.

- Linear, ideal MHD stability analysis\(^9\) predicts the onset of the column mode instability.
- However, it doesn’t directly address the accessibility of the equilibria or poloidal flux amplification, which are determined by nonlinear plasma evolution, motivating our formation study.
- Magnetic helicity increases after the column mode, and the effect is more pronounced at higher bias flux.

The computations successfully reproduce the magnetic evolution of multi-pulse shots in SSPX.

The computational and experimental injector voltages agree, despite the electric field in our numerical model lacking an electrostatic component.

This agreement indicates that the dominant contribution to the injector voltage is stretching of the magnetic field-lines during expansion of the poloidal flux bubble, an inductive effect.

Both our computational results and experimental magnetic diagnostics indicate that the column mode instability (and poloidal flux amplification) did not occur during every injector current pulse.

This suggests that changing the operation of SSPX could have lead to better performance.
Calculations explore how the rate of change of the injector current affects both spheromak performance and injector voltage requirements.

- The voltage requirements of an experiment affect the design and cost of the power supply.
- $I_{inj}$ linearly ramps from 0 to 500 kA over a time $t_0$ and is then held constant.
- We’ve found two effective limits for the ramp rate:
  - Too slow $\rightarrow$ gradual diffusion, less coherent flux bubble
  - Too fast $\rightarrow$ current filaments form along the expanding flux bubble

![Graph](image)

(a) Injector Voltage

(b) Flux Amplification

$V_{\max} = 1.4, 14.3, 31.3, 32.6$ V, respectively. For the initial bias magnetic field, $\tau_A \sim 10^{-5}$ s.
Current filaments along the expanding flux bubble drive MHD activity prior to the column mode.

- The MHD activity lowers the threshold for the column mode.
- The current filaments are undesirable, because they require large injector voltage, but don’t increase the poloidal flux amplification.
The column mode onset ($\Phi$) and poloidal flux amplification ($A_\Psi$) are largely insensitive to the injector current ramp rate.

- $A_\Psi$ asymptotes toward a constant value, regardless of the injector current rise time $t_0$.
- The poloidal flux amplification produced initially by the column mode ($A_\Psi \approx 2.3$) is much greater than the relative increase later for the same injected flux.
- At $\Phi_{inj} = 0.7 \, \text{Wb}$, $A_\Psi \approx 2.5$, or only 4–9 % more than produced initially.
- From our multi-pulse calculation, we know this trend holds during refluxing.

Injected Flux: $\Phi_{inj} = \int_0^t V_{inj}(t) \, dt$
The threshold ($\Phi$) of the column mode instability and amount of poloidal flux amplification scale with the injected current.

- $I_{inj}$ must be large enough to drive the column mode.
- If $I_{inj}$ is too large, then could drive current filament.
- Between those limits, the amount of injected energy retained in the plasma as magnetic energy ($\eta_M$) increases with injected current.

\[
\eta_M = \frac{\int_V \frac{B^2}{2\mu_0} dV}{\int_0^t |I_{inj}V_{inj}| dt}
\]

\[
\eta_H = \frac{1}{2\mu_0L_c} \frac{\int_V \mathbf{A} \cdot \mathbf{B} dV}{\int_0^t |I_{inj}V_{inj}| dt}
\]

Constant $\Psi_{bias} = 40 \text{ mWb}$
When exploring poloidal flux amplification at different values of $\psi_{bias}$, we scaled the $I_{inj}$ to keep $\lambda_{inj}$ constant.

- We chose intermediate ramp rates to avoid a diffuse flux bubble and current filamentation.
- The threshold of the column mode, $\eta_M$, and $\eta_H$ increase with the bias flux and injected current.
- Poloidal flux amplification doesn’t strongly scale with the bias flux and injected current.

$$\eta_M = \frac{1}{2\mu_0} \int_V \frac{B^2}{dV} \int_0^t |I_{inj} V_{inj}| \, dt$$

$$\eta_H = \frac{1}{2\mu_0 L_c} \int_V A \cdot B \, dV \int_0^t |I_{inj} V_{inj}| \, dt$$

constant $\lambda_{inj} = 15.7 \, m^{-1}$
The highest temperatures in spheromaks are typically observed when the injector current is reduced after the column mode instability.

- The formation of closes flux surfaces is aided by:
  - toroidal current produced by the column mode instability
  - reduction of the injector current, which perturbs the magnetic field
- However, the line-tying of the injector current current stabilizes the spheromak to the tilt instability.
- The injector current is also a major contributor to the force balance, so reducing it too quickly degrades confinement.
- We’re simulating different injector current traces to explore these competing processes.
Poloidal flux amplification affects spheromak lifetime and peak temperature during decay.

- For the longest decay times, \( I_{\text{inj}} \) stays above the threshold for current-driven instability long enough to produce additional poloidal flux amplification.
- The highest peak plasma \( T \) observed occurs with a decay time of 1.0 ms.
- For even longer decay times than shown, the peak temperature remains low for 100's of \( \mu \)s.
- The peak temperature is maintained for the longest when \( I_{\text{inj}} \) is reduced and held below the threshold for instability.
The onset of the column mode (in terms of injected flux) and poloidal flux amplification are largely insensitive to the injector current ramp rate, which relaxes power supply design requirements.

Increasing the bias flux significantly increases operational efficiency in terms of the ratio of magnetic-to-injected energy.

The bias fluxes are easily achievable with non-superconducting magnetic field coils, e.g. water-cooled copper coils.

Results suggest that during the multi-pulse sustainment/relaxation phase, the injector current should be held slightly below the threshold for the column mode instability.
Flux Compression
The goal of these calculations is to evaluate spheromak performance during magnetic flux compression, in particular compressive heating.

- To achieve fusion temperatures, it's necessary to maintain sufficient thermal confinement during compression.
- Therefore, it’s desirable to maintain plasma stability as much as possible during compression.
- In this section, we present results from two distinct sets of compression calculations.
- First, we model compression of ‘constructed’ spheromak equilibria in a bowtie flux conserver, which is consistent with our design point for a neutron source.
- Next, we model compression of our earlier spheromaks formed and sustained through coaxial helicity injection.
For direct comparisons between our initial and compressed states, the computations must start with a stable equilibrium, consistent with the transport model.

- Otherwise, the early evolution will be dominated by the rapid equilibration of the plasma to a new equilibrium state, skewing any comparisons.
- We generate the initial conditions for our bowtie spheromaks by generating a Grad-Shafranov equilibrium.
- Then, we apply a steady-state current source to the initial equilibrium (for sustainment) and allow it to evolve with our transport model to a new steady-state equilibrium state.
- These calculations require greater computation expense than direct solution methods, but much less development.

\[
J_r = J_{\phi,0} \exp(-r^6/a_6)
\]
By varying our Grad-Shafranov equilibrium and the sustainment current, we are able to generate many different initial conditions for our compression calculations.

- First, we allow each case to freely decay, i.e. without being sustained by the current source, in order to verify that each initial state is not immediately unstable without sustainment.
- Often, broad-spectrum MHD instability occurs when the plasma current decays to around half of its initial value, though some cases stably decay to zero plasma current.
- The plasma is compressed with two coils at $R = 60$ cm, $Z = \pm 35$ cm with a linearly ramping coil current.
- For each case, we explore two rates of compression: $B_0/\text{ms}$ and $10 \times B_0/\text{ms}$, where $B_0$ is the field strength at the magnetic axis.
The amount of compressive heating observed in our preliminary calculations is very encouraging.

- The case shown uses the $10 \times B_0$/ms compression rate and starts with $B_0 = 0.2$ T, $I_p = 59$ kA, and a peaked current profile.
- With a volumetric compression ratio $\sim 10$, the plasma achieves significant amplification of the magnetic field ($\sim 6.1$) and plasma temperature ($\sim 6.4$) at the magnetic axis.
- Eventually, the plasma succumbs to an $n_\phi = 2$ instability at $\Delta t \sim 120 \mu s$. 
The instabilities that limit plasma lifetime are typically only observed when the spheromak plasma is near or in direct contact with the central conductor.

It’s plausible that slowing the rate of compression as the spheromak approaches the central conductor could extend its lifetime.

However, in order to get the best possible performance during compression, it’s critical to have a high initial rate of compression.

With the $B_0/\text{ms}$ compression rate, the same initial plasma only achieves about a third of the amplification of the magnetic field ($\sim 2.1$) and plasma temperature ($\sim 2.4$) at the magnetic axis.
Our preliminary calculations of flux compression of a spheromak formed and sustained through coaxial helicity injection.

The figures above correspond to the ‘hold’ formation case compressed at the $10 \times B_0/\text{ms}$ compression rate, starting at $t = 500 \mu\text{s}$.

- As the spheromak compresses, current density in the annular current column increases, periodically triggering the $n_{\phi} = 1$ column mode instability.
Summary & Discussion

Bowtie spheromak
- Preliminary results for compression yielded substantial, and encouraging, amounts of plasma heat.
- Despite our numerical model greatly over-predicting thermal transport, temperatures in excess of 1 keV are achieved in several cases.
- Higher relative gains in field and temperature on axis are generally achieved with lower $B_0$, lower $I_p$, and hollow current profiles.
- This trend relaxes the requirements on the pre-compressed spheromak and is compatible with spheromaks produced with CHI.

CHI spheromak
- Injecting current during compression helps stabilize the spheromak to the tilt instability.
- By gradually lowering the injected current during compression, we may be able improve performance by avoiding both the column and tilt mode instabilities.
Acknowledgements

This study uses computational resources from the following facilities:

- The U.S. Army Engineer Research and Development Center (ERDC) DoD Supercomputing Resource Center (DSRC), which is operated by the DoD High Performance Computing Modernization Program (HPCMP).

- The UMBC High Performance Computing Facility (HPCF), which is supported by the US National Science Foundation through the MRI program (grand nos. CNS-0821258 and CNS-1228778) and the SCREMS program (grant no. DMS-0821311), with additional substantial support from the University of Maryland, Baltimore County (UMBC).

- The National Energy Research Scientific Computing Center (NERSC), which is supported by the Office of Science of the US Department of Energy under Contract No DE-AC02-05CH11231.

Access to Sustained Spheromak Physics eXperiment (SSPX) data is provided through the Lawrence Livermore National Laboratory (LLNL). Special thanks to Harry McClean and Bill Meyer for their support.